Plato's *Timaeus* and the Missing Fourth Guest

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Plato's *Timaeus* and the Missing Fourth Guest

Finding the Harmony of the Spheres

By

Donna M. Altimari Adler

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To Lucia Elena Adler, my daughter,
Sam (Sonto Altimari) and Martha Altomere, my parents, and
Anne Marie and Hans Schuster, my dearest friends

Most people simply do not realize that without this kind of detailed ranging and wandering through everything, it is impossible to meet with truth and gain intelligence.

PLATO *Parmenides* 136 E.

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Preface

This study presents a new *Timaeus* analysis demonstrating how the *Timaeus* articulates a musical paradigm of the cosmos. It is a complete, rigorous analysis of each element of the *Timaeus* text from 35 A–36 D. It both discovers the primary musical scale dwelling hidden within Plato's recipe for constructing the world soul and breaks new ground, *inter alia*, by finding the exact "fabric" that Plato cut to form the χ ("chi") figure instrumental to making the world soul. It characterizes the fabric as a particular matrix of numbers intimately connected with Plato's musical cosmology and far more extensive than ancient or modern commentators have allowed. It is open to the possibility that the ancient commentators knew what they were about in their attempts to find a "scale" in the text, over against the efforts of some modern philosophers to discount a musical interpretation of the work.

The position of Francis Cornford, Luc Brisson, and Walter Meyerstein that Plato could not primarily have been constructing a musical scale, and so, a musical cosmology, in the *Timaeus*, seems ultimately untenable.[1] It too easily discounts the existing musicological scholarship on Plato's *Timaeus* and some of his other works, such as the *Myth of Er*, in the *Republic*, and sections of Book V of the *Laws* from 737 E–741 A and 745 B–745 E. It also largely neglects the ancient evidence documenting the emergence of the octave scale in the West; the cosmological significance of music in Plato's time; and the place of music education as a vehicle for the formation of the soul in the ancient Greece of Plato's day.[2] These factors are important to a proper evaluation of the *Timaeus* because they reveal the musical issues and ethos extant in Plato's time.

Although Luc Brisson has given some interesting alternative interpretations of the construction of the world soul that are not without validity for a multivalent text, they do not rule out a musical interpretation.[3] In addition, although

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1 Francis Cornford, *Plato's Cosmology, The Timaeus of Plato Translated with a Running Commentary* (London: Routledge and Kegan-Paul, 1951), 68–70; Luc Brisson and Walter Meyerstein, *Inventing the Universe, Plato's Timaeus, The Big Bang, and the Problem of Scientific Knowledge* (Albany: State University of New York, 1995), 35; Luc Brisson, *Le même et l'autre dans la structure ontologique du Timée de Platon, un commentaire systématique du Timée de Platon*, International Plato Studies, eds. Luc Brisson, Tomás Calvo, Livio Rossetti, Christopher J. Rowe, and Thomas A. Szlezák, no. 2 (Sankt Augustin: Academia Verlag, 1994), 314–332.

2 François Lasserre, "L'Education musicale dans la Grèce antique," in *De la musique* par Plutarque, texte traduction commentaire précédés d'une étude sur l'éducation musicale dans la Gréce antique (Olten & Lausanne: Urs Graf-Verlag, 1954) 53–74.

3 Luc Brisson, *Le même et l'autre dans la structure ontologique du Timée de Platon, un commen-*

Cornford raised an interesting issue for the long-established view that Plato's text elaborates a musical cosmology, it is hardly an insurmountable one. He claimed that, if Plato had really meant to articulate a harmonic conception of the cosmos, he would not have stopped at four octaves and a major sixth, the limitation that commentators universally apply to Plato's harmonic construction of the world soul. Such a limitation, he argued, does not reflect a closed system and seems arbitrary from a musical standpoint.[4] It is, indeed, an odd limitation; but Timaeus warns one from the outset that his likely account of the origin and structure of the universe may not be wholly self-consistent (*Timaeus* 29 C). Accordingly, the lack of complete consistency in a musical metaphor can hardly be taken to argue against Plato's intent to articulate a musical cosmology.

Taking the strangeness of the limitation as hard and steadfast proof that the whole body of ancient tradition acknowledging a musical cosmology in Plato's text is wrong in its fundamental instincts, as though the ancient commentators were not themselves very educated men much more deeply steeped in a living tradition than any modern observer of Plato, also seems misplaced. It appears more appropriate to assess that, being only human, they did not see the whole picture.

Perhaps some modern scholarship, also, simply does not see the whole picture. Maybe, as Handschin suggested, Plato primarily furnished the principles of a musical scale and only secondarily articulated a particular one.[5] As long as all relevant principles are articulated in the text, the musical interpretation remains viable and cosmologically important.

Alternatively, Plato may not, in fact, have limited the harmonics to four octaves and a major sixth, despite the universal assumption, otherwise. The current study indicates that such a limitation depends upon an unnecessary assumption concerning Plato's termination of the numerical sequence at twenty-seven. It demonstrates how that termination possibly has a different meaning than commentators have previously given it that is, nonetheless, exactly in line with the Pythagorean mode of thinking that many persons, ancient and modern, have attributed to Plato in the *Timaeus*. Previous commentators have missed the possibility suggested by this study at least partly because of the onerous investment of time that the discovery requires and the particular mathematics required to reveal it.

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taire systématique du Timée de Platon, International Plato Studies, eds. Luc Brisson, Tomás Calvo, Livio Rossetti, Christopher J. Rowe, and Thomas a. Szlezák, no. 2 (Sankt Augustine: Academia Verlag, 1994), 33–54.

4 Cornford, *Plato's Cosmology*, 68–69.

5 Jacques Handschin, "The Timaeus Scale," *Musica Disciplina* 4 (1950):31–35.

The study claims that the commentators of the ancient Academy did not unpack Plato's riddle at *Timaeus* 35 A–36 D, completely, at least in any form that survives, though it appears that at least one commentator of Augustine of Hippo's day, Favonius Eulogius, knew how the riddle ended. The study heeds the work of Jacques Handschin, a prominent musicologist fascinated with the problems that an effort to find a *Timaeus* scale poses; it takes seriously John Dillon's suspicion, deriving from Speusippus, that the Decad may be the "All Perfect Animal"; it follows-up Mitchell Miller's intuition about a particular "God-given method" at work in the *Timaeus*; and it gives significant weight to Plato's link to the Pythagorean community at Tarentum.[6]

The new interpretation argues that Plato delineated the proposed musical paradigm for the *Timaean* cosmos against a more fundamental harmonic conception of the universe. It also hypothesizes a structure of human perception governed isomorphically by the proportions proper to the movement of the whole cosmos. It demonstrates how the pattern of the Decad not only describes the *ordo* of human perception in the *Timaeus* but also constitutes the pattern according to which all things are generated, including the form of the world soul and the primary diatonic musical scale that is its symbol. It establishes the primary diatonic musical scale against a large but finite set of competing, secondary scale possibilities.

The analysis is compelling in its articulation of the matrix comprising Plato's fabric and in its illumination of a number of other problems of textual inter-

6 See Handschin, "Timaeus Scale," 3–41; John Dillon, "The Timaeus in the Old Academy," in *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils (Notre Dame: University of Notre Dame Press, 2003), 82–84; Mitchell Miller, "The Timaeus and the 'Longer Way': God-Given Method and the Constitution of Elements and Animals," in Reydam-Schils, *Cultural Icon*, 23–50; Carl A. Huffman, *Philolaus of Croton, Pythagorean and Presocratic, A Commentary on the fragments and Testimonia with Intepretative Essays* (Cambridge: Cambridge University Press, 1993), 21–25 (discussing the dialogue of Plato and the Old Academy with Pythagoreans); Dominic O'Meara, *Pythagoras Revisited: Mathematics and Philosophy in Late Antiquity* (Oxford: Clarendon Press, 1989), 146–149, 179–183 (discussing Proclus' attribution of Pythagorean inspiration to Plato's science of the divine, as expressed in *Timaeus* and other texts). Speusippus was Plato's sister's son, a member of the Academy, and Plato's immediate successor as its head upon his death. Leo Tarán, *Speusippus of Athens, A Critical Study with a Collection of the Related Texts and Commentary* (Leiden: Brill, 1981), 5. Speusippus, however, conceived of numbers only as a collection of units, according to Tarán, and spurned the idea of ideal number. Ibid., 16. A similar contempt is not to be found in the *Timaeus*. *See* Sarah Klitenic Wear trans., *The Teachings of Syrianus on Plato's Timaeus and Parmenides*, Studies in Platonism, Neoplatonism, and the Platonic Tradition, eds. Robert Berchman and John Finamore, vol. 10, Ancient Mediterranean and Medieval Texts and Contexts (Leiden: Brill, 2011), 40 (for the notion that the dyad contained the intelligible Decad of forms in the much later neoplatonic tradition).

pretation. The latter include the precise identity of the primary *Timaeus* scale; the manner of forming the "chi" from the fabric and the soul sphere from the "chi"; the creation of bands from the soul sphere; the meaning of the differences among the subsets of three and four bands resulting from the splitting of the band of difference at *Timaeus* 36 D; the reasons for the differing motions and speeds of the band of same (36 C) and the bands of difference, as well as the significance of those differences; the relationship of the motions of all bands to the motions of the cosmos and the planets, in their orbits; the reasons for the appearances of "chromatic" elements among the numbers corresponding to the dominant musical scale emerging from Plato's text; the specific order of their emergence; and their relevance to identifying the band of "same" at 36 C.

The study illustrates that the chromatic distortions critical to identifying the band of "same" at 36 C arise, as the octave sequences of the primary *Timaeus* scale repeat, because Plato's original world soul divisions at *Timaeus* 35 B–36 A created continuously overlapping octaves and musical twelfths (octave plus fifth) in line with the predictions of both Jacques Handschin and Ernest McClain.[7] This continuous overlapping causes a corresponding overlapping and interference among the fourths included within those two series of intervals as they continue running together, with the result that numbers properly belonging to one fourth interval among the musical twelfths begin to show up as extraneous additional numbers in octave sequences and *vice versa*. The two series are out of synchronization with each other, so to speak. Handschin explained that "if one follows Plato strictly," one will, therefore, soon "transgress the limits of diatonicism."[8]

In the dominant diatonic *Timaeus* scale found by this analysis, the chromatic distortions arise in an orderly sequence following the same logic of fourfold progression pertaining to every other scheme of generation in Plato's text. They also arise in an orderly fashion in all of the secondary diatonic *Timaeus* chains.

The invasion of the octave scale by chromatic elements is exactly what Jacques Handschin predicted would happen to octave periodicity with continued iterations of the diatonic scale, under the particular conditions for the scale set by Plato in the *Timaeus*. Handschin also predicted that the octave periodicity would eventually give way to fifth periodicity but expressed doubt that Plato

7 Handschin, "Timaeus Scale," 21; Ernest McClain, *Pythagorean Plato, Prelude to the Song Itself* (York Beach, Maine: Nicolas Hys, Inc. 1978), 61 and 62.

8 Handschin, "Timaeus Scale," 24; see also, McClain, *Pythagorean Plato*, 59 (identifying the triple intervals with musical twelfths).

understood the implications attached to his text.[9] Handschin did not prove from the text his predictions about the degeneration of octave periodicity to fifth periodicity. The analysis offered here, however, accomplishes exactly that proof. It arrives at its conclusions independently, based upon a detailed and rigorous examination of Plato's text, departing from Handschin only in his suggestion that Plato was clueless about the degeneration of octave periodicity to fifth periodicity.

The present interpretation suggests that Plato could plausibly have known about the decay of octave periodicity to fifth periodicity, entailed by his text, and, perhaps, even entertained a pedagogical purpose for allowing it. One sees fifths fall out, in proper order, too, as the fifth periodicity replacing octave periodicity itself disappears with the completion of Plato's divisions at *Timaeus* 36 B. It is possible that Plato deliberately intended to set up conditions in the *Timaeus* under which octave periodicity would give way to a fifth periodicity which would, then, itself give out, just so that he could show his reader the *ordo* according to which the fifths fill out the octave. One might at least so hypothesize in connection with the shadowy figures whose tradition the *Timaeus* reflects.

The primary scale emerging and degenerating in the *Timaeus* is an ascending Lydian diatonic scale, if one interprets the numbers resulting from Plato's divisions of the world soul stuff as numbers indexed to string vibration or impacts on air, or, alternatively, a descending Dorian scale, if one interprets them as string lengths (384 as an index to string vibration or impacts on air is a low pitch given the set of mostly larger numbers comprising Plato's set; but as a string length, it is a high pitch, representing a relatively short length in comparison with those other numbers).[10] Plutarch elected for the Lydian, rather than Dorian option, and that is my preference as well.[11] The results of this study are valid, in either case, because the two scales are exact reciprocals of each other. An ascending Lydian scale, that is, has the same tone/*diesis* pattern as a descending Dorian scale.[12] Tone/*diesis* sequences are inverse for the ascending

9 Handschin, "Timaeus Scale," 23–24.

10 Ibid., 24 and 26.

11 Ibid.; see also, Plut., *Moralia*, 13.1.2.13.1018D, 13.1.2.18.1021E (manifesting that Plutarch assigned smaller numbers to lower notes and higher numbers to higher notes; the implication is that Plutarch interpreted Plato as having defined an ascending Lydian scale); see James Haar, "Musica mundana: Variations on a Pythagorean Theme" (Ph.D. diss., Harvard University, 1960), 17–21, for a good discussion of the ancient and modern debates on the ascending or descending scale question and 1–70, more generally, for an excellent account of the history of interpretation of the *Timaeus* scale from ancient times.

12 Gustave Reese presented the seven different possible diatonic arrangements of tones and

Lydian and ascending Dorian and for the descending Lydian and descending Dorian. Plato or those whose tradition the *Timaeus* reflects may have meant for ambiguity to remain pertaining to an ascending or descending interpretation of the scale just to emphasize that the octave scale emerges according to the same principles whether it ascends or descends. One simply interprets the scale in reverse order, for a rising and declining scale, identifying the initial number as the lowest pitch for the ascending Lydian scale and as the highest one for the descending Dorian.[13]

The numbers resulting from Plato's divisions of the world soul are sufficient to account for the following phenomena in relation to the primary diatonic scale built in the text. They show the rise to the scale from its elements; then they articulate a perfect *disdiapason* (two-octave sequence) followed by four chromatically distorted *disdiapasons* for a total of a decad of octaves; then they exhibit a decad of incomplete octave sequences that accomplish the replacement of octave periodicity by fifth periodicity and end in the demise of fifth periodicity, as well. The decad of octaves stands to the decad of incomplete sequences as monad stands to duad; and so, this monad/duad relation in Plato's text reproduces the octave proportion *vis-à-vis* its members.

At the same time, each of the decads, inasmuch as each is structurally a decad of elements, marks the fourfold progression (monad to tetrad 4:1) normally associated with the *disdiapason* (two octave sequence); so the duad of decads marks the eightfold progression proper to a duad of *disdiapasons*. A duad of *disdiapasons*, however, is a tetrad of *diapasons* (octaves). A tetrad is never achieved in Plato's text except through a fourfold progression in the pattern of the Decad; so the tetrad of *diapasons* points to a higher order decad standing behind the monad/duad pair made out by octave periodicity/other periodicity. The higher order decad governing the pair works like any other decad. The fourfold progression marked by the pattern of the Decad terminates in a tetrad just as it gives rise to a *disdiapason*. Accordingly, imitating the Decad constituting the exemplary pattern and "All Perfect Animal," the higher order decad standing behind the primary monad/duad pair of the *Timaeus* text

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semitones in the Greek octave interval, thus, in descending order of pitch: TTTSTTS (Mixolydian, B or D); STTTSTT (Lydian, C or C#); TSTTTST (Phrygian, D or B); TTSTTTS (Dorian, E or A); STTSTTT (Hypolydian, F or G#); TSTTSTT (Hypophrygian, G or F#); TTSTTST (Hypodorian, A or E). Gustave Reese, *Music in the Middle Ages*, with an introduction on music of ancient times (New York: W.W. Norton & Co., 1940), 28 and 30.

13 Handschin stated that it would have been characteristic of Plato's style to have left the question open. Handschin, "Timaeus Scale," 15. He observed that Pseudo-Timaeus, Proclus, and Psellus, did, in fact, leave the choice between the Dorian or Lydian *tonos* open for the *Timaeus* scale, just as this study has done. Ibid., 21.

(octave periodicity/other periodicity) structures the cosmos as a *disdiapason*, just as Favonius Eulogius had written much later in his *In somnium Scipionis*.[14]

In addition to revealing phenomena pertinent to the primary scale and structure of the cosmos, identified above, the analysis demonstrates how an entire system of ancient Greek music—seven dominant octave species in all three genera, diatonic, chromatic, and enharmonic—can be derived from the matrix of numbers hidden within the *Timaeus* text. This system recalls the fifth century Eratocles, in its demonstration of cyclical variations to achieve different octave species, as well as the spirit of fourth century experimentation with perfect systems against fifth century developments, otherwise. The study sheds new light on possible avenues of development of the Greater Perfect System ("GPS"), Lesser Perfect System ("LPS"), and Unmodulating Perfect System ("UPS") in the fourth century.

Modern scholarship affirms that Greek writers were agreed on the basic shape of GPS by the late fourth century; and certainly the *Sectio Canonis*, a late fourth century Pythagorean text attributed to Euclid, provides direct evidence of GPS, LPS, and UPS.[15] In relation to the *Timaeus*, one might place the *Sectio Canonis* at the end of the experimental period for UPS. It explicitly discusses and demonstrates the immutable system and reflects all of the concerns that one might expect of a Pythagorean thinker for the importance to music of superparticular ratios in the mathematical sense.[16]

Stefan Hagel has assigned the beginning of interest in modulating music to the second half of the fifth century, remarking upon Pythagoras of Zacynthus' early success in tuning an entire set of strings to the Dorian, Phrygian, and

14 Eulogius *somnio Scipionis* 25.

15 For the development of UPS, including GPS and LPS, in the fourth century, see Andrew Barker, *The Science of Harmonics in Classical Greece* (Cambridge: Cambridge University Press, 2007), 12–17 (noting, *inter alia*, that by the late fourth century, all Greek writers on harmonics were agreed on the basic shape of the system); Donald Creese, *The Monochord in Ancient Greek Harmonic Science* (Cambridge: Cambridge University Press, 2010), 21 (GPS was a fourth century construct). Stefan Hagel, *Ancient Greek Music, A New Technical History* (Cambridge: Cambridge University Press, 2010), 5–6 (defining "Unmodulating System").

16 See André Barbera, ed. and trans., *The Euclidean Division of the Canon: Greek and Latin Sources*, Greek and Latin Music Theory, eds. Thomas J. Mathiesen and Jon Solomon (Lincoln: University of Nebraska Press, 1991), 21, 129–131, 134–135, 150–159, 170–171, 174–175, 178–179, 186–187; see also, Andrew Barker, "Early *Timaeus* Commentaries and Hellenistic Musicology," in *Ancient Approaches to Plato's Timaeus*, eds. Robert W. Sharples and Anne Sheppard, Bulletin of the Institute of Classical Studies Supplement, ed. Geoffrey Waywell, no. 78 (London: Institute of Classical Studies, School of Advanced Study of the University of London, 2003), 76 (for dating of the text).

Lydian *harmoniai* in such a way that one could proceed from one to another without interruption. He has also mentioned Pronomos' invention of the modulating aulos before 400, integrating at least the Dorian, Phrygian, and Lydian *harmoniai*, stating that the incorporation of several scales on one instrument was indubitably a milestone in the evolution of a modal system.[17]

Both Andrew Barker and Stefan Hagel have discussed the importance of the fifth century Eratocles' efforts to the development of UPS in the fourth century.[18] Barker noted that octaves can be organized without a change in genus in seven different ways. In his opinion, compelling evidence shows that Eratocles recast the *harmoniai* under which fifth and early fourth century melodies were classified as the seven octave species: Dorian, Phrygian, Lydian, Mixolydian, Hypodorian, Hypophrygian, and Hypolydian. He observed that, for Eratocles, each of the seven *harmoniai* or species of octave is generated by removing the interval at the top of its predecessor and replacing it at the bottom.[19] Hagel and Gustave Reese have affirmed the existence of the octave species identified, above, by the fourth century.[20]

Evidence that the seven octave types identified were the primary octave types recognized in the fourth century is derived from the writings of Plato himself and Aristoxenus, Plato's younger fourth century contemporary, who commented on earlier musical practice.[21] At *Republic* 398 E–399 A, Plato expressly mentioned the Mixolydian, various other "Lydian" modes, including "tense" and "lax (Hypolydian?)" varieties, the Dorian, the Phrygian, and the "Ionian" but protested, from Socrates' mouth, that he did not know the modes. At *Laches* 188 D, he named the "Ionian," Phrygian, Dorian, and Lydian. Barker has drawn particular attention to the Mixolydian, Lydian, Dorian, and Phrygian in Plato, while Reese has clarified that the label "Ionian," was sometimes a label for the Phrygian.[22]

17 Hagel, *Ancient Greek Music*, 378–379.

18 Barker, *Science of Harmonics*, 83, 224; Hagel, *Ancient Greek Music*, 373–374, 378, 387.

19 Barker, *Science of Harmonics*, 83, 224.

20 Hagel, *Ancient Greek Music*, 4–5 (mentioning all but the Mixolydian, but referring to a seven tonoi system predating Aristoxenus); Reese, *Music*, 35 (dating the "high" Mixolydian to 475 B.C.).

21 See Thomas J. Mathiesen, "Greek Music Theory," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002; reprint, Cambridge: Cambridge University Press, 2004), 113 (establishing Aristoxenus as one of Plato's younger fourth century contemporaries).

22 Barker, *Science of Harmonics*, 309; Reese, Music, 32; see also, Hagel, Ancient Greek Music, 430.

Stefan Hagel acknowledges the Hypophrygian, Hypodorian, Dorian, Phrygian, Lydian, Mixolydian, and (probably also) the Hypolydian as having predated Aristoxenus, whose writings imply their earlier use, and opines that the "Ionian" had no roots in traditional pre-Aristoxenian musical practice comparable to those of the others mentioned.[23] Thomas Mathiesen mentions all modes except the Hypolydian as having existed among the "harmonicists" predating Aristoxenus, but recognized Aristoxenus' knowledge of all seven octave species preceding his own experimentation in association with the *tonoi*.[24]

Barker has observed that Aristoxenus worked from the foundation that Eratocles laid, noting that Aristoxenus' notion of the role of the *tonoi* was essentially tied up with their relation to the octave species.[25] Mathiesen has also mentioned both Aristoxenus' knowledge of Eratocles' efforts and his understanding that Eratocles' primary interest was the possible cyclic orderings of octave intervals.[26] It may interest some readers that Ptolemy, a much later musical theorist, far from following Aristoxenus, essentially reverted to an approach stressing seven *tonoi* in accord with the seven diatonic tunings for octave species.[27]

Hagel has opined that Eratocles' tuning diagrams certainly represent a system of *tonoi* reflecting a highly abstract conception of tonal relations. Like Barker, he has observed that Eratocles analyzed different scales as successions of intervals which can be transferred from one end to the other; thus his seven octave species are certainly possible sources for the "canonical" seven *tonoi* widely acknowledged by the time of Aristoxenus or not long after.[28]

Hagel has pointed out that a demonstration of Eratocles' seven octave species, assigning the same pitch to similar functional notes, required a structure of two octaves such as GPS.[29] He has opined that such a two octave system was probably already known to Plato and was important, practically, for the boring of the single-holed aulos.[30] He states also that Plato treated the Pythagorean diatonic as a given (though the Eratoclean school was concerned only with the enharmonic), notes that the *Timaeus* presupposes the

23 Hagel, Ancient Greek Music, 4–5, 8.
24 Mathiesen, "Greek Music Theory," 119, 125.
25 Barker, *Science of Harmonics* 224, 227.
26 Mathiesen, "Greek Music Theory," 119.
27 Hagel, *Ancient Greek Music*, 5, 387.
28 Ibid., 8, 378, 387, 430–431; see, also, Barker, *Science of Harmonics*, 297 (for fourth century placement of Aristoxenus).
29 Hagel, *Ancient Greek Music*, 387–388.
30 Ibid., 387–388.

fixed notes skeleton of a musical scale, and observes that it is but a minor step from the *Timaean* framework of fixed notes to the conception of a perfect system.[31]

This study indicates that Plato's *Timaeus* may provide indirect evidence of GPS, LPS, and UPS, perhaps while there was still some controversy concerning their structures. The text may also demonstrate a concerted interest in Eratocles' seven octave species, noted above, and the means by which they can be generated from each other by interval transference. It proceeds from a Pythagorean rather than Aristoxenian standpoint, in its approach to previous developments, and stresses the diatonic, rather than the enharmonic, in contrast to the Eratoclean school, interested solely in the enharmonic.[32]

The study adopts use of the word *tonos*, in connection with the musical preoccupations of the *Timaeus*, as appropriate. Reese clarified the legitimacy of using the word "tonos" in association with the thinking of various ancient Greek writers, including Plato, as follows:

> In many of the Greek treatises, the word *harmonia* (tonal structure) appears … [W]hen high tuning came into use and when the eleven- and twelve-string kitharas yielded, as we have seen, a range of two octaves, "harmonia" was employed in at least two senses: as a synonym for tonos and as a name for the individual octave-species as projected upon the Greater Perfect System. It is often difficult to determine which meaning is intended, but it seems fairly certain that such writers as Plato, Aristotle, and Herakleides used it in the sense of tonos.[33]

The study claims, in addition, that, although Plato was primarily interested in diatonic phenomena in the *Timaeus*—certainly, as James Haar has noted, all ancient writers agreed that the *Timaeus* scale is in the diatonic genus,—the system arising from the *Timaeus* numbers accommodates the chromatic and enharmonic octave genera, as well, also within a UPS structure.[34] Diatonic, chromatic, and enharmonic interests would certainly all have been possible for Plato.

Andrew Barker, for example, points to Philolaus fragment 6A as evidence for the provenance of the diatonic genus before Plato; and he marks the fourth cen-

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31 Ibid., 430–431, 448–449.

32 Hagel, *Ancient Greek Music*, 431.

33 Reese, *Music*, 44 (relying upon Bonaventura Meyer, "ARMONIA," *Bedeutungsgeschichte des Wortes von Homer bis Aristoteles*, 1931).

34 Haar, "Musica Mundana," 15.

tury Archytas' diatonic, chromatic, and enharmonic divisions of the fourth.[35] Hagel notes Aristoxenus' specific acknowledgment, in the fourth century, that the diatonic genus was older than the other two genera, evidencing knowledge of all three by his time. In relevance to the plausibility of Plato's diatonic interests, Hagel relates that the diatonic genus considerably predates Hellenic culture, probably deriving from Sumerian music. He marks its witness in Old Babylonian cuneiform tablets; observes that Near Eastern music had codified the complete cyclic system of seven diatonic tunings in the second millennium B.C. and probably earlier; notes that most or all of these tunings were actually used in musical practice; and states that, in the Sumerian tradition, they were exemplarily constructed and construed on a stringed instrument. He opines that the diatonic division of tonal space similar to scales known for the ancient Near East was, in some respect, the basis for Greek lyre music.[36]

Hagel further makes reference to an early preference, in pre-Aristoxenian musical practice, for enharmonic music, that gave way in the fourth century to a chromatic preference and, even later, to a diatonic preference; and Mathiesen specifically notes Aristoxenus' awareness, in the fourth century, of the diatonic, enharmonic, and chromatic genera.[37] Surely other sophisticated fourth century thinkers were also aware of all three genera.

Although it is somewhat speculative to suggest that Plato or the persons whose work informed the *Timaeus* sought to experiment with accommodating all three genera within a unified UPS framework, the study supports the suggestion and points scholars to further researches in that direction.

Assuredly, the interpretation of the *Timaeus* offered by this study depends upon a much larger set of numbers than is usually attributed to Plato's divisions of the world soul. Accordingly much of the analysis is devoted to exhibiting just how one arrives at the set. It makes manifest that the smaller sets of numbers achieved by commentators to date is partially due to a misstep that they make at *Timaeus* 36 B. That passage directs the reader to put sesquioctave intervals into all sesquitertian parts produced when harmonic and arithmetic means are inserted between members of the original set of numbers defined at 35 B–36 A. Commentators err, as the text shows, by failing to consider all of the possibili-

35 Barker, *Science of Harmonics*, 38, 264, 292, 293–307; see, also, Hagel, *Ancient Greek Music*, 135 (noting that diatonic heptatony, associated with stringed instruments has origins beyond the second millennium B.C.) and 143 (recognizing Philolaus as evidence of the diatonic genus among Greeks prior to Plato).

36 Hagel, *Ancient Greek Music*, 10 (n. 35), 106, 414, 436, 442.

37 Ibid., 44–52; Mathiesen, "Greek Music Theory," 123.

ties for textual interpretation of the sesquitertian parts that Plato intended his reader to fill. Indeed, they miss the ambiguity in the text that gives rise to possibilities beyond the most obvious. Plato gave his reader a veiled hint at the very beginning of the *Timaeus* that correctly identifying the sesquitertian parts is the key to his whole riddle. The sesquitertian part, correctly elaborated, could certainly be Plato's missing fourth guest mentioned at the beginning of the dialogue.[38]

Some modern commentators have moved in particularly promising directions. Luc Brisson, for example, argued in *Le même et l'autre* that the *Timaeus* articulates a mathematical model for the universe, but he did not consider that the text itself could be a kind of number generating machine.[39] That the text is a number generator makes sense, as a good working hypothesis, from the standpoint of Plato's close friendship with the Pythagorean community at Tarentum. Certainly, cosmogony, cosmology, and number generation would have walked hand-in-hand for them.

The beauty of the present analysis is its beginning from a theory of text as number generator and not from musical presuppositions. Its method is simply to find the maximum set of numbers that Plato's divisions legitimately allow one to construct on the basis of some principle already implicit in the *Timaeus* text. Only after the set is identified is there an attempt at an interpretation. The entire set of numbers is a completely nonarbitrary set, yielding the results skeletally described above. The best argument for the method and the set of numbers it generates is that they yield an interpretation making sense of Plato's *Timaeus* cosmogony down to the fabric and the "chi" figure, in line with the best instincts of the ancients; the formation of the sphere from the "chi" figure; and the further divisions from the soul sphere of the bands of same and difference. The study also yields a new bridge to musical material in the *Laws*. In the end, the study's best justification for itself is its delivery of a complete, cogent exposition of *Timaeus* 35 A–36 D.

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38 Plato *Tim*. 17 A.

39 See Brisson, *Le même et l'autre*, generally, for Brisson's explanation how the *Timaeus* posits a mathematical model of the universe.

Acknowledgments

This work was originally prepared as chapter nine of a fifteen hundred page genetic study of Augustine's doctrine of God, entitled "The Triune Creator and the Rhythm of Creation: Augustine's Emerging Apprehension of the Trinity as God." The Augustine study emerged during my dissertation effort at the University of Notre Dame; and I submitted a relatively freestanding part of it as my dissertation. I stumbled across the relevance of the *Timaeus* to Augustine's work in the course of discerning how Augustine articulated a figure expressing the Trinity/creature relation of Catholic tradition in his *De Musica*. It had occurred to me that Augustine had found his precedent for using the liberal discipline of music as a vehicle for expressing metaphysical and cosmological ideas precisely in the *Timaeus*.

Augustine used the *dispondee* (·--·/·--·/·--·/·--·), a rhythmic foot comprising the lowest level mathematical analogue, in rhythm, of the two octave sequence, i.e., the *disdiapason*, in harmony, as a symbol expressing the bounds of the order of reality. The notion that the order of reality comprises a periodic interval represented by the *disdiapason* appears to have derived from the *Timaeus*. Favonius Eulogius, the rhetor, for example, commenting upon Cicero's *In somnium Scipionis*, a text known to have been in dialogue with Plato's *Timaeus*, specifically limited the cosmic expansion to two octaves.[1] It is eminently probable that Augustine knew of this tradition sufficiently well both to have constructed a rhythmic analogue of the idea in *De Musica* and to have recast the notion to fit a Christian metaphysical scheme. He spoke of one of his students of rhetoric, Favonius Eulogius by name, in his little work *De Cura Pro Mortuis*, mentioning, generally, that he had taught Eulogius his Cicero and alluding, specifically, to Cicero's *In somnium Scipionis*.[2]

1 Eulogius *somnio Scipionis* 25. Pierre Courcelle noted the influence of Calcidius on Eulogius. Indeed, he claimed that Eulogius' commentary draws from Calcidius' *In Timaeum*. Pierre Courcelle, "La postérité chrétienne du songe de Scipion," *Revue des Études Latine* 36 (1958): 211 (and note 3).

2 Augustine *De cura pro mortuis* 13. Roger-E. Van Weddingen, translator of the Eulogius commentary into French, speculated that the Eulogius of the commentary was one and the same as the Eulogius of Augustine's treatise on the dead. Roger-E. Van Weddingen, "Introduction," in the *Disputatio de somnio Scipionis* of Favonius Eulogius, edition et traduction de Roger-E. Van Weddingen, *Collection Latomus*, vol. 27 (Bruxelles: Latomus revue d'études latines, 1957), 5–6 and 8. Aimé Soulignac, Pierre Courcelle and Peter Brown went farther, positively identifying the figures of the two references. See Aimé Soulignac, "Doxographies et manuels chez S. Augustin," *Recherches Augustiniennes* 1, *Supplement à la revue des etudes Augustini-*

One can only speculate, of course, whether Eulogius' commentary reflects Augustine's teaching.[3] The commentary shows, at least, that knowledge of the particular *Timaeus* tradition setting the cosmic limit at the interval of a *disdiapason* was widely available to the rhetorical world that Augustine inhabited.

Once having discovered the plausibility of a link between Augustine's *De musica* and Plato's *Timaeus* through the medium of Favonius Eulogius' commentary on Cicero's *In somnium Scipionis*, I was curious to discover how the view that the order of reality is limited to the *interval* of a disdiapason could have arisen from Plato's *Timaeus* and what it could possibly mean. My journey has been fascinating, and the present study is its result. It could not have been what it has become without the patience and encouragement of David Burrell, C.S.C., in allowing me to go off on what originally appeared to be a frolic of my own in a dissertation on Augustine; Gretchen Reydams-Shils' *Timaeus* conference at the University of Notre Dame in the spring of 2000, as well as her questions about the first draft of the *Timaeus* analysis; some early advice of Ernest McClain on musical points that convinced me to make a thorough study of the issues in Greek music current when Plato wrote the *Timaeus;* and the inspection, suggestions, and encouragement of Calvin Bower, John Dillon, Thomas Mathiesen, and Andrew Barker, regarding various revised versions, in correspondence, and occasionally, in person. I am also indebted to Luc Brisson for the insights inspired by a diagram depicting the results of Plato's "means" operation at *Timaeus* 36 A, in his *Le même et l'autre*, and for his permission to use a similar diagram and variations thereon in my own presentation.

My acknowledgments would be incomplete if I did not mention Anne Marie Schuster and the late Alan Tybor, good friends who expressed a great interest in ancient philosophy when I told them of my project. In an unparalleled test of our friendship, they patiently sat through a full presentation of my then existing, long, intricate *Timaeus* analysis and assisted me with the mechanics of preparing to present it, in a shortened format, to the annual conference of the International Society of Neoplatonic Studies ("ISNS") in New Orleans, Louisiana in June of 2003. I also thank my lovely daughter, Lucia Elena Adler, for her patience with the long hours and many years it took to bring this work

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ennes (1958): 131–132; Courcelle, "La postérité chrétienne du songe de Scipion," 212–213; Peter Brown, *Augustine of Hippo*, 2d ed. (New York: Dorset Press, 1986), 131 and 302.

3 Soulignac went so far as to propose that the Eulogius commentary was a joint work of Eulogius and Augustine, with Augustine taking the position as master. Soulignac, "Doxographies," 131–132. Courcelle maintained that Augustine both possessed and used Eulogius' commentary. Courcelle, "La postérité chrétienne," 213.

to completion, for her proofreading assistance, and for her continued enthusiasm and encouragement all along the way. I am abundantly blessed with her presence in my life.

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The reports of my anonymous reviewers at Brill, on a recent version of this work, have certainly been invaluable. The limited, additional research that they suggested I do sparked insights allowing a much more efficient way to present some of my material, as well as a convenient means to eliminate a number of appendices. The analysis has surely gone through many permutations and variations in some of its parts, over the years, as I have striven to address Plato's riddle convincingly. It is finally complete to my present satisfaction and ready to be shared with *Timaeus* enthusiasts more generally.

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Introduction: Plato's Missing Fourth Guest

The *Timaeus* reflects a living Pythagorean tradition of musical and mathematical philosophizing, current at the time of its authorship.[1] Purporting to be an account of the creation of the universe, it is at once a strange and fascinating text. Socrates opened the dialogue counting (17 A): "One, two, three, but where, my dear Timaeus, is the fourth of those who were yesterday my guests and are to be my entertainers today?"[2] This beginning is not fortuitous. It is, perhaps, a particularly appropriate clue to unpacking a work as intensively preoccupied with the Decad, as diligently in search of the cosmic fourth, and as ardently impassioned about an octavally constituted cosmic *harmonia* as is the *Timaeus*.[3]

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1 Francis Cornford described "the Pythagorean doctrine of Numbers as the real being of all things" as one of the roots of Platonism. Francis MacDonald Cornford, *Plato's Theory of Knowledge, The Theaetetus and the Sophist of Plato*, trans. with a running commentary (London: Routledge & Kegan Paul, Ltd., 1935; reprint, London: Routledge & Kegan Paul, Ltd., 1949), 9. He also acknowledged that much of the doctrine in the *Timaeus* is "no doubt Pythagorean." Cornford, *Plato's Cosmology*, 3. See also, Thomas Taylor, "Introduction to the Timaeus," in *The Works of Plato*, vol. 2, trans. Thomas Taylor and Floyer Sydenham, *The Thomas Taylor Series*, vol. 10 (England: Antony Rowe, Chippenham, Wiltshire, 1804; new, revised edition, Dorset: The Prometheus Trust, 1996; repr., Dorset: The Prometheus Trust, 2007), 375–378, 392, 401, 408, 414; A.E. Taylor, *A Commentary on Plato's Timaeus* (Oxford: Clarendon Press, 1928), ix; Michael B. Allen, "The Ficinan Timaeus and Renaissance Science," in *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils (Notre Dame: University of Notre Dame Press 2003), 239, 243, John Dillon, "The *Timaeus* in the Old Academy," 82–85; Kenneth Sayer, "The Multilayered Incoherence of Timaeus' Receptacle," in *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils (Notre Dame: University of Notre Dame Press 2003), 74; Mathiesen, "Greek Music Theory," 114–115. See also, Henry George Farmer, "The Music of Mesopotamia," in *Ancient and Oriental Music*, ed., Egon Wellesz, *The New Oxford History of Music*, vol. 1 (London: Oxford University Press, 1957; repr. London: Oxford University Press, 1969), 252–253; O'Meara, *Pythagoras Revisited*, 146–149, 179–183 (discussing Proclus' attribution of Pythagorean inspiration to Plato's science of the divine, as expressed in *Timaeus* and other texts); and John Curtis Franklin, "Diatonic Music in Greece: A Reassessment of its Antiquity," *Mnemosyne* 55, no. 6 (2002): 669 (for the probable Mesopotamian roots of the Pythagorean musical consonances).

2 Plato *Tim.* (trans. Benjamin Jowett in *The Collected Dialogues of Plato, Including the Letters*, eds. Edith Hamilton and Huntington Cairns, Bollingen Series, vol. 71 [Princeton: Princeton University Press, 1963; repr. Princeton: Princeton University Press, 2009]) 17 A. The Greek text reads: "Εἷς, δύο, τρεῖς· ὁ δὲ δὴ τέταρτος ἡμῖν, ὦ φίλε Τίμαιε, ποῦ, τῶν χθὲς μὲν δαιτυμόνων, τὰ νῦν δὲ ἑστια· τόρων;" Plato *Tim.* (direttore Giovanni Reale [Milano: Rusconi Libri, 1994]) 17 A.

3 Concerning Speusippus' equation of the *Timaeus*' paradigm of all things with the Decad and the plausibility of the same, see Dillon, "The *Timaeus* in the Old Academy," 82–85.

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The next line of the text develops the clue for a musical mind. Referring, it seems, to disputes among Pythagoreans over calculating the ratio for the remainder (*leimma*) beyond two whole tones in the fourth proper to the diatonic scale, Timaeus responds that the fourth interlocutor "labours—under a certain infirmity; for he would not willingly be absent from such an association as the present" (17A).[4]

The infirmity of the fourth interlocutor may involve the *leimma* (also called *diesis*) in the fourth and fifth intervals of the diatonic scale. The *leimma*, designating a halftone or semitone, was actually something less than a halftone. Archytas of Tarentum, a figure of the fourth century B.C., contemporary with Plato and much younger than Philolaus (more contemporary with Socrates), discovered, using reliable procedures, that the whole tone, an interval of the size 9/8, could not be divided into two equal halves. For purposes of this study, this means that the number corresponding to a whole tone just preceding the *leimma*, in a fourth or fifth, when multiplied by the square root of 9/8 (corresponding to half the tone interval), produces not the whole number representing the closure of the fourth on the other side of the *leimma*, but no whole number at all—rather a number having something left over. Such a number cannot represent any element in an ancient Greek musical scale; thus, the *leimma*, whatever it might be, cannot be a halftone, strictly speaking.[5]

The 9/8 (1.125, in modern decimal terms) interval was not the true size of a whole tone in an octave, either, but the closest approximation that the ancients could practically achieve as a ratio of whole numbers. Six whole tones approximated in this way exceed the size of an octave (2.02728653, expressed in our modern decimal terms). The true size of a whole tone mathematically would be the sixth root of two (an irrational number, approximated here to 1.122462048); and the true size of a semitone would be the square root of the sixth root of two (also irrational, approximated here to 1.059463094; cf. 256/243, approximated here to 1.053497942 and the square root of 9/8, approximated here to 1.060660172). Controversy surrounded the correct calculation of the *leimma* in the fourth century ostensibly because the whole tone calibrated at 9/8 was not only not mathematically a true whole tone, but it also was not a perfect square, and so, no true halftone based upon it could be calculated. Boethius attributed the calculation of the *leimma* at 256/243 to Philolaus, although the first instance

4 Plato *Tim.* (T. Taylor) 17 A.

5 Burkert, *Lore and Science*, 198; M.L. West, *Ancient Greek Music* (Oxford: Clarendon Press, 1992), 168 and 237; Levin, *Manual*, 136, 175.

of the ratio in a text appears in *Timaeus* 36 B.[6] This *leimma* comprises the left-over portion of a sesquitertian (4/3) interval divided by the sesquioctave (9/8) ratio in Plato's recipe for the world soul (*Timaeus* 36 B).

Philolaus had allegedly established whatever his "halftone" ratio may actually have been, unscientifically, with a resulting error in his calculation of the intervals of another genus of scale on principles analogous to those he had used for the diatonic scale. He is also thought to have been unaware that the whole tone of the diatonic scale could not be equally bisected.[7] Archytas corrected him on the latter point and surpassed his musical efforts by determining a consistent system of ratios for each genus of scale, diatonic, chromatic, and enharmonic.[8] This study claims that the *Timaeus* allows an equally coherent, alternative system of ratios, also within a Pythagorean framework.

It is not surprising that Plato should have chosen to study the octave interval. The octave was still a relatively recent discovery in the Greece of his day and not yet thoroughly understood. Nicomachus of Gerasa (2nd century A.D.) attributed the discovery of the octave to Pythagoras, describing it equivalently as either two fourths separated by a whole tone or a fourth conjoined without separation to a fifth. The fourth in the octave had two whole tones and a *leimma*, while the fifth had three whole tones and a *leimma*.[9] Ptolemy (2nd cent. A.D.) ascribed the octave to Pythagorean tradition, rather than to Pythagoras, himself, similarly explaining that the fifth is greater than the fourth by a tone.[10] His separate description of Archytas of Tarentum's division of the fourth further also indicates a *leimma* therein.[11] Pseudo-Plutarch (circa 170–300 A.D.) gave credit to Pythagoras for promoting octave analysis as the sole proper task of music study and judging musical intervals, not by hearing, but by the laws of harmony. He used the sections of Plato's *Timaeus* concerning the division of the world soul by the demiurge, to illustrate the octave as a union of a fourth and a fifth constructed from two disjoint fourths separated by a whole tone.[12]

The earliest available Greek written evidence purporting to document the knowledge that a fourth and fifth comprise an octave appears to be fragments of Philolaus' *Physis*, although tradition has it that Hippasus of Metapontum,

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6 Boethius *De institutione musica* 3.5–3.10 (especially 3.8); Andrew Barker, *The Science of Harmonics in Classical Greece* (Cambridge: Cambridge University Press, 2007), 269, 290 n. 10.

7 West, *Ancient Greek Music*, 235–236, 237; Burkert, *Lore and Science*, 398–399.

8 West, *Ancient Greek Music*, 236–237.

9 See, e.g., Nicomachus *Manual of Harmonics* 6.1–7.3; Levin, *Manual*, 73.

10 Ptolemy *Harmonicorum libri tres* 1.5–1.7.

11 Ibid., 1.13 and 2.14 (table III); Burkert, *Lore and Science*, 198.

12 Pseudo-Plutarch *On Music* 22, 37.

a Pythagorean preceding Philolaus by a generation or two, had demonstrated or even discovered the appropriate ratios for its two constituent intervals by working with bronze discs of varying thickness.[13] Lasus of Hermione, a contemporary of Hippasus, apparently also studied the ratios of the octave.[14] Modern musicologists, however, view the *Timaeus* as one of the earliest reliable written sources directly documenting the octave scale in ancient Greece. Leo Treitler, in fact, included the *Timaeus* sections concerning the structure of the world soul (34b–37c) in his authoritative revised version of Oliver Strunk's *Source Readings in Music History*.[15]

Problems bound up with the octave, in Plato's time, were not limited, in any case, to the size of the *leimma*. They included difficulties in defining the relationship between the two parts of the octave vis-à-vis the *leimma's* optimal placement and the acceptability of the resulting scale.

Problems for present students of the *Timaeus* arising from obscurity in the commentary tradition may or may not have been independent issues for the musical tradition of Plato's day. For example, Nicomachus of Gerasa implied that the harmonic divisions described in the *Timaeus* psychogony define a scale identical both to the one that he attributed to Pythagoras and the one that he ascribed to Philolaus.[16] It is, in fact, difficult however, for modern scholars to say whether the *diapason* characteristic of the *Timaeus* is exactly the same kind of *diapason* as the one that Pythagoras is alleged to have discovered, since *inter alia*, the ancient sources are not definitive about exactly where the *leimmata* stood in the Pythagorean scale. It is also difficult to claim certainty about an identification of the scale in the *Timaeus* with Philolaus' scale, relying on the scant testimony that scholars possess about the Philolaus scale. Nicomachus claimed that the Pythagorean octave progressed *via* fourth, then fifth as follows: *diesis*, whole tone, whole tone for the fourth, then three whole tones and a *diesis*. He did not specify exactly where the *diesis* occurred in the fifth but maintained, instead, that it shifts in position, being able to occupy any of four possible places in the fifth.[17]

Thomas Mathiesen interpreted the description of the Philolaus scale that Nicomachus presented as a seven note, rather than eight note octave. Speaking

13 West, *Ancient Greek Music*, 167, 234–235; Burkert, *Lore and Science*, 229, 237–249, 250–298; Huffman, *Philolaus*, 17–35, 44, and 73–74.

14 West, *Ancient Greek Music*, 234.

15 See Oliver Strunk, ed., *Source Readings in Music History*, revised by Leo Treitler (New York, London: W.W. Norton & Company, 1998), 19–23.

16 Nicomachus *Manual of Harmonics* 8.1–9.2; Levin, *Manual*, 107–108, 125–126.

17 Nicomachus *Manual of Harmonics* 7.1 and 7.3; Levin, *Manual*, 98.

in order of ascending pitch he suggested a lower fourth with the *diesis* (*leimma*) in the first position and an upper fifth with a whole tone in the first position, an undivided one and one-half tone interval in the next position, and a whole tone in the final position.[18] Clearly the span of the upper fifth in Mathiesen's hypothetical arrangement is the three whole tones and one halftone needed; but there is a note missing in the upper part of the scale, somewhere in the one and one-half tone interval between the second and third notes. Carl Huffman agreed that, although the same diatonic attunement governs the *Timaeus* and the Philolaus fragments, Philolaus worked with a heptachord spanning an octave, while Plato worked with an octave.[19] The conclusions necessarily remain somewhat speculative as to the "sameness" of the diatonic attunement.

Flora Levin has concluded that Nicomachus' Pythagorean scale is actually markedly Aristoxenian in character, particularly in view of the movable *leimma* in the fifth interval.[20] Aristoxenus, a fourth century B.C. figure one generation younger than Archytas, proposed a system challenging the mathematically rigorous Pythagorean approach to *harmonia*. He calculated new interval sizes and adopted a different general approach to determining them.[21] According to Ptolemy, Aristoxenus arranged the octave as follows: STTTSTT.[22] Ptolemy's vertical diagram does not list the names of the tones; so it is unclear whether he was proceeding from the lowest position on the lyre or from the lowest pitch beginning from the bottom of his diagram. If the "bottom" S is the lowest position on the lyre, then it is the highest note, and the sequence given is the descending order of pitch, representing a Lydian octave. If the "bottom" S is the lowest pitch, then the descending order of pitch is TTSTTTS, representing the Dorian octave. Accordingly, Ptolemy's text is ambiguous as to whether the diatonic arrangement he represented for Aristoxenus was Dorian or Lydian. Bear in mind that because of Aristoxenus' new approach, his Dorian or Lydian octaves were bound to sound peculiar to Pythagorean ears.

The issue whether the octave *harmonia* proceeds in ascending or descending order, i.e., from lowest pitch to highest pitch or from highest pitch to lowest pitch may or may not have existed for Plato. This study suggests that he (or those whose work *Timaeus* 35 A–36 D reflects) was sensitive to the direc-

18 Mathiesen, *Apollo's Lyre*, 402.
19 Huffman, *Philolaus*, 376.
20 Levin, *Manual*, 171–174.
21 Ibid., Burkert, *Lore and Science*, 198; Mathiesen, *Apollo's Lyre*, 292, 294.
22 Ptolemy *Harmonicorum* 1.9.

tionality of the scale. Gustave Reese noted that, at least early on, the Greeks calculated series of tones in descending, rather than ascending order.[23] Nicomachus, again by way of example, assumes an order from the lowest to the highest note.[24] The "low" and "high" assumed by the names of the notes in the octave scale had reference not to pitch, however, but to the positions of the strings on the lyre. The lowest note on the lyre, for the Greeks, was actually the highest pitch because ancient lyre players tilted their instruments when playing, with the result that the highest pitched strings were in the lowest positions, physically, and the lowest pitched strings were in the highest positions.[25]

Without additional context, it is difficult to say whether Nicomachus' usages of "high" and "low" do or do not refer to pitch. One can hazard a guess that they probably do not and judge that his description proceeded in descending order of pitch. Depending, however, upon the denouement of the question, Nicomachus was talking about one of four possible octave scales, the Mixolydian, Lydian, Hypolydian, or Dorian, known to the ancient Greeks, taking into consideration the movable *diesis* in the fifth interval of his Pythagorean scale. The named alternatives involve modern scholars in further debates concerning the note ranges to which these ancient scales most closely corresponded.[26] Gustave Reese warned that the *tonoi* corresponding to the octave types are not exactly equivalent to modern major or minor keys.[27] In any case, the Dorian and Mixolydian possibilities mentioned assume that Nicomachus' list reflects an ascending order of pitch. The other two possibilities assume that it reflects a descending order of pitch.

As noted, Thomas Mathiesen analyzed Nicomachus' text in terms of an ascending order of pitch (from *hypate* to *nete*), apparently assuming that Nicomachus meant that he was beginning from the lowest pitch in his recitation of the intervals, as follows: STTTSTT.[28] In the descending order of pitch, one

23 Reese, *Music*, 21.

24 Nicomachus *Manual of Harmonics* 7.1; Levin, *Manual*, 98.

25 Reese, *Music*, 22.

26 Cf., Reese, *Music*, 30; West, *Ancient Greek Music*, 230.

27 Reese, *Music*, 30.

28 Note that a prior starting tone number (STN) is assumed in all arrangements presented in this manner, as well as in Nicomachus' description. The letters are intervals rather than notes and so they are always fewer than the number of notes in the scale by one. There are seven intervals in an octave; so there are only seven letters. Mathiesen, *Apollo's Lyre, Greek Music and Music Theory in Antiquity and the Middle Ages*, Publications of the Center for the History of Music Theory and Literature, vol. 2 (Lincoln, Nebraska and London, England: University of Nebraska Press, 1999), 398.

represents the scale that Mathiesen suggested as TTSTTTS.[29] This scale was generally known as the Dorian octave.

M.L. West pointed out the different logical possibilities for the structure of the ancient octave that would have been preoccupations for Plato, as well as for scholars studying the ancient Greek musical tradition today: fourth-tone/ fourth; fourth/ tone-fourth; tone-fourth/fourth; fourth/fourth-tone.[30] Clearly, these variations dictate different possibilities for the locations of the *leimma* in the two halves of the octaves. Gustave Reese presented the seven different possible diatonic arrangements of tones and semitones in the Greek octave thus, positing, also, in general compatibility with M.L. West, that they represent a descending order of pitch ranges in the *ordo* given: (T)(TTS)(TTS) (Mixolydian); (STT)(T)(STT) (Lydian); (TST)(T)(TST) (Phrygian); (TTS)(T)(TTS) (Dorian); (STT)(STT)(T) (Hypolydian); (T)(STT)(STT) (Hypophrygian); (TTS)(T) (TST) (Hypodorian).[31] The acceptability of each of these scales lies in the preservation of units of identical fourths and independent tones. Only some placements of the *leimma* allow such units.

This new *Timaeus* analysis sheds light on many of the problems mentioned, even as it articulates a harmonic cosmogony (or psychogony, as one may prefer). This study supports the theory that, regardless of how the "Pythagorean" and Philolaic octaves may have been structured, the *Timaeus* defines a primary diatonic scale with a very definite *ordo:* TTS/T/TTS. If the scale is understood in ascending order of pitch, then the pattern represents an ascending Lydian diatonic scale. If it is understood in descending order of pitch, then it represents a descending Dorian diatonic scale. The scale exhibits uniform whole tone intervals of 9/8 and a *diesis* of 256/243. Musicologist Ernest McClain, long a student of Plato's mathematical *excursuses* and allegories, believes that the *diesis* has to be at the termination of the fourth in the *Timaeus* scale.[32] Plutarch characterized the *Timaean* fourth in terms of an ascending Lydian transposition of the fourth interval (even as he noted the work of others considering an ascending Phrygian or Hypodorian transposition of the fourth).[33]

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29 Reese, *Music*, 28 and 30.

30 West, *Ancient Greek Music*, 161.

31 Reese, *Music*, 28 and 30; West, *Ancient Greek Music*, 230 (parentheses added).

32 Ernest McClain, *Pythagorean Plato, Prelude to the Song Itself* (York Beach, Maine: Nicolas Hys, Inc. 1978), 60 (emphasis added; the order of the proportion, as he states it, indicates the descending interval between the top note of the fourth and the tone immediately below it; one could equally validly speak of 256/243 as the ascending interval between the penultimate note of the fourth and its last tone).

33 Handschin, "Timaeus Scale," 15 and 26; Plutarch *De Procreatione in Timeo*, 1019 E–1022 C, particularly, 1018 D and 1021 E (ascending Lydian scale presupposed in Plutarch's commen-

The *Timaeus* scale's possibly Lydian character and the disagreement over whether the Lydian *tonos* most closely corresponded to C or C# explains the indecision of some modern scholars about whether to assign the number corresponding to the first tone in the *Timaeus* scale a note value of D or C. The entire debate may be misconceived, as ancient Greek music did not embrace our modern system of keys. The structures of the scales native to various *tonoi* are themselves independent of pitch ranges, though ancient Greeks may have been accustomed to employing them in certain ranges for particular voices, instruments, and purposes. In any case, the ancient texts apparently do not specify a tone number that would correspond to the modern C#. Handschin assigned 384, the first number, in an analysis of the *Timaeus*, from which a full octave scale can be articulated, according to the ancient commentaries of Proclus and Plutarch and many other analyses, to the note D.[34] Ernest McClain did the same by implication in assigning 768, the number beginning the octave above (assuming an ascending scale), to D, while Levin implied that 384 is C, since she took 192 (the octave below, assuming that smaller numbers represent lesser vibration rates) as C, based on Nicomachus' evidence suggesting the equation.[35] It matters little, for the purposes of this study how one assigns pitch ranges to *tonoi*.

As noted, the *Timaeus* scale emerging from this study could surely have a Dorian character. As Handschin explained, Plutarch's assessment that the scale is an ascending Lydian one depends upon the assumption that the tone numbers in the *Timaeus* scale are indexed to "frequency" and so point to the number of vibrations per unit time of a string on a standard instrument.[36] It is not clear how the ancients might have measured "frequency" in this sense. They seem to have had a notion that higher string tension produces a higher sound. See Plutarch, *De animae procreatione in Timaeo* 1021A. Whether they knew how to measure string tension with any accuracy is another issue.

Donald Creese has observed, in his careful history of the monochord, that when strings of the same thickness and length are tuned together, the ratios of their different tensions are not the same as the ratios of their string lengths. Rather ratios of string tensions are the squares of the ratios of string lengths;

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tary treats the numbers corresponding to the first fully articulated diatesserson emerging from an analysis of the *Timaeus*, beginning with 192, as indexed to string tension and so string vibration or impacts on air, rather than string length).

34 Plutarch *De Procreatione* 16 (reporting Crantor's identification of the first number); Plutarch *De Animae Procreatione in Timaeo*, 1020 C (reporting Crantor's identification of the first number); Proclus *in Timaeum* 3.2.177.1–179.5, 2.185.3–2.187.15, 2.191.1–10.

35 Handschin, "Timaeus Scale," 18; McClain, *Pythagorean Plato*, 61; and Levin, *Manual*, 135.

36 Handschin, "Timaeus Scale," 26.

and the application to pitch is inversely proportional to those ratios. It is necessary to increase tension four times to raise a string by an octave. One need only double a string length to produce a difference by an octave (one actually halves a string length to raise a string by an octave).[37] Creese has opined that a means of measuring tension seems not to have been known in antiquity.[38] Ancient Greeks appear to have known that differences in pitch were related to the temporal density of impacts of a moving body, such as the impacts of a string on air.[39]

Donald Creese's conclusion that the ancients had no technique for measuring string tension seems questionable, in spite of the lack of evidence, if one just considers the practicality of making music with strings. If an instrumentalist were faced with tuning according to string lengths over any modest period of time, he might be expected to have discovered quite early on that basing tuning on string length, alone, could get cumbersome. Tightening a string or two would certainly have been within the realm of his practical experimentation; and the result would have been a practical and, probably, fairly precise knowledge of the way in which increased tension was tied to higher pitch.

Creese has explained that, in the fifth century, musicians achieved pitch variation primarily by varying string thickness.[40] Note that increasing string thickness is one way to increase string tension. In harps, he has noted, at least those known to have been in Greece in the late sixth century, instrumentalists achieved pitch variation through a combination of string lengths and string thickness. String division as a means of achieving pitch variation was not attested, he has noted, until the end of the fourth century.[41]

Many modern commentators assume that the tone numbers in ancient commentaries represent string lengths.[42] In that case, the longer the string length, the lower the pitch; so a scale beginning from relatively small numbers and progressing to larger ones would be a descending scale. One can see why the conclusion makes sense, in view of the foregoing discussion. However, without some ancient reason justifying this modern descending departure from the ancient opinion of Plutarch, favoring an ascending scale, for the *Timaeus*, there is no compelling reason to insist upon the Dorian octave over the Lydian in its

37 Donald Creese, *The Monochord in Ancient Greek Harmonic Science* (Cambridge: Cambridge University Press, 2010), 83.

38 Ibid., 83, 164–165.

39 Ibid., 164–165, 170–171, 219, and 243.

40 Ibid., 100.

41 Ibid., 100–102.

42 Ibid., 24.

case, especially, as Jacques Handschin has noted, since Plato seemed to refer to "frequency" numbers at *Timaeus* 67 B.[43] The *Timaeus* could present both scales, reciprocally.

One might, of course, take issue with Handschin's characterization of *Timaeus* 67 B. The passage at *Timaeus* 67 B actually refers to sound as a blow that passes through the ears and is transmitted by means of the air, brain, and blood to the soul; it characterizes hearing as the vibration of a blow of sound that begins in the head and ends in the region of the liver. The sound "that moves swiftly" is acute, the passage notes; and the sound "that moves slowly" is grave. The actual wording of the passage testifies to conceptions falling short of the modern notion of frequency, which presupposes vibrations per unit time. It does indicate a belief that sound creates impact and causes vibrations, cognizable upon entrance to the ears; further, some of these vibrations are such as to seem acute and others to seem grave. "Moving swiftly" is certainly a perception that might attach to what was, in fact, a larger number of vibrations per unit time and so, an acute sound; but it is not a scientifically precise formula and is no guarantor that the ancients of Plato's time had any scientific notion of frequency in the modern sense.

To the extent that larger numbers arising from the analysis of Plato's construction of the world soul were linked to more numerous vibrations caused by the initial impact of a sound on a string, then ascending interpretations of the *Timaeus* scale, in which low numbers indicate low pitch and high numbers higher pitch, are to be expected, even if one cannot say that numbers are direct measures of vibrations per unit time, exhibited by a string upon a discrete impact. As long as they can be indices, in some sense, to the relative vibratory behavior of strings in relation to known concords, they can pertain to an ascending scale. One must remind oneself, as Creese has noted, that the *Timaeus Scale* is not, ultimately, about any particular kind of practical music making.[44] It is about a *harmonia* established between and among numbers that is, in some way, a standard for all other harmonious relations and that is, above all, descriptive of the *harmonia* of the macrocosmic order.[45]

The *Timaeus* scale posited by this study could be unique to Plato in his time.[46] It does not utilize Philolaus' octave, if Philolaus worked with a heptachord, as some scholars suggest. It uses no octaves like those of the Pythagorean Archytas of Tarentum. Archytas' transposition of the fourth, as mentioned

43 Handschin, "Timaeus Scale," 15.

44 Creese, *Monochord*, 160–161.

45 Ibid.

46 Mathiesen, *Apollo's Lyre*, 402; Huffman, *Philolaus*, 376.

above, had two whole tones and a *diesis* but the whole tones were different sizes and he adjusted the *diesis* so that, in his scale, it became less than the 256/243 ratio that Plato had recognized. Archytas placed it at 28/27, so that the structure of his transposition of the fourth was $9/8 \times 8/7 \times 28/27$.[47] How Archytas' fiddling kept him within the parameters of a Pythagorean paradigm will become apparent much later in this analysis. The system emerging from this study represents a genuine alternative approach to that of Archytas within a Pythagorean paradigm.

The octaves of Plato's scale have no complementarity to those of Aristoxenus' system, as it stands within a rival tradition. Plato's approach to the octave in the *Timaeus* produced a venerable tradition. Early on, it may have influenced Eratosthenes of Cyrene, a scholar at the Alexandrian library in the last half of the third century B.C., who recognized exactly the same size intervals as the *Timaeus* for the diatonic scale.[48]

The *Timaeus* may also have influenced the *Sectio canonis*, attributed to Euclid (300 B.C.) by Porphyrius in his commentary on Ptolemy's *Harmonicorum* 1.5. Although the *Sectio canonis* evolved over time, it is, according to Thomas Mathiesen, "the fullest and most systematic surviving application of Pythagorean mathematics to very specific musical topics: consonance, the magnitudes of certain consonant intervals, the location of movable notes in an enharmonic fourth, and the location of the notes of the Immutable System on a monochord."[49] Its musical theory represents one of the two basic positions in Greek musical theory. Aristoxenus' system was the other.[50] Archytas' system, as noted, was a Pythagorean variation. In any case, the *Sectio canonis* also recognizes the same ratios, as the *Timaeus*, since it clearly specifies a uniform sesquioctave whole tone (9:8), while acknowledging that the fourth and fifth comprising the octave are actually less than two and one-half tones and three and one-half tones, respectively.[51]

The affinities among Eratosthenes' thought, the *Timaeus*, and the *Sectio canonis* may, of course, reflect a common source, rather than any direct impact of the *Timaeus*. The only candidate for a common Greek source, among these figures, would appear to be Philolaus.

Considering the preoccupations of Plato's day with the ideal structure of the harmonic fourth and the higher level structures including it, Plato's spe-

47 Ptolemy *Harmonicorum*, 1.13 and 2.14.
48 Ptolemy *Harmonicorum*, 2.14; West, *Ancient Greek Music*, 237.
49 Mathiesen, *Apollo's Lyre*, 344.
50 Ibid., 352.
51 Ibid., 344–349.

cial relationship with the Pythagoreans at Tarentum, his probable awareness of Archytas and Philolaus (much older), and his own placement of the octave *harmonia* at the heart of his construction of the world soul, it is plausible to propose that the infirmity of the fourth interlocutor at *Timaeus* 17 A is an infirmity of the octave *harmonia* stemming from its fundamental indetermination to intervals of equal size.[52] The system of music derived from the *Timaeus*, in this study, has an additional marked infirmity in the eventual degeneration of its octave periodicity to fifth periodicity. Intriguingly, the weakness both reinforces the text's message about the structure of being and generates important insights about the character of the ordo appropriate to a universe of changing things. The ordo that reigns, as the study will show, ensures a finite universe subsisting within definite bounds.

There are, of course, other interpretations of the missing fourth guest. Iamblichus, for example, apparently believed that the missing guest was a contemplative of the noetic realm, rather than someone inclined to the study of natural philosophy. The deficiency under which he labored was his disinclination to that study.[53] Syrianus believed just the opposite. The fourth person dropped out because he was unfit for a discourse that was purer and more intellectual than the discourse of the previous day concerning the state.[54] These viewpoints, not being clearly rooted in problems of the text itself, seem unduly speculative. In keeping with the spirit of this study, A.E. Taylor argued that the missing fourth guest represented the kind of doctrine current in Sicily and Italy contemporary with the conversation presented by the text; so it was suitable for Timaeus, who hailed from those parts, to take the guest's place.[55] It is at least plausible to think that musical doctrine is at issue, since the problem of the *leimma* is so unavoidable by reference, in the text, and since the rules of the state in *Laws*, as Chapter 8 of this study discloses, ideally operate on harmonic principles shared with the *Timaeus*. Musical preoccupations and matters of state, such as those with which the *Timaeus* was set to begin, go together.

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52 Burkert, *Lore and Science*, 198 and 201; Plato *Epistles* 9 and 12 to Archytas of Tarentum; Nicomachus *Introduction to Arithmetic* 2.18 (trans. D'ooge, n. 2 re Philolaus *floruit*).

53 See John Dillon, "Iamblichus' Commentary on the *Timaeus* of Plato: A Collection of the Fragments with an Attempt at Reconstruction" (Ph.D. diss., University of California, Berkeley, 1969), 183–184.

54 Syrianus *in Timaeum* (Sarah Klitenic Wear trans., in *The Teachings of Syrianus on Plato's Timaeus and Parmenides*, Studies in Platonism, Neoplatonism, and the Platonic Tradition, eds. Robert Berchman and John Finamore, vol. 10, Ancient Mediterranean and Medieval Texts and Contexts [Leiden: Brill, 2011]), fr. 1 (and accompanying annotations).

55 A.E. Taylor, *Commentary*, 25.

Even so, the path is not an easy one to rigorous proof that a Pythagorean musical analogy, expressing a harmonic cosmogony grounded in a defined octaval ordo, is really at the core of the *Timaeus*. Many have assumed similar notions from the harmonic divisions of Plato's construction of the world soul and have even elaborated variously upon a *Timaeus* scale. Jacques Handschin and Ernest McClain, for example, each discussed the *Timaeus* scale in great detail.[56] Handschin observed that, in the *Timaeus*, "a scale, musical as well as cosmical, is expounded."[57] Edward Lippmann spoke at length about Plato's belief that harmony and music have cosmological implications and related how both the Republic's *Myth of Er* and the *Timaeus* constituted evidence for his expression of "the musical constitution of the world."[58] M.L. West likewise asserted: "Plato's harmony of the spheres is not some unimaginable, transcendental passacaglia or fugue, but the naked glory of the diatonic octave."[59] Walter Burkert pointed out that, although the theory of elements in the *Timaeus* is non-Pythagorean, the overall cosmological scheme certainly betrays Pythagorean influence, particularly since a denizen from Magna Graecia is made its mouthpiece. Besides, the numerical conception of harmonic intervals was unique to the Pythagoreans.[60]

Walter Burkert and John Dillon have both agreed that the Old Academy interpreted the *Timaeus* from very early on as providing a musical metaphor for the structure of being in the Pythagorean tradition.[61] In addition, C.J. Vogel opined:

> That the World-Soul is divided according to harmonic intervals brings one back to Pythagorean theory. That the circles of the planets are fitted into the framework of the circles of the Same and the Different, which are the World-Soul's elements, is doubtless Plato's adaptation. But again, that the planets are the heavenly clock, the movements of which are the measure of time, was a Pythagorean theory.[62]

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56 Handschin, "Timaeus Scale," 4–42; McClain, *Pythagorean Plato*, 57–70. Ernest McClain also affirmed in electronic correspondence that an octaval *harmonia* of being is at the heart of the *Timaeus*. Ernest McClain, Electronic mail conversation with Donna M. Altimari Adler, October 25, 2001.

57 Handschin, "Timaeus Scale," 5.

58 Edward A. Lippman, *Musical Thought in Ancient Greece* (New York and London: Columbia University Press, 1964), 23–44.

59 West, *Ancient Greek Music*, 234.

60 Burkert, *Lore and Science*, 85.

61 Ibid., 84–85; Dillon, "The *Timaeus* in the Old Academy," 82–83.

62 C.J. Vogel, *Pythagoras and Early Pythagoreanism, An Interpretation of Neglected Evidence on the Philosopher Pythagoras* (Assen: Van Gorcum & Company, N.V., 1966), 193.

François Lasserre explained that certainly by fifth century B.C., ancient Greeks regarded musical education to be an integral vehicle for the methodological formation and education of the soul to a reflective character. Lasserre concentrated, in his work, particularly on the figure of Damon of Athens, mentioned in several of Plato's writings.[63] In light of music's accepted formative influence on the soul in ancient Greece, the psychogenic implications attending Plato's construction of the world soul on harmonic principles in the *Timaeus* cannot be ignored.

Evanghélos Moutsopoulos spoke of *Timaeus*' musical cosmology and the role of harmony in it by explaining a nexus of ideas that Plato shared with Pythagoreans. He explained that, for Philolaus, the center of the universe was formed from a fire around which the ten celestial bodies gravitated from East and West, held by a bond. This center of gravity was the measure, the bond, and the harmony of the universe, as well as the living spirit and the soul of the world providing life.[64]

The nature of the bonds in the universe were mathematical and unique in their quality. Moutsopoulos noted that the octave ratio ½ came to represent harmony in the metaphysical sense just because of the numerical relationship that it expresses. Limit is represented by unity ("1") and Unlimit by the undetermined number two ("2"). "Two" reached determination only in relation to unity. The particular determination binding "2" to unity is a unique mathematical relationship, the octave, belonging only to "2" and "1." Such a conception, Moutsopoulos opined, was equally valuable and operable for Plato and Pythagoreans.[65]

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63 Lasserre, "L' Education musicale," 53–74.

64 Evanghélos Moutsopoulos, *La musique dans l'oeuvre de Platon*, (Paris: Presses Univeristaires de France, 1959), 331–333; see, also, ibid., 345–347, and 351–390 (for a full discussion of the musical cosmology of the *Timaeus* and its connection both with the *Myth of Er* in Plato's *Republic* and certain passages of the *Laws*). Others have also appreciated the musical and musico-cosmological significance of the Republic's *Myth of Er*. As M.L. West observed: "in the cosmology of Plato's *Republic* the music of the spheres is made by eight Sirens, each responsible for a different note of the diatonic scale." West, *Ancient Greek Music*, 224. Huffman cited the myth as a source independent of Philolaic astronomy and alternative to it for the conception of a music of the spheres. Huffman, *Philolaus*, 281. Pseudo-Plutarch discussed the musical significance of the *Republic*, in general, in his treatise on music; and Leo Treitler included sections of the *Republic* in his revised edition of Strunk's *Source Readings in Music History*. Plutarch *De musica* 17; Treitler, *Source Readings*, 9–19; see also, McClain, *Pythagorean Plato*, 41–55 (for more on the musical significance specifically of the *Myth of Er*).

65 Moutsopoulos, *Musique*, 331–333; see, also, ibid., 345–347, and 351–390.

Moutsopoulos maintained that once such ideas of harmony were established, one could easily free oneself from speculation by applying mathematical laws of harmony to geometry and stereometry. Geometry, applied to metaphysical speculations, in turn, led directly to cosmology. The metaphysical understanding of harmony, communicable by means of number, thus became central and primary for Pythagoreans constructing models of the cosmos.[66]

Moutsopoulos noted that Plato utilized his understanding of metaphysical harmony in his exposition of the genesis of the regular solids and his extension of it to other realms. The same principle of harmony applied to the individual soul, as a psychological principle, surely as much for Plato as for the Pythagoreans; further, for Plato, harmony is a bond existing not only between the soul and body but also among different parts of the body and the soul, although the extension of the harmonic principle to the domains of morals and religion seems to have been conditioned by popular belief. In Moutsopoulos' assessment, Plato owed much to the circle of Pythagoreans for his conception of harmony.[67]

The present study bears out the claims of the distinguished scholars who have advocated that Plato constructed a musical cosmogony/psychogony in the *Timaeus*, decidedly influenced by Pythagorean thought. The total context of the dialogue supports such a notion; but a detailed proof takes patience.

For the reader's convenience and the sake of the argument, the discussion occupies eight chapters. The first is a general overview of the *Timaeus* giving special attention to Plato's emphasis on a fourfold *ordo* of generation mediated by the Decad; the second through fourth chapters comprise a detailed examination of Plato's construction of the world soul according to the pattern of the Decad, taking the text in small portions to identify all numbers emerging from the divisions of the world soul. The fifth and sixth chapters are analyses of the musical significance of the set of *Timaeus* numbers, derived solely in accord with mathematical principles. The seventh relates the musical data to the cutting of the fabric, the making of the "chi," and the cosmic orbits. The eighth and final chapter explores the relationship between Plato's project to construct an ideal *polis*, in the *Laws*, and his effort to construct an ideal cosmos in the *Timaeus*.

66 Ibid.
67 Ibid.

CHAPTER 1

The *Timaeus*, the Decad, and the *Harmonia*: an Overview

The *Timaeus* begins as a dialogue among Socrates, Critias, Hermocrates, and Timaeus, its host, with the distinction between being that is always real and without generation and generated things that always become but never truly are real (27 D–28 A). Real being, Timaeus claimed, is apprehended by intelligence and reason, "since it always subsists according to the same" (28 A).[1] Generated things are, at best, objects of opinion and irrational sense, since they are subject to corruption and change (28 A). All things visible and embodied are sensible; and human beings apprehend them by opinion and sense (28 B and 28 C); so they seem generated. Indeed, they must be generated, since they subsist in coming to be. Because the universe is sensible, being visible and having a body, it must, then, be generated (28 B and C).

As something generated, Timaeus said, the universe must have some cause (28 C); and so he attributed it to "the artificer and father of the universe" (28 C).[2] Timaeus maintained that "the father" must have looked toward an eternal exemplar in the world's fabrication, since it would not otherwise be beautiful (29 A). This exemplar is comprehensible to reason and intelligence and subsists immutably in an abiding sameness of being (29 A). The world is the image of the immutable exemplar but is not immutable itself (29 C). It can only be a basis for probable, analogous arguments *vis-à-vis* the exemplar (29 C). Timaeus offered just such an argument in considering the nature of the universe (29 C and 30 B). He asserted that it is an animal endowed with intelligence, explaining that only such an effect is sufficiently beautiful to have derived from the best of causes (30 A and 30 B). It must, therefore, also be endowed with soul, since "it was impossible for intellect to accede to any being, without the intervention of soul" (30 B).[3] As he stated:

1 Plato *Tim.* (T. Taylor) 28 A. The Greek text reads: "ἀεὶ κατὰ ταὐτὰ ὄν …" Plato *Tim.* (Reale) 28 A.

2 Plato *Tim.* (T. Taylor) 28 C. The Greek text reads: "ποιητὴν καὶ πατέρα τοῦδε τοῦ παντὸς …" Plato *Tim.* (Reale) 28 C.

3 Plato *Tim.* (T. Taylor) 30 B. The Greek text reads: "νοῦν δ' αὖ χωρὶς Ψυχῆς ἀδύνατον παραγενέσθαι τῳ." Plato *Tim.* (Reale) 30 B.

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> [29 E] The artificer, indeed, was good: but in that which is good envy never subsists about any thing which has being. Hence, as he was entirely void of envy, he was willing to produce all things as much as possible similar to himself. If therefore anyone receives this most principal cause of generation and the world from wise and prudent men, he will receive him in a manner the most perfect and true. [30 A] For as the Divinity was willing that all things should be good, and that as much as possible nothing should be evil; hence, receiving every thing visible, and which was not in a state of rest, but moving with confusion and disorder, he reduced it from this wild inordination into order, considering that such a conduct was by far the best. *For it neither ever was lawful, nor is, for the best of causes, to produce any other than the most beautiful of effects*. [30 B] In consequence of a reasoning process, therefore, he found that among the things visible, there was nothing the whole of which if void of intelligence could ever become more beautiful than the whole of that which is endued with intellect: and at the same time he discovered, *that it was impossible for intellect to accede to any being, without the intervention of soul*. Hence, as a result of this reasoning, placing intellect in soul and soul in body, he fabricated the universe; that thus it might be a work naturally the most beautiful and the best. In this manner, therefore, according to an assimilative reason, it is necessary to call the world an animal, endued with intellect, and generated through the providence of Divinity.[4] [References in brackets added; italics introduced for emphasis.]

As an image of the immutable exemplar, the world is the most similar to "that animal, of which other animals, both considered separately and according to their genera are nothing more than parts" (30 C and 30 D).[5] In other words, the exemplar is the immutable idea of the ultimate animal. The exemplar is "that which comprehends all intelligible animals whatever" (31 A).[6] It is "All Perfect Animal" (31 B).[7] The exemplar, of course, unlike the universe, is not corporeal; so a question arises, how the universe acquired corporeality. Timaeus did not explain. Instead, the passage, 29 E, just quoted seems to indicate that seeds

4 Plato *Tim.* (T. Taylor), 29 E–30 B.

5 Ibid., 30 C. The Greek text reads: "οὗ δ' ἔστιν τἆλλα ζῷα καθ' ἕν καὶ κατὰ γένη μόρια, τούτῳ πάντων ὁμοιότατον αὐτὸν εἶναι τιθῶμεν." Plato, *Tim.* (Reale) 30 C.

6 Plato *Tim.* (T. Taylor) 31 A. The Greek text reads: "τὸ γὰρ περιέχον πάντα ὁπόσα νοητὰ ζῷα …" Plato *Tim.* (Reale) 31 A.

7 Plato *Tim.* (T. Taylor) 31 B. The Greek text reads: "ᾗ τῷ παντελεῖ ζῴῳ …" Plato *Tim.* (Reale) 31 B.

of corporeality were, in some way, already incipiently or inchoately in place before the world's formation, bound up with chaotic motion. The father made the universe by "receiving everything visible" which was not in a state of rest but "moving with confusion and disorder" and reducing such "wild inordination" to order.[8] Nonetheless, corporeal elements do not seem to have existed prior to the world soul, since Timaeus specified elsewhere that the demiurge had fashioned (ἐγεγένητο) all that is bodily within it, after its construction (36 D–E).[9]

The creation of the universe was, indeed, a two-fold process in which the soul's creation preceded the body's generation (34 B–C). As Plato wrote:

> [34 B] … As for the world's soul, even though we are now embarking on an account of it *after* we've already given an account of its body, it isn't the case that the god devised it to be younger than the body. For the god would not have united them and then allowed the elder to be ruled by the younger. We have a tendency to be casual and random in our speech, reflecting, no doubt, the whole realm of the casual and random of which we are a part. The god, however, gave priority and seniority to the soul, both in its coming to be and in the degree of its excellence, to be the body's mistress and to rule over it as her subject.[10] [Section references in brackets added.]

Soul was a necessary first step in creation because, as already noted, the intellect requisite to the life of the image of "All Perfect Animal" could not "accede to any being, without the intervention of soul" (30 B).[11] An intermediary, in other words, was needed between intellect and corporeal being to endow the latter with life. Endowed with intellect, the soul became a living being; so the investiture of the bodily with soul endowed it, as well, with intellect and life. The coming to being of something bodily, however, to be thus invested, itself required intermediaries, namely, fire and earth; and these, in turn, could relate to one another only through the additional intermediaries, air and water (31 B–32 B).

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8 Plato *Tim.* (T. Taylor) 30 A. The Greek text reads: "πᾶν ὅσον ἦν ὁρατὸν παραλαβὼν οὐχ ἡσυχίαν ἄγον ἀλλὰ κινούμενον πλημμελῶς καὶ ἀτάκτως …" Plato *Timaeus* (Reale) 30 A.

9 Ibid. (Reale), 36 D. Cornford agreed, noting a consensus of scholarly authority on the point. Cornford, *Plato's Cosmology*, 203.

10 Plato *Tim.* (trans. Donald J. Zeyl, in *Plato: Complete Works*, eds. John M. Cooper and D.S. Hutchinson [Indianapolis, Cambridge: Hackett Publishing Co., 1997]) 34 B and 34 C.

11 Ibid. (p. 478), 30 B. The Greek text reads: "νοῦν δ' αὖ χωρὶς ψυχῆς ἀδύνατον παραγενέσθαι τῳ." Plato *Tim.* (Reale) 30 B.

Much of the rest of the *Timaeus*' first half (through 40 D) reads like a lesson in cosmology for a theoretical physicist, despite its mythical character, even espousing views on the basic spherical shape of the universe echoed in recent years by an eminent theorist.[12] This study devotes much time to cosmogony and cosmology, below, as it explores *Timaeus*' construction of the World Soul. Following 40 D, Plato's text gropes toward an account, as closely analogous to the preceding cosmogenesis as possible, of the origin of the human soul and body, the state of the soul in an incarnate condition, and the mechanisms of the body. Then, it devolves more fully into the purely mythical (through 47 D). In both the narratives of the *Timaeus*' first half, two factors are primary: the divine exemplar and its generated copy.

The second half of the text (beginning at 47 E) shifts from the macrocosmic arena to a micro level consideration of the genesis of the primary bodies, "fire, air, water, and earth" and an extended reflection on the character of the medium or nature in which the becoming of the exemplar's generated copy occurs. A stereoscopic chemist could have a field day with large segments of the *Timaeus*' last half in view of some valid insights it contains about how the shapes of elements can effect physical properties and how those shapes might play a role in what are essentially chemical interactions.[13] Like the first part of the text, the last half eventually encompasses matters pertinent to human nature within its broader interests (beginning *circa* 61 C).

An overarching concern with motion ties the two halves of the dialogue, with their two levels of analysis, into one harmonious theory of everything. Timaeus represents the motions governing the World Soul as an outer circular band of motion called "same (τῆς ταὐτοῦ φύσεως)" and an inner band of motion called "difference (ἐντὸς τῆς θατέρου)" (36 C).[14] The band of difference is further

12 Stephen Hawking, *A Brief History of Time From the Big Bang to Black Holes*, with an introduction by Carl Sagan (Toronto, New York, Sydney, Auckland: Bantam Books, 1988), 133–141. Hawking's ideas do not have the negative ramifications for a creator god that he thought they have. If the reader appreciates Hawking's book just on the plane of physical theories, however, he may find it interesting.

13 It is a well established observation of stereoscopic chemistry and molecular biology that both organic and inorganic molecules, as well as atoms themselves, have definite geometrical configurations; further, those configurations help determine combinatorial possibilities. It is also well known that opposite symmetries of the same microgeometrical structures and slight variations of molecular and atomic shape can significantly alter chemical properties, even accounting for differences in color. See, e.g., Bruce H. Mahon, *University Chemistry*, 3rd ed. (Reading, MA: Addison-Wesley Publishing Co., 1975), 470–534, 719; Richard E. Dickerson and Irving Geis, *The Structure and Action of Proteins*, (Menlo Park, CA: 1969) (generally).

14 Plato *Tim.* (Reale) 36 C.

subdivided six times into seven subsidiary bands of motion (36 D). Its motion, as a whole, is oblique in direction to that of the outer band of sameness and is further differentiated within by opposite directions of motion among its subsidiary bands and varying speeds of motion among and within two subsets of them (36 D).

One can think of the macro level relations among the differentiated motions of the outer and inner bands as generating stable patterns of motion with definite geometrical characteristics. These patterns are not only circular but spiral, according to Timaeus, because of the oblique cross-motion of the outer band *vis-à-vis* the inner one (36 C and 39 A).

The motions of the World Soul extend to the physical "sphere" of the universe, its body, because the demiurge fashioned the latter within the former and fit the two "center to center (μέσον μέση)" (36 D–E).[15] Thus, the seven planets eventually came to inhabit the seven bands of difference in Timaeus' creation story; and their orbits have a spiral twist (38 C–D and 39 A).

The relation of the ordered motions of the universe to the motions within the "Receptacle (ὑποδοχὴν)" and the things passing in and out of it is not all together certain in all details from Plato's text, partly because the status of the Receptacle itself is unclear.[16] Plato called the Receptacle a form "difficult and obscure (χαλεπτὸν καὶ ἀμυδρὸν)," the "nurse (τιθήνην)" of all becoming (49 A).[17] It has no character of its own, yet ever remains what it is; it is invisible and all-receiving, participating very obscurely in the intelligible (50 B and 51 A). The passing impressions of real things, i.e., copies of eternal things, come to be situated within it and depart from it (50 C–D); and so, the Receptacle seems to have different qualities at different times. It does not really adopt the character of anything that enters it (50 C). It is just a matrix (50 C), a container, one might say, for everything.

Plato called the Receptacle a nature that receives all bodies (50 B) and, then, actually defined that nature, generally, as "space (χώρας)," everlasting, indestructible, providing a seat for all things that come into being, and "tangible

15 Ibid., 36 D–E. Although the world soul is spherical, the body of the universe may not be a perfect sphere. Cornford speculated that the latter is actually a dodecahedron, the last of the regular solids and the only one that Plato did not generate as a primary body in the *Timaeus*. Cornford, *Plato's Cosmology*, 218.

16 Plato *Tim.* (Reale) 49 A; see, also, 48 A (speaking of motion due to an errant cause), 43 B (speaking of six chaotic rectilinear motions, presumably to be assigned to the Receptacle upon Plato's later direction), and 50 B through 50 C (for references to things passing in and out of the Receptacle).

17 Ibid., 49 A; Plato *Tim.* (T. Taylor) 49 A.

without tangent perception" (52 A and B).[18] One must, of course, be careful not to attach unwarranted preconceptions to an understanding of "χώρας" as "space."

As John Sallis has observed, many of the connotations that one might ordinarily attach to "space" are unfitted to communicate the meaning of "χώρα."[19] These include associations of it with the "isotropic" space of post-Cartesian physics; with empty space, i.e., the void of Greek atomism, called τὸ κενόν; and with the notion of "place" or "locus," advanced by Calcidius. Sallis has noted that the last association collapses the distinction between χώρα and τόπος and confuses Plato's chorology with Aristotle's topology in the *Physics*.[20] He warns, in the end, that the Greek term χώρα is intrinsically untranslatable, as there is no available term comprising its semantic correlate.[21] Sallis acknowledges that the connections and affinities of the word with other terms in the Greek language allow "a kind of lateral translation" in view of other contexts.[22] He states that the term χώρα is not independent, in the *Timaeus*, of associations with "place," "land," and "country."[23]

Interestingly, Plato never used the term, "ὕλη," to describe the Receptacle. Cornford greatly emphasized this point.[24] He stated:

> There is no justification for calling the Receptacle 'matter'—a term not used by Plato. The Receptacle is not that 'out of which' (ἐξ οὗ) things are made it is that 'in which' (ἐν ᾧ) qualities appear, as fleeting images are seen *in* a mirror. It is the qualities, not the Receptacle, that constitute 'the bodily' (τὸ σωματοειδές). The term was used at 31 B: 'That which comes to be must be bodily and so visible and tangible; and nothing can be visible without fire or tangible without earth.' The contents of the Receptacle will presently be called 'bodies' (σώματα 50B), but we must beware of taking this to mean 'particles', as if the qualities had already received shapes.[25]

18 Plato *Tim.* (T. Taylor) 52 A; Plato *Tim.* (Reale) 52 A. The Greek corresponding to the quoted portion of the text reads: "δε μετ' ἀναισθησίας ἁπτὸν λογισμῷ τινὶ νόθῳ, μόγις πιστόν …"

19 John Sallis, *Chorology: On Beginning in Plato's Timaeus* (Bloomington and Indianapolis: Indiana University Press, 1991), 115.

20 Ibid.

21 Ibid.

22 Ibid., 115–116.

23 Ibid., 117.

24 Cornford, *Plato's Cosmology*, 181.

25 Ibid.

R.E. Allen also has stressed that the Receptacle of the *Timaeus* is "space," in whatever proper sense one is to take it, and not matter.[26] Lloyd Gerson has argued, however, that Aristotle interpreted the *Timaeus*' Receptacle of becoming, as matter, claiming that Plato implicitly identified matter and space. He further observed that the much later Neoplatonist, Plotinus, who received Plato through an Aristotelian lens, "actually" accepted Aristotle's interpretation. Gerson has admitted however, that the Receptacle cannot finally be identified with matter in an Aristotelian sense, pointing out that "the complexes resulting from the demiurge's imposition of forms and numbers on the Receptacle are not sensible substances."[27]

Experts clearly disagree about the precise meaning of "χώρα." This study takes a middle road among them concerning how to understand it as "space," without venturing to take a definitive stand, since real ambiguities appear to attend interpretation. The "χώρα" is, perhaps, really nothing in itself. It seems indeterminable except as a duad brought to unity by a limiter. It is, more precisely, the shadow attendant upon a particular kind of limiter that gives it both kinetic potentiality and the capacity to host the formation and transformation of the bodiliness proper to particular things.

If, for example, one assumes that Plato made a thorough and reciprocally exclusive identification of the Receptacle and "space," however one is to understand "space," it seems that he associated the Receptacle not only with "space," generally, but also with the space and sometimes the enabling capacity for bodiliness proper to particular things. In 58 A, explaining why the four primary kinds of body in the Receptacle do not entirely coagulate with others of their kind and so cease moving, he spoke of the particular space circumscribed by the circulation of the universe:

> [58 A] ... Once the circumference of the universe has comprehended the [four] kinds [of primary bodies], then, because it is round and has a natural tendency to gather in upon itself, it constricts them all and allows no empty space to be left over ....[28] [Reference in brackets added; other bracketed material added for clarity.]

The Receptacle, defined by a limiter (the circumference of the universe) and thus conceived as a kind of "space" or "locus," apparently having kinetic poten-

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26 R.E. Allen, "Comment on Plato's *Parmenides*," in *Plato's Parmenides*, translated with comment by R.E. Allen, Revised Edition (New Haven; London: Yale University Press, 1997), 320.

27 Lloyd Gerson, *Plotinus* (London and New York: Routledge, 1994), 109.

28 Plato *Tim.* (Zeyl) 58 A.

tiality in its role as a passive *situs* of commotion, also emerges in the *Timaeus*' notion of elements. Describing the result of the transformations of the primary bodies into one another as a change of the places proper to them within the Receptacle (the proximate agent of change is really the limiter) Plato stated:

> [57 C] ... And what is more, as they undergo these processes, they all exchange their territories: for as a result of the Receptacle's agitation the masses of each of the kinds are separated from one another, with each occupying its own region, but because some parts of a particular kind do from time to time become unlike their former selves and like the other kinds, they are carried by shaking towards the region occupied by whatever masses they are becoming like to.[29] [Reference in brackets added.]

The idea of containment seems associated with the Receptacle. In connection with the effect of sensible things upon the human soul, Plato's usage of the notion of containment, a Receptacle-like idea, in the image of the soul and its vessel, seems to suggest some relation of "vessel" to "bodiliness":

> [44 A] ... And so when certain sensations come in from outside and attack them, they sweep the soul's entire vessel along with them ...[30]

The same containment notion, in Plato's discussion of the formation of the human soul and body by the junior gods at 69 C and D and 73 C and D, carries overtones of both "space" and "bodiliness." "Bodiliness" seems to emerge as the dominant idea until one notes Plato's association of "bodiliness" with geometric shape:

> [69 C] ... They imitated him: having taken the immortal origin of the soul, they proceeded next to encase it within a round mortal body [the head], and to give it the entire body as its vehicle. And within the body they built another kind of soul, as well, the mortal kind, which contains within it [69 D] those dreadful but necessary disturbances: pleasure, first of all, evil's most powerful lure; then pains, that make us run away from what is good; besides these, boldness also and fear, foolish counselors both; then also the spirit of anger hard to assuage, and expectation easily led astray. These they fused with unreasoning sense perception and all-venturing lust, and

29 Ibid., 57 C. See Sallis, *Chorology*, 127–128 concerning the paradoxical character of the χώρα's kinetic actuality.

30 Plato *Tim.* (Zeyl) 44 A.

> so, as was necessary, they constructed the mortal type of soul. In the face of these disturbances they scrupled to stain the divine soul only to the extent that this was absolutely necessary, and so they provided a home for the mortal soul in [69 E] another place in the body, away from the other, once they had built an isthmus as boundary between the head and the chest by situating a neck between them to keep them apart …[31] [References in brackets added.]

> [73 C] … He then proceeded to mold the "field," as it were, [73 D] that was to receive the divine seed, making it round, and called this portion of the marrow, "brain." Each living thing was at its completion to have a head to function as a container for this marrow. That, however, which was to hold fast the remaining, mortal part of the soul, he divided into shapes that were at once round and elongated, all of which he named "marrow." And from these as from anchors he put out bonds to secure the whole soul and so he proceeded to construct our bodies all around this marrow, beginning with the formation of solid bone as a covering for the whole of it.[32] [References in brackets added.]

The cosmic Receptacle and the human body are not, of course, the same. They are comparable analogously, however; so it is useful to make a comparison. Kenneth Sayre has noted the aptness of the containment metaphor in connection with the Receptacle. Indeed, he has identified the following names and ideas, suggesting containment, associated with the Receptacle: Imprint bearer (ἐκμαγεῖον, 50 C); Container (δεχόμενον, 50 D, 53 A, 57 C); Winnowing Basket (πλοκάνων, 52 E), Receptacle (ὑποδοχὴν, 49 A, 51 A); Universal Recipient (πανδεχές, 51 A); Nurse (τιθήνην, 49 A, 52 D, 88 D), Foster Mother (τροφόν, 88 D), Mother itself (μητρί, 50 D, 51 A); Space (χώρας, 52 A; 52 D, 53 A); and Place (ἕδραν 52 B).[33]

An idea of containment on the cosmic plane, connoting both the place where things are and the material upon which things are impressed, arises in Timaeus' account of the creation of the four species of animal:

> [39 E] … Prior to the coming to be of time, the universe had already been made to resemble in various respects the model in whose likeness the god

31 Ibid., 69 C–D.

32 Ibid., 73 C–D.

33 Sayre, "The Multilayered Incoherence of Timaeus' Receptacle," in *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils (Notre Dame: University of Notre Dame Press, 2003). 62.

> was making it, but the resemblance still fell short in that it didn't yet **contain all the living things that were to have come to be within it**. This remaining task he went on to perform, **casting the world into the nature of its model**. And so he determined that the living thing he was making **should possess** the same kinds and numbers of living things as those which, according to the discernment of Intellect, are contained within the real Living Thing …[34] [Reference in brackets added; emboldening provided for emphasis.]

Clearly the dialogue characterizes the Receptacle variously. The ambiguities it generates merely reflect that Plato did not really succeed in explaining how corporeality arises, just that it arises, apparently in association with mathematical entities. The concept of "participation" does not appear to be available for assistance. Kenneth Sayre has emphasized that, although the *Timaeus* is the last of the Platonic dialogues to contribute meaningfully to the notion of participation, the idea does not get beyond the level of image in the text. It does not, in his opinion, provide an adequate account of the relationship between the forms and sensible things.[35]

Regardless of its exact nature as place, matter, or capacity for facilitating bodiliness, the Receptacle certainly partakes of the motions governing the world soul and its body to the extent that the Receptacle inseparably belongs to the universe. Its inseparability from the universe, however, itself seems to be a bone of contention, thereby drawing into question the claim that it participates in the larger cosmic motions. Kenneth Sayre has spoken of a precosmic state of the Receptacle, as though it were something original upon which the demiurge operated to form the universe, once he had formulated a plan for it.[36] Moreover, the *Timaeus* speaks of a motion, bound up with an Errant Cause (48 A), that is intrinsic to the Receptacle itself and not among the ordered motions of the universe. It is a chaotic motion, associated with the heterogeneity of what is in the Receptacle (52 D–E and 57 D–E). This heterogeneity causes the Receptacle to shake and, in turn, violently shake all within it (30 A and 52 D–53 A).

One may surmise that the shaking motion expresses a certain indeterminacy preventing the Demiurge from imposing a rigid order upon the moving image of eternity that is the being of the universe (37 D) and causes persuasion to be the mode of its governance (48 A). However, it is dubious that this motion, peculiar to the Receptacle, indicates some chronologically "precosmic" state.

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34 Plato *Tim.* (Zeyl) 39 E.

35 Kenneth Sayre, *Plato's Late Ontology* (Princeton: Princeton University Press, 1983), 14.

36 Sayre, "Multilayered Incoherence," 62–63, 66, 71, and 77.

Plato did assert that Being, Space, and Becoming existed even before Heaven came into being (52 D); but the statement makes more sense construed in terms of logical priority than in terms of temporal priority, considering other passages already discussed. The chaotic motion intrinsic to the Receptacle (52 D–53 A) does, after all, seem dependent upon the larger ordered motions of the universe. Indeed, Plato spoke of the Receptacle's peculiar motions as though they would cease without those larger ordered motions. For example, in connection with the question why primary bodies of similar kinds do not simply coagulate into homogeneous masses that would bring them to rest (57 E and 58 A–C), Plato discussed the "saving" effect of a condensing action by the circuit of the whole of the universe. He continued the passage at 58 A, already noted above, as positing such an action, stating:

> [58 B] ... This is why [on account of the compression of all things within the spacious receptacle of the universe by the circuit of the whole] fire, more than the other three, has come to infiltrate all of the others, with air in second place, since it is second in degree of subtlety, and so on for the rest. For the bodies that are generated from the largest parts will have the largest gaps left over in their construction, whereas the smallest bodies will have the tiniest. Now this gathering, contracting process squeezes the small parts into the gaps inside the big ones. So now, as the small parts are placed among the large ones and the smaller ones tend to break up the larger ones while the larger tend to cause the smaller to coalesce, they all shift, up and down, into their own [58 C] respective regions. For as each changes in quantity, it also changes the position of its region. This, then, is how and why the occurrence of nonuniformity is perpetually preserved, and so sets these bodies in perpetual motion, both now and in the future without interruption.[37] [References in brackets added; other bracketed material introduced for clarity.]

The chaotic motions in the Receptacle, then, while as original as the ordered motion of the universe, are nonetheless subsidiary to that ordered motion and coincident with it; so they could not be chronologically prior to the universe. The ordered motion of the universe, it seems, guarantees the very heterogeneity in the Receptacle that Plato said causes its characteristic shaking (52 D–E).

Perhaps Plato meant to suggest that, inasmuch as it governs movement, order itself contains the seeds of an indeterminacy (the Errant Cause) that is,

37 Plato *Tim.* (Zeyl) 58 B and C.

nonetheless, permitted to stray only within certain bounds just because there is an *ordo*. Such indeterminacy seems intrinsically proper to an order required to be nonrigid because the milieu it governs entails change. There can be no change without indeterminacy, and so, no order in a changing realm without provision for it. The *ordo* of change helps one to see the limits of chaos and because chaos, too, has limit, it, too, participates in the form that is the *ordo*. In itself and by itself, though, it is not real. Rather, the grammar of form and chaos, dualistic though it is, simply helps one to comprehend the kind of unified order befitting a changing realm.

Cornford's analysis of the relation between cosmic order and the disordered motions of Plato's Receptacle seem to drive in the direction, suggested above, that the Receptacle is not a negative entity but an inseparable element of the universe, ultimately subordinate to the cosmic order. He pointed out that body does not exist prior to soul in the *Timaeus* and argued convincingly that the chaos of the Receptacle is not precosmic but reflects something that is actually always present in the universe. In his opinion, Plato merely attempted to focus on it more closely by considering it in abstraction from the cosmic order. He wrote:

> Bodily motion cannot exist without a soul to cause it. The World-Soul was a creation of the Demiurge, who put reason in soul, and soul in body. When soul was fitted to body, the world, as a living creature containing soul and reason, began its 'unceasing and intelligent life for all time.' Plato clearly means that there never was a time when the body existed without the soul, or the soul without the body. We must also, I think, rule out the notion, favored by some ancient Platonists, that the soul of the world was at first irrational, having only irrational motions, and then the Demiurge endowed it with reason and reduced it to order.
>
> It follows that chaos is, in some sense, an abstraction--a picture of some part of the cosmos, as it exists at all times, with the works of Reason left out, 'such a condition as we should expect for anything when deity is absent from it'. Now if you abstract Reason and its works from the universe what is left will be irrational Soul, a cause of wandering motions [and so the Errant Cause] and an unordered element of the bodily, itself moving without plan or measure. [Material in brackets added for clarity.][38]

38 Cornford, *Plato's Cosmology*, 203.

The Receptacle, then, just because it inseparably belongs to the universe, shares in the ordered cosmic motion, with its own proper shaking movement being strictly subordinate to the circuit of the whole.

To the extent that the Receptacle and what it receives partake of the ordered cosmic motions, it follows that those motions harmonize with whatever occurs within the Receptacle. Indeed, Plato's demiurge seems to go out of his way to ensure such harmony. According to the text, he directly formed the primary bodies, fire, air, water, and earth by conferring distinct configurations through shape and number on vague characters within the Receptacle (53 B). In addition, the relations of priority among these "shapes and numbers," i.e., the elemental isosceles and scalene triangles and the geometric shapes that they generate corresponding to the primary bodies, mirror the relations of priority in the generation of the bands of soul stuff that underlie the cosmic motions. The demiurge, in other words, employed the same formative harmonic pattern to generate the ideal structures underlying body on the microcosmic level as he did to generate ideal structures underlying body on the macrocosmic plane. To see how this is true, it is easiest to begin with the macrocosmic plane.

To generate bands of soul stuff at 36 B the demiurge began with a complex monadic composition. This monad had a triadic character because of its generation at 35 A from the intermixture of same, difference, and essence (a kind of middle subsistence between the immutable and mutable). The mixture was an ordered multiplicity inasmuch as it united and related three distinct natures. Timaeus 35 A–36 D is simply the elaboration of what was implicit within the original complexity.

The original mixture underwent a separation of its parts, by means of ordered numerical divisions at 35 B–36 B reflecting one or another of its distinct constituent natures. The resulting matrix of numbers, still subsisting as a mixture, might be analogized to a chemical suspension, wherein different kinds of particles coexist separately—like sodium and chloride when salt is dissolved in water. The demiurge cut this fabric of numbers lengthwise at 36 B to generate two bands that became the circular bands at 36 C having different motions and directions *vis-à-vis* one another. One of the bands, the band of same, had primacy because it moved uniformly. The demiurge made it revolve to the right by way of the side. The other band, the band of difference, moved to the left by way of the diagonal.

The 1:2 proportion names the relationship whereby a monad determines a duad to unity with it; the relationship is harmonic just because unity is the object. For this proportion properly to designate the relationship between the triadic monad of soul stuffness and the fabric that became the duad of bands, the fabric had to have features revealing the kind of same/different complexity

already implicit in the triadic monad. The duad could not otherwise have been brought to unity, i.e., harmony, with the monad.

As Chapter 7 of this study proves in detail, the band of difference was a submatrix of numbers in the whole fabric of numbers, collected in a distinct strip of it. The nature of "different" suited this band because, when set in motion to the left by way of the diagonal (36 C) (that is, following the direction of the double proportions in the number matrix), it gave rise to a variety of fully differentiated octave chains coexisting unconfusedly within their own proper bounds.

The band of same was a different submatrix of numbers, in the whole fabric of numbers, collected in its own distinct strip adjacent to the strip that became the band of difference. It contained overlapping but wholly undifferentiated double and triple intervals.[39] Cut off from the band of difference, none of the double intervals within the band of same could be fully elaborated; so the demiurge caused this band to move laterally toward the right, i.e., in the uniform direction of the 3/2 ratio of the whole number matrix as it had existed before the division of the fabric, but, as the study will show, in the direction of its own line of triples, running in the same direction as the 3/2 ratios of the band of difference after the formation of the χ. Plato characterized this triple movement as movement according to sameness (36 C). The ratio of the motions of the bands of same and different was 3/2, reflecting their adjacent positions in the undivided fabric of the World Soul and also calling to mind the resultant fifth periodicity to which the fabric of numbers was subject.[40]

Note that the ordo of fifth periodicity is like the "essence" of the primary complex monad of soul stuffness at 35 A, defined as a middle subsistence or compromise between an "essence impartible and always subsisting according to sameness of being" and "a nature divisible about bodies" (35 A).[41] It

39 Plato's divisions at 35 B–36 A ensure that the double intervals in the number matrix he generated are continuously overlapped by independent series of triples and that triple intervals are continuously overlapped by independent series of doubles.

40 Jacque Handschin noted, particularly, that the chromatic distortions in the double intervals, arising because of the interfering fourths created when Plato divided the musical twelfths (the triples), lead to the degeneration of octave periodicity to fifth periodicity. Handschin, "Timaeus Scale," 19–24. As this study makes clear below, these independent fourths derive not only from Plato's division of the original independent series of triples (not just the original doubles) through the insertion of means but also from his creation, through that same instrumentality, of specific, wholly new independent doubles and triples continuously overlapping both each other and the original order of doubles and triples.

41 Plato *Tim.* (T. Taylor) 35 A.

represents a compromise with double motion between an original uniformity of triple motion in the soul stuffness, existing before the insertion of means at 36 A, that would have allowed double motion to subsist differentiated without distortion, and a disruptive alien order of fourths, within the triple motion, independent of the double motion and interfering with it, after the insertion of means. Fifth periodicity is uniform like the undistorted octave and derivative of it; but it is the product of difference acting upon sameness (resulting in the division of the triple motion) acting upon difference (the double motion in relation to the triple motion) and, so, is sourced in both sameness and difference. Accordingly, the duad of bands made from the fabric revealed, in their relationship, all of the complexity of the triadic monad.

In the further divisions at the macrocosmic level, the same/difference duad of bands progressed, first, to a triad and, then, to a tetrad when the band of difference was divided six times, alternately, to produce seven bands, three of which followed the triple motion and four of which followed the double motion. The ratio of the double and triple motions of the two new subsets of the band of difference was 3/2, mirroring the higher level relationship of the motions of the bands of same and difference; thus there was sameness within this new difference. At the same time, there was a new element representing the relationship between the triad of three and tetrad of four in the band of difference, the 4/3 ratio; and this represented the difference setting the limit for the sameness within difference just observed. It was a real difference, as the motions of the subsets were truly distinct vis-à-vis one another for reasons rooted in the manner of derivation of the subsets. The difference within sameness within difference arising with the tetrad completed a fourfold pattern of macrocosmic differentiation clearly described by the Decad, lending support to John Dillon's statement that "the way lies open" to equating this mathematical entity with Plato's eternal paradigm.[42]

The decadic pattern of generation claimed by this study to be so typical of the *Timaeus* can be represented by a simple dot diagram, wherein the successive horizontal rows represent the four different levels of generation. The proportions of the elements of one horizontal row to another variously represent the following intervals driving the generation: the double (second level vis-à-vis the original monad), triple (third level vis-à-vis the original monad) (source of mediation and interference between the second and fourth levels), *sesquialter* (3/2, defining the relation of the triple to the double, i.e., third to second level),

42 John Dillon, "The *Timaeus* in the Old Academy," 82–83.

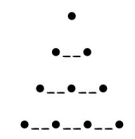

FIGURE 1
Pattern of the Decad[43]

quadruple (fourth level vis-à-vis the original monad), and sesquitertian (4/3, defining the relation between the quadruple and triple ratios, i.e., the fourth and third levels).

Note that the dot pattern simultaneously terminates in the quadruple ratio of 4:1, representing the limit of the double expansion of the monad, and the tetrad permitting the 4/3 proportion between the fourth and third rows. In this mode of termination, the overall structure of cosmic generation reflects a harmonic cosmic movement to a disdiapason (two-octave sequence) through the fourfold progression of a cosmic fourth interfering with the movement and *limiting* it.

The prominence of the disdiapason in the mode of macrocosmic generation perhaps reflects Plato's dialogue with a stream of thought relevant to the development of the Greater Perfect System ("GPS") of ancient Greek music. This mode of generation does not derive from GPS; rather, GPS, in its eventual elaboration, set the parameters for a true music on the human plane that would reflect a fully elaborated, essentially harmonic, cosmic order.

The Decad, vis-à-vis the *Timaeus*, is an "ideal number" comprised of a *tetractys*, i.e., a set of four other ideal numbers, the "figurate" representation and addition of which symbolize the Decad's ideal structure.[44] It is not a mathe-

43 The Decad was clearly associated with the octave in Pythagorean thought. As M.L. West wrote in reference to a dot representation of the Decad identical to the one set forth in this study:

> "The earliest Pythagoreans appear to have been entranced by the simple ratios of the octave, fifth, and fourth, and to have treated them as an exclusive set with a mystical significance. One of their catechistical propositions ran: "What is the oracle at Delphi?—Tetractys, which is the octave (harmonia), which has the Sirens in it." Tetractys (τες δεκάδος), "tetradizing (of ten)," was the name they gave to an arrangement of the numbers 1, 2, 3, and 4 in a triangular figure … Among other properties, it embodied the concord ratios 4:3, 3:2, 2:1; that is why it is identified with the octave." (West, *Ancient Greek Music*, 235; see, also, Burkert, *Lore and Science*, 72.)

44 Burkert, *Lore and Science*, 72; Edna E. Kramer, *The Nature and Growth of Modern Mathematics* (United States: Hawthorn Books, 1970; reprint, Princeton: Princeton University Press, 1982), 20–22; B.L. Van Der Waerden, *Science Awakening*, translated by Arnold Dresden with additions of the author (Groningen, Holland: P. Noordhoff, Ltd., 1954), 95 and 98. For Pythagoras, number was not a separate essence but inseparably united to singular realities. Ivan Gobry, Pythagore, *Collection les Grande Leçons de Philosophie*, dirigée par

matical number (e.g., a counting number, "plane" number, "solid" number, or "figurate" number) of any kind. Its image is a figurate number, a shape, that is not produced through an additive accumulation of ten equal monadic "one-place" units, e.g., 1 + 1 + 1 + ... = 10, but by an addition of the figurate images of four other ideal numbers, represented as monadic units of different kinds. One might represent the contribution of these figurate images to the composition of the Decad's figurate image by symbolizing with figurate numbers that a tetrad in relation to a triad in relation to a duad in relation to a monad are the Decad.[45] Similarly to the figurate image of the Decad, the figurate images of other ideal numbers have their own possibilities for illustrating relations that are unique to them.

Certainly, Pythagoras had known of the *tetractys*; but for him number was not a separate essence. It was something inseparably united to singular, sensible realities.[46] Walter Burkert has suggested that, for Pythagoras, numbers were spatially extended shapes.[47] Exactly what that might mean would be difficult to pin down without better evidence, but one could speculate, that for Pythagoras, reality, including the sensible realm, was comprised of living figurate numbers. These numbers accounted for the shapes of things, and so, captured their essences. Whether there was a realm, for him, beyond the sensible is unclear. For Plato, in contrast, the Decad seems to have been a transcendent pattern revealing itself as a living structure of reality through its expression in the cosmos. Accordingly, it was an "ideal number."

One might call the Decad "ten-ness." Scholars have stated that an ideal number "n," represents "n-ness" but rarely elucidate the idea altogether satisfactorily. Leo Tarán's mostly helpful explanation, for example, nonetheless, leaves room for disagreement:

Henri Hude (Paris: Editions Universitaires et Editions Mame, 1992), 37; Kramer, *Nature and Growth of Modern Mathematics*, 20 (regarding addition of figurate numbers as a union of patterns).

45 Kramer, *Nature and Growth of Modern Mathematics*, 20 (re addition of figurate numbers as a union of patterns). Therefore, the Pythagorean maxim, that "the Decad is complete at four," could not be based, as Keith Critchlow has suggested, on the simple additive idea of 1 (dot) + 2 (dots) + 3 (dots) + 4 (dots) = 10 dots. Keith Critchlow, "Foreword" in *The Theology of Arithmetic, On the Mystical, Mathematical and Cosmological Symbolism of the First Ten Numbers Attributed to Iamblichus*, translated by Robert Waterfield (Grand Rapids: Phanes Press, 1988), 9–10.

46 Ivan Gobry, Pythagore, *Collection les Grande Leçons de Philosophie*, dirigée par Henri Hude (Paris: Editions Universitaires et Editions Mame, 1992), 37.

47 Burkert, *Lore and Science*, 79.

> ... These ideal numbers are not congeries of units, for each as an idea is a perfect unity which, like every other idea, has no parts, is not derived from any principles, and is not in any sense whatever the product of any other idea or element. Each exists by itself, outside the limitations that space and time impose upon phenomenal existence. There is no single idea of number as such; ideal number is simply all the ideal numbers from two to infinity. The ideal two and the ideal three, for example, are not, respectively, two units and three units, nor is the number five the sum of two and three. These numbers are just Twoness, Threeness, and Fiveness, each being a unity which is irreducibly itself and nothing else. This means, in Aristotle's words, that each number is *essentially different* from every other number. Plato's ideal numbers are then the necessary consequence of the theory of ideas; but *qua* numbers they are really the natural numbers.[48]

This study's examination of the Decad, as it manifests in the *Timaeus*, rejects Taran's claim that, as numbers, ideal numbers are really the natural numbers. Ideal numbers, *qua* numbers, seem, in fact, for Plato, to be no such thing; they are transcendent. It would be better to say that they, in some fashion, regulate the natural numbers. One might suggest that each natural number, represents a set of binary patterns, such that the very number of binary patterns comprising the set is named by the number; further, it is the logical order of generation of these patterns that gives rise to the whole natural numbers series. Further explanation would take the discussion too far afield, but the interested reader can verify for himself why the suggestion is interesting.

One must certainly not gloss over difficulties in ascertaining exactly how ideas and numbers are connected in Plato. In the *Timaeus*, idea and ideal number seem to be one; but, as Walter Burkert noted, this question is one of the most controversial in Platonic studies.[49] The scholarly state of understanding concerning the relationship between mathematical and ideal number is also not settled. As Walter Burkert stated:

> The relation of these numbers [ideal numbers] to the mathematical numbers which are used in calculation is hard to establish. Speusippus and Xenocrates put forth different solutions for this question: the former eliminated ideas and only recognized mathematical numbers as the ground of

48 Leo Tarán, *Speusippus*, 14.

49 Burkert, *Lore and Science*, 16.

> reality; Xenocrates equated ideal and mathematical numbers. The main concern of these philosophers, however, is not to lay the foundations of mathematics, but to explain the world by means of its principles. The ideal numbers are not only the ideas of particular numbers—"twoness," "threeness," etc. but somehow govern the structure of reality: they are ideas themselves. It is not clear how this connection of ideas and numbers is to be understood, in detail. While Aristotle says simply that the ideas are numbers, Theophrastus speaks of an "attaching" (*anaptein*) of the ideas to certain numbers, and thus allows one to imagine a looser relationship. [Material in brackets added for clarity.][50]

The present study aims only to ascertain how the relations between ideal number and mathematical number appear to be working in Plato's text. No one has yet devised a better way of determining the relations indicated than to read the text. Opinions clearly abound, and there is often little necessary basis for preferring one opinion over another; so the task of this study obviously cannot be to find the definitive noncontroversial reading of the *Timaeus* from the standpoint of its number theory. It can only purport to offer the one that best accounts for as many features of the text as possible.

In the sense of "ten-ness" most compatible with the *Timaeus*, the Decad is an immutable, transcendent, pattern, multivalent in its expressions, setting, limiting, and regulating generative cycles of emanation and return. In that capacity, it is an "agent responsible for bringing all things to realization" and has an "active demiurgic role in the universe."[51] It is responsible, particularly, for bringing all things to realization in the human mind, *ergo*, its role, from the human perspective, of bringing all things, to realization.

The Pythagorean commonplace, reported by Aetius, that "the nature of Number is the Decad" may provide the key to understanding how the Decad has so important a role. This cryptic saying may mean, *inter alia*, that the Decad generates and manifests itself through mathematical number, absent which, human perception would be impossible.[52] Number itself, that is, in all of its expressions (e.g., counting, plane, solid, figurate) advances, in some fundamental way, under the governance of the idea of ten-ness. Counting number, for example, issues forth in discrete periods of ten, returning to itself at a higher

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50 Ibid., 22–23.

51 John Dillon, "The *Timaeus* in the Old Academy," 82; Kramer, *Nature and Growth of Modern Mathematics*, 19 (stating in connection with Pythagorean number theory: "pattern was the objective …"); Ivan Gobry, *Pythagore*, 37 (for the metaphysical significance of the Decad).

52 Critchlow, "Foreword," 9.

level to begin all over again in a way appropriate on a new plane. The decadic pattern sets the parameters for the behavior of counting number and, so, makes it available and perceptible to the human mind.

If mathematical number orders all things for human understanding, and humans actually perceive mathematical number because of its governance by the Decad, then the Decad simply does order the cosmos for the mind of man from a *Timaean* point of view. It sets the grammar of perception.

Human beings number and order on many different levels and planes, of course, to perceive anything as a phenomenon and to come to understanding. Comprehending requires numbering, that is, ordering, the numbers whereby one strives to reach both objects of thought and objects thought to be; and the ordering must be in harmony with the rule that makes number available to the mind in the first place. The Decad, in other words, encodes the proper, ordering scheme for analyzing anything in the unified and consistent way required to find a match with something real.[53]

To the extent that human analysis fails, it emanates from intelligence but does not return to itself in the proper manner to provide a constructive basis for further thought. This study shows that Plato's generation of mathematical numbers, at *Timaeus* 35 A through 36 D, makes a moving image of eternity, where one understands the Decad as the eternal paradigm, because it describes a universe adhering to the pattern of emanation from one and return to unity, native to the Decad.

The Decad most certainly links the microcosmic plane of Plato's account to its macrocosmic plane. The logical relations of priority among the elemental triangles and planar figures entering into the composition of bodies respect a decadic grammar with harmonic significance just like the generation of bands of soul stuff.

Plato began his account of primary bodies with a consideration of body surfaces (53 C); but he quickly focused attention on rectilinear surfaces, no doubt because the irrational rectilinear motions native to the Receptacle (cf. 34 A, 40 B, 43 B–E), made the rectilinear surface, as opposed to the spherical surface, characteristic of the Receptacle.

Understanding the account depends upon discerning how Plato managed to give the primary bodies a nature reflecting the Receptacle in its rectilinear character, while at the same time subjecting them to the harmonic order of the circuit of the whole, as must be the case for the motion of the Receptacle to be

53 The point is parallel to the one that Sayre has made in connection with the *Philebus*: "… the order of knowledge must correspond to the order of creation." Sayre, *Plato's Late Ontology*, 130.

subordinate and subject to the circuit of the whole. He noted that all rectilinear surfaces are composed of triangles and so pursued his study of plane surfaces from a triadic monad (triadic because it is a triangle) that one might call, for the sake of convenience, "the quintessential triangle" (53 C). He made a duad of the monad when he maintained that all triangles are derived from two kinds of right triangles: the scalene right triangle and the isosceles right triangle (53 D). Note that an isosceles triangle has two congruent sides; and a scalene triangle has no congruent sides. There can be "right" and "non-right" varieties of each type.

The isosceles right triangle clearly had logical priority, in Plato's account; for he stressed that there is an endless variety of scalene right triangles but only one kind of isosceles right triangle (54 A). The isosceles right triangle, then, appears to relate to the element of "same" in the "same-different" duad it forms with the scalene right triangle.

It stands to reason that differentiation from a monad, as opposed to the return, on a higher plane, to an original monad, always proceeds *first*, in order, from the element most closely corresponding to the different within a duad. Plato's manner of division of the soul stuff at 35 B and C seems to confirm this choice and suggests a general pattern. He made the first double interval (1:2), representing differentiation, first, and only afterward the first triple interval (1:3), the triad returning the monad to itself. Then he returned to the double (the nature of 2 in the 1:2 pair); and, after that, he returned to the triple (the nature of the 3 reflecting the monad, on another plane, in the 1:3 pair). He proceeded, in this manner, until he reached "27," the final number of his primary sequence. This study will start as Plato did, with the differentiation of the "isosceles right triangle–scalene right triangle" duad from the "most different" element of the pair, the scalene right triangle.

Differentiation of the "isosceles right–scalene right" duad, according to the procedure adopted in the previous paragraph, gives rise to an *isosceles right–scalene right–equilateral* triad (54 D and E). These triangles form a true triad, though the equilateral derives only from a right triangle of the scalene type, because they all participate in the right triangular idea; moreover, a real bond in the principle of congruency joins the isosceles and the equilateral, though the equilateral does not derive from the isosceles. The progression to the triad thus leads to the manifestation of a hidden potential among the scalene natures, similar to the most prominent feature of the isosceles.

The elements of the triad belong together, as well, though the equilateral triangle is not a right-angled triangle, because it, nonetheless, participates in the nature of a right triangle. As Figure 2, below, shows, the equilateral triangle is a triadic monad, composed of three pairs of duads, themselves individually

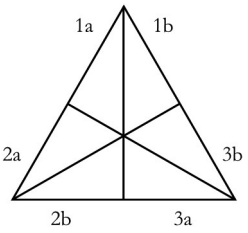

FIGURE 2
Equilateral triangle as triadic monad

consisting in two right-angled scalene triangles, each of which has sides and hypotenuses equal to one another (54 D and E). Because right-angled triangles compose it, the equilateral triangle participates in the nature of a right triangle.

The elements of the new *isosceles right–scalene right–equilateral* triad belong together most importantly because the equilateral explicates something deeply intrinsic to right triangularity, not previously evident. The three pairs of its scalene right triangles are each joined at their hypotenuses with bases oriented to the center and double apexes pointed outward to form the apexes of the equilateral.[54] All lines of construction converge at the center, revealing that right triangularity accommodates circularity. This makes sense, since the triad is the return of the monad to itself. Because the equilateral enhances the articulation of right triangularity, the progression from the isosceles right–scalene right duad, to the triad including the equilateral triangle, is a progression of sameness within difference.

The triad represents a stage of progression in which the scalene right triangular notion has not yet been fully played out in its possibilities for the construction of plane figures. For example, it can be used to construct the special kind of non-right isosceles triangle needed to make the pentagonal faces of the dodecahedron, just as it is used to construct the equilateral triangles that constitute the faces of the tetrahedron.[55] A further two-level generation is, however, required in the former case. First the special isosceles triangles must be constructed from the scalene triangle and then the pentagonal faces from the special isosceles triangles. This indicates that the pentagon is not as primary a plane figure as the equilateral triangle and also that the dodecahedron is not as

54 One could make different pairings of the scalene duads with the same result that all lines of construction converge at the center to accommodate a circular idea. These different pairings should be obvious from Figure 2.

55 *Note bene:* this does not mean that the scalene right triangle is more primary than the isosceles right triangle. They are equally original, being constructed neither of one another nor of any other kind of triangle, though the isosceles is logically prior.

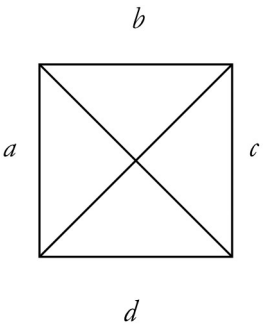

FIGURE 3
Square constructed from isosceles triangles

primary a regular solid as is the tetrahedron. In any case, because of its logical posteriority to the equilateral triangle, the pentagon could not have been constructed first. Taking first things first, the generation of a triad from the isosceles right triangle–scalene right triangle duad selected only the kind of scalene triangle needed to construct the more primary equilateral triangle, rather than the pentagon or other plane figure.

If Plato's alternating method of divisions at 36 B and C is part of the grammar of generation, then one can reason that the generation of a plane figure from the right scalene idea means that generation now has to pass to the other pole of the primary duad. A new logical progression, this time linked to the isosceles right triangle is in order. The final differentiation of "the quintessential triangle," relevant to Plato's generation of primary bodies, results in an "isosceles right triangle–scalene right triangle–equilateral triangle–equilateral quadrangle (square)" tetrad, reflecting the level of difference within sameness in difference (55 B).

It appears that the primary duad was the level of exhaustion for the specifically triangular notion deriving from the isosceles right triangle; so the square emerges from the isosceles right triangular idea, like a new duad proceeding from a new monad. Because it emerges from the isosceles right triangular idea, it shares something that was implicit within that idea. The square, as well, has an orientation toward the center, since it is composed of four isosceles right triangles with right angle apexes turned in, as in Figure 3, so that all lines of construction converge at a central point (55 B). Further, like all of the triangles at issue, it participates in the nature of the right-angled triangle, being a composition of them. By virtue of these features, the square constitutes a kind of image of the idea of the right triangle--a tetradic image, since it is composed of four isosceles right triangles. It is truly something different, as well, however, since it is no longer something specifically right triangular in itself.

The relation of the triad to the tetrad clearly repeats the original movement from sameness to difference made in the differentiation of isosceles and scalene right triangles from "the very idea of a triangle." The mirroring of that original movement is enhanced by the specific relation of the generating triad to the last element of the tetrad it generates. As noted, this last element, the square, is itself a tetradic monad (being composed of four isosceles right triangles). Just as the triad represents the monad returned to itself, so likewise, the square comprising the last element of the tetrad represents the duad returned to itself. The overall structure of the tetrad, however, is itself a double duad: isosceles right triangle–scalene right triangle/equilateral triangle-equilateral quadrangle. It reflects the primary monad–duad pair progressed to a duad of pairs; so, on a higher plane, it creates a monad/duad relationship, just as the relation between the generating triad as a unit and the tetrad as a unit reflects a monad/duad relationship on another plane.

Having completed one round of generation of plane figures, Plato stopped. He had done enough to identify the elements needed to construct his primary bodies because he had completed an entire decadic pattern of generation. The grammar is exactly the same as the grammar employed in the formation of the world soul sphere. There is a movement from a complex triadic monad (the quintessential triangle in this case) to a duad of "same–difference" to a triad of "sameness within difference" to a tetrad of "difference within sameness in difference." The basic octave proportion of 2:1, made by the initial progression of "sameness" to a duad of "same–different," repeats itself, in the pattern of generation, with the attainment of the level of difference within sameness in difference. The generation progresses to a disdiapason in the pattern of the Decad.

Reinforcing the harmonic connection, the emergence of square and triangle from an original triadic monad comprised by "the very idea of a triangle" parallels the demiurge's production of double (analogous to the square) and triple (analogous to the triangle) harmonic intervals within the triadic monad of soul stuff at 35 B. Further, the last element in the pattern of generation, the equilateral quadrangle (square) of the tetrad, allows a 4/3 ratio vis-à-vis the generating triad, similarly to the last element in the pattern of generation of the musical intervals. Just as the quadruple and triple intervals, in that case, were in the proportion of 4/3, so, in this case, the sides of the square and the sides of the triangle form a 4/3 proportion. In addition, it takes four isosceles right triangles to make one equilateral quadrangle (square); so the magnitude of the square, as the fourth element of the tetrad, compared to the magnitude of the triad (*isosceles right–scalene right–equilateral* triad) generating it, might be represented as a 4/3 proportion. The number of angles of the figures can similarly be compared.

The *ordo* of generation of the primary bodies reflects the very same decadic and harmonic grammar governing lower level relations among planar figures. Plato constructed all of the solids with triangular faces prior to the solid with the square face (54 D–55 C). He clearly proceeded analogically. Just as triangles are logically prior to the square, so solids with triangular faces are prior to solids with square faces; further, the solids with triangular faces arise in the order of least to greatest complexity. For example, Plato's order of progression of solids is as follows: tetrahedron, octahedron, icosahedron, and cube, corresponding, respectively, to fire, air, water, earth (55 D–56 A).

One cannot just rest with an analogy to the planar dimension to comprehend the *ordo* on the level of the solids because the cube does not logically arise from the preceding series of triangular solids in precisely the same way that the square logically arises from a preceding series of triangles among the planar figures. There are no isosceles elements among these triangular solids as there were in the corresponding triad on the planar level. The triad of triangular solids is an utterly scalene triad; but the single quadrangular solid rests upon the isosceles element alone (54 C).

Some mediating idea or set of mediating ideas is required to explain why the octahedron and icosahedron are prior to the cube. Otherwise generation is skewed in favor of a scalene element, in seeming defiance of the alternating procedure that ensures a duad's uniform differentiation on a given conceptual plane. In addition, some umbrella concept or set of umbrella concepts is needed to explain the conceptual rise from the planar to the solid level; the inclusion of all four kinds of solids in a series, given their disjunct character; and the restriction of the series to a specifically discrete set of four solid objects.[56]

It is easiest to approach the mediating notions first. Since the only mediating idea between the isosceles and scalene elements dividing the solids is "the very idea of a triangle," that has to be one mediating notion. For the purpose of understanding the progression of solids in this discussion, in connection with that mediating idea, "most triangular" will mean most triangular in outward form and "least triangular" will mean least triangular in outward form. The other mediating notion, degree of complexity, is suggested by the *ordo* in

56 Plato himself noted the seemingly disjunct character of the series, stemming from the disjunction in its triangular components. Three of the primary bodies, those composed of the scalene elements, can be resolved and compounded into one another; but the fourth cannot, precisely because it is composed of isosceles and not scalene elemental triangles (54 B and C).

which Plato lists the triangular solids (54 D–55 C). One can measure complexity objectively by registering the combined number of vertices, faces, and edges of each of the solid figures.

An overarching umbrella concept for all of the relevant solids arises if one reflects upon a new idea that the progression to a tetrad among the planar figures made possible. Plato actually built all of his four primary bodies from only two kinds of plane figures: the two composite figures of the equilateral triangle and the equilateral quadrangle. Since these composite plane figures ultimately derive from the two simplest kinds of plane figures, however, he spoke truly of building the solids from the primary scalene and isosceles triangles (54 B and C). The fact that complex planar figures have to be constructed before a solid figure becomes possible shows that the idea of a solid figure is logically posterior to the idea of a plane figure; further, it is an idea that becomes possible, rationally, only because of ordinary developments on the planar level.

The "equilateral" notion is the ordinary development on the planar level that first permits the idea of the solid figure, in addition to the plane figure; for equilateral symmetry is not confined to the planar dimension. In contrast, "triangle" and "square" are specific references to "plane" objects. The equilateral notion arises because the final tetrad in the plane progression, i.e., "Isosceles right triangle–Scalene right triangle–Equilateral triangle–Equilateral quadrangle," can be reconceived as the following duad of duads: Nonequilateral plane figure–Equilateral plane figure. The first member of this superduad preserves an internal *isosceles–scalene* same-difference distinction, and the last member preserves a double *triangle–quadrangle/**scalene–isosceles*** same-difference distinction. The *scalene–isosceles* inversion specially emphasized in the last description reflects the emergence of the equilateral idea from the differentiation of the duad, as demonstrated, above, in the discussion of the plane figures.

Clearly, Plato's four primary bodies belong in one series because they are all equilateral solid figures; however, they are not the only equilateral solid figures. There is one more—the dodecahedron; but this fifth body naturally arises later in the *ordo* of generation, as the culmination of the equilateral solid idea. It is not, therefore, a primary equilateral solid object. Plato did not designate his primary bodies as "primary" for nothing. He reserved the dodecahedron, possibly, for the body of the universe, because it is the most complex possible object.[57] Being complex, it obviously cannot be built up without more primary constituents. It is logically the shape most appropriate to the *body* of the universe because it most closely approximates the shape of the soul sphere, without

57 Cornford, *Plato's Cosmology*, 218.

TABLE 1 *Ordo* and relations of the primary bodies

Tetrahedron (fire)	**Octahedron (air)**	**Icosahedron (water)**	**Cube (earth)**
Four vertices	Six vertices	Twelve vertices	Eight vertices
Four faces	Eight faces	Twenty faces	Six faces
Six edges	Twelve edges	Thirty edges	Twelve edges
14 combined features	26 combined features	62 combined features	26 combined features
C 1	C 2	C 3	C 2
T 4	T 3	T 2	T 1

being a sphere, so as to preserve the same–difference distinction of soul and body. "Primary," then, is the second umbrella concept (the "equilateral" notion is the first) needed to ensure the integrity of Plato's *ordo* of solids.

The mediating ("triangularity" and "complexity") and umbrella concepts ("primary" and "equilateral") delineated above allow one to discern the grammar describing Plato's generation of the primary bodies. Classifying the primary equilateral solids according to the mediating notions identified engenders a decadic grammar sufficient to explain the *ordo:* tetrahedron, octahedron, icosahedron, and cube, i.e., fire, air, water, and earth. The table above lays out the relevant distinctions. For purposes of the comparisons it contains, please note that C 1–C 3 is the order of least complex to most complex; and T 1–T 4 is the order of least triangularity of shape to most triangularity of shape.[58] A few observations about the fitness of this mediating scheme are in order before laying out the decadic pattern appropriate to the generation of the primary solids from the equilateral idea.

The symmetries of the proportions of triangularity to complexity, triangularity to triangularity, and complexity to complexity in and among elements along the vertical, horizontal, and diagonal axes in the table above are the same as those that emerged for the musical scale. This, of course, reflects the fact that Plato gave his reader exactly four elements (fire, air, water, earth) to compare in terms of shared features (triangularity and complexity) classifiable to indicate degree by the numbers 1, 2, 3, 4. If those shared features could not be measured

58 See Robert Lawler, *Sacred Geometry*, (New York: Crossroad, London: Thames & Hudson, 1982), 97 (for the geometrical data). Triangularity/complexity indexes visibility/tangibility. Note that the dodecahedron, if measured on the same basis, would be properly classified C3T1. It shares the exact complexity level of the icosahedron but not its triangularity. Like the cube it has no triangular face and so is similar in triangularity to that figure.

by degree, there would be no genuine progression; further, the characteristic harmonies of a fourfold progression would not appear. Plato did not provide his readers with a fourfold progression of solids gratuitously. Part of his purpose was to show that a *bona fide* progression to four always works the same way harmonically.

Upon inspecting Table 1, one can see that the ratio of triangularity to complexity for the tetrahedron is the quadruple proportion of the disdiapason, while the ratio of complexity to triangularity for the cube is the octave proportion. Similarly, the vertical proportion of the degrees of these qualities in the cases of both the octahedron and the icosahedron is the 3/2 ratio of the fifth, though they possess that proportion in the reverse orders, respectively, of T:C and C:T compared to one another. The identity proportion in the relation of triangularity to complexity as between these two solids characterizes the cross diagonals between them, as well. Considered jointly, these relations strongly support the mediating status of the octahedron and icosahedron.

The diagonal cross proportions in the degrees of complexity and triangularity, as between the tetrahedron and octahedron, are the triple and octave proportions, though in reverse order, while the horizontal proportions between like qualities for these same figures are, though not in the same order, the octave proportion and the 4/3 ratio of the fourth. A further comparison of the respective vertical relations of triangularity to complexity for these two figures, yielding the quadruple proportion and the sesquialter relation of the fifth, shows that the progression from the tetrahedron to the octahedron yields all essential ratios relevant to the decadic pattern for the disdiapason. Thus, the same harmonic intervals govern the progression of solids as govern the evolution of the scale.

The diagonal cross proportions in the degrees of complexity and triangularity as between the icosahedron and the cube are the triple proportion and the identity proportion, while the horizontal proportions in the degrees of like qualities are the sesquialter proportion of the fifth and the octave ratio. A comparison of the respective vertical relations of complexity to triangularity for these two element yields the sesquialter proportion of the fifth and the octave ratio. Comparing all relevant relations among degrees of complexity and triangularity of the first two figures with those of the last two, one notes that the relation between the tetrahedron and octahedron encompasses all relevant harmonic proportions except the identity proportion; and the relation between the icosahedron and the cube encompasses all except the quadruple proportion and the sesquitertian proportion of the tetrachord. However, in exhibiting the identity proportion from which all the other proportions spring, the relation between the icosahedron and cube implicitly possesses the sesquitertian

ratio (4/3) and the quadruple proportion. Likewise, in possessing the quadruple proportion that completes the disdiapason, the relation between the tetrahedron and octahedron implicitly possesses the identity proportion. Consequently, the relation between icosahedron and cube is equivalent to the relation between tetrahedron and octahedron.[59]

The diagonal cross proportions in the degrees of triangularity to complexity of the two outermost figures are identity and the 2:1 proportion of the octave. The horizontal proportions in the degrees of like qualities are the octave and the quadruple ratio of the disdiapason in reverse order. These relations establish the nodes of a disdiapasonic expansion, i.e., its starting point, its midpoint and its termination. A "disdiapason" mediated by reciprocal sesquialter relations in the degrees of complexity and triangularity of the two middle elements, i.e., the octahedron and icosahedron, links the first and fourth solids.

The diagonal cross proportions of the degrees of complexity and triangularity of the first and third figures are the octave ratio and sesquitertian proportion of the fourth, though in reverse order, while the horizontal proportions in the degrees of like qualities are the triple proportion and the octave, though, again, in reverse order. A comparison of the vertical relations, as well, yielding the quadruple and sesquialter proportions, in reverse order, completes all essential harmonic intervals of the octave.

The diagonal cross proportions of the degrees of complexity and triangularity of the second and final figures are the octave ratio and the sesquialter ratio of the fifth, though in reverse order, while the horizontal proportions in the degrees of like qualities are identity and the triple. A comparison of the respective vertical relations of complexity and triangularity for these two elements yields the sesquialter proportion of the fifth and the octave ratio, though in reverse order. The identity proportion in the degrees of complexity of the octahedron and cube locates each as the end-term of a duad.[60]

The fourfold symmetries are as pervasive. Zigzagging through the chart from bottom left to top right of the diagram, one sees that first and fourth elements are linked, through other elements, by an octave proportion, an identity proportion, and another identity proportion, with the twoness of the identity

59 This is as it should be, since the tetrahedron and icosahedron can be generated within each other by connecting the midpoints of their respective faces, and the octahedron and cube are related in the same fashion. Lawler, *Sacred Geometry*, 55. Observe the unique character of the dodecahedron, not among the primary solids, in this respect. It can be used to generate a copy of itself within itself by the method discussed. Ibid.

60 See immediately preceding note for their further relation.

PES

C1T4–C2T3

C1T4–C2T3–C3T2

C1T4–C2T3–C3T2–C2T1

FIGURE 4
Decadic pattern expressing relations among the primary solids

proportions explicating the octave idea in the first link. Working in reverse from the bottom right to the top left, one sees that the first and fourth elements are linked by a triple proportion, an identity proportion, and a triple proportion, with the twoness of the triple proportions in relation to the identity proportion recalling the octave proportion. In essence, then, the first fourfold diagonal cross symmetry expresses two octave proportions and the second expresses two triple proportions that, in relation to each other, return to the octave idea. The duplication signifies Plato's interest in constructing a relation between the first and fourth elements governed by mutually interpenetrating double and triple proportions under the umbrella of an overarching octaval idea.

The overall decadic grammar for the progression of solids from the equilateral idea works as follows. The equilateral idea spawns a duad C1T4–C2T3, differentiated on two levels by a minimal same–difference distinction. This duad spawns a triad C1T4–C2T3–C3T2. This is a true triad of elements belonging together because, jointly, its elements reflect an ascending degree of complexity, from left to right, beginning with a monad, and an ascending degree of triangularity from right to left proceeding from a duad. Because this triad itself actually contains two interpenetrating sub-triads (complexity 1-2-3 and triangularity 2-3-4), it internally mirrors the monad/duad distinction. At the same time, the two triads vis-à-vis each other express sameness in difference because they jointly describe (scalene) triangular equilateral solids. This triad spawns another element C2T1 that is a true fourth because it issues in a new kind of equilateral solid grounded in a different version of the triangular idea, while beginning a new order of complexity proceeding from a duad (scalene equilateral solid/isosceles equilateral solid), instead of a monad.

Note that the cube has to begin a new order of complexity with the change in the character of its triangularity (from scalene triangularity to isosceles triangularity) because change starts a new series. Further, it has to start from C2 and not C1 because the tetrahedron is objectively simpler. T1 is a proper characterization of a cube because, although it does not look like a triangle, its sharp corners are directly due to its triangular components. The cube genuinely belongs in the tetrad because it shares in the overarching equilateral and triangular ideas and also has a cognate level of complexity vis-à-vis the solid, the

octahedron, the mediator of the tetrahedron and the icosahedron.[61] It does not, however, belong to the tetrahedron-octahedron-icosahedron triad because of its truly duadic character. It is duadic in the resultant "different" face of its triangularity, as well as duadic in its order of complexity; so the fourth element arrives at the level of generation that fills out a disdiapasonic interval, described above as difference within sameness in difference. The overall decadic pattern expressing all of the relevant relations and ratios, then, is as specified in Figure 4, where "PES" indicates Primary Equilateral Solid.

The concepts derived from Plato, in this analysis, for mediating the primary solids relate analogically to the visibility and tangibility that Plato says are necessary for bodies (31 B) as follows. Visibility is indexed by the triad; and the true fourth, tangibility, in the sense Plato associates with earth, arises only with the final element of the tetrad. Sharp simple objects are the most visible because they are clearest and best defined. More complex and so, more blunted or roundish objects are less visible because they are less definite.

Tangibility is a function of stability. The cube is constructed of the stablest kind of right triangle, the isosceles. As Plato pointed out, there is but one kind of isosceles right triangle (54 A). Compared to the scalene triangle entering the composition of the other solids, it is quite impervious to change. Because it is the only isosceles (and, indeed, right isosceles) equilateral solid, earth has tangibility as its special quality. Note that it has little visibility because of its level of complexity, but it has some visibility because it participates in the idea of a triangle by possessing triangular components.

It should not be surprising that a "decadic" analysis nicely accounts for the *ordo* of progression of the primary bodies in Plato's *Timaeus* in view of the Pythagorean influences at work in the text. As Edna Kramer noted, concerning the Decad, in her work on the nature and growth of mathematics:

> It was considered the sacred "fourfoldness," because its four rows represented a totality of the reason and justice of man and woman, and also, in Pythagorean metaphysics, cosmic creation through the four basic elements—fire, water, air, and earth.[62]

Clearly, the approach to the "God-given method," in this analysis, differs from Mitchell Miller's.[63] He has provided an interesting analysis of the mediation

61 See immediately preceding note for the further relation of the octahedron and cube.

62 Kramer, *Nature and Growth of Mathematics*, 20.

63 The notion of the "God-given method" actually comes from *Philebus* 16 C–D and 18 B–C,

among the primary solids based on the ideas of visibility and tangibility but has not rooted his analysis as firmly in the geometrical characteristics of the primary bodies as is evident above.[64] He has begun, instead, from a generic idea of "the bodily" that gives rise to a duad of visibility and tangibility. He has rightly assigned these elements to fire and earth as the extremes of sameness and difference, in view of Plato's alignment of tangibility with earth and his claim that nothing is tangible without earth. In characterizing these contrasting ideas as the duad ordering the primary solids, instead of understanding them as labels appropriate as the result of a more original order on a more fundamental plane, he has accorded earth an immediate place that, in the above scheme, is appropriate only to the level of difference within sameness in difference.[65]

This analysis, unlike Miller's has proceeded on the basis that elements as extreme as fire and earth cannot stand in relation to one another until mediating features are in place between them. Such an assumption seems consistent with Plato's account. Plato reminded his reader why intermediaries are required as follows:

> [31 B] ... But since it is necessary that a corporeal nature should be visible and tangible, and since nothing can be visible without fire, and nothing tangible without something solid, and nothing solid without earth—hence the Divinity, beginning to fabricate, composed the body of the universe from fire and earth. But it is impossible for two things alone [31 C] to cohere together, without the intervention of a third; for a certain collective bond is necessary in the middle of the two. And that is the most beautiful of bonds which renders both itself and the natures which are bound remarkably one. But the most beautiful analogy naturally produces this effect. *For when either in three numbers, or masses, or powers, as is the middle to the last, so is the first to the middle; and again as is the last to the middle, so is the middle to the first: then the middle becoming both first and last, and the last and the first passing* [32 A] *each of them into a middle position, they become all of them necessarily the same, as to relation to each other. But, being made the same with each other, all are one*. If, then, it

as Socrates' epithet for dialectic. The epithet suggests that the proper *ordo* of dialectic is itself governed by the Decad in Plato.

64 Mitchell Miller, "The *Timaeus* and the 'Longer Way,'" 33–39.

65 Ibid (see figures on 34–35, particularly). In seeming contradiction to himself, Miller has made an observation actually parallel to the point made above in a previous portion of his text. He has noted that the *Philebus*' Unlimit is a continuum of the more and the less. Ibid., 26. The terms characterizing the opposite poles are clearly comparative, not superlative.

> were necessary that the universe should be a superfiecies only, and have no depth, one medium would indeed be sufficient, both for the purpose of binding itself and the [32 B] natures which it contains. But now it is requisite that the world should be a solid; and solids are never harmonized together by one, but always with two mediums. Hence the Divinity placed water and air in the middle of fire and earth, and fabricated them *as much as possible* in the same ratio to each other, so that fire might be to air as air to water; and that as air is to water so water might be to earth. And from this conjunction and composition he rendered the world visible and tangible. [32 C] *Hence from things of this kind, which are four in number, it must be confessed that the body of the universe was generated through analogy*, conspiring into friendship with itself from their conjunction, and so aptly cohering in all its parts, as to be indissoluble except by its artificer, who bound it in their union and consent.[66] [References in brackets added; italics introduced for emphasis.]

Intermediaries are required, as the above passage suggests, because unity, being intrinsic to perfection in its implications for self-sufficiency, is that by analogy with which the universe was generated; and unity is intrinsically characterizable as a whole having three terms, beginning, middle, and end. The wholeness of the universe entails that no part of what was originally visible, but moving in a state of confusion and disorder, remain external to the world. Because the universe is a solid, the analogy to unity made by its wholeness requires two middle terms in a proportion of four terms. A proportion progresses from beginning to end through the middle and not otherwise; so the middle terms arise before the end term.

Consequently, the duad with which to begin is a duad between fire and air. Generation advances from lesser degrees of difference to greater degrees of difference; it does not immediately beget the maximum level of difference. Miller has not observed such a rule; so, he has had to make difficult claims about the degrees of tangibility of air and water, in spite of Plato's insistence that nothing is tangible without earth. One might argue that Miller's analysis preserves a more perfectly exact proportion among the elements than the analysis provided above, so that fire is precisely to air, as air is to water; and air is to water, as water is to earth: p^3: p^2q: pq^2: q^3 (where p is visibility and q is tangibility).[67] However, Plato did not tell his reader in 32 B that the Demiurge made the pro-

66 Plato *Tim.* (T. Taylor) 31 B–32 C.

67 Miller, "The *Timaeus* and the 'Longer Way,'" 34–35.

portions exact. He went out of his way to specify that he made them as exactly analogous *as possible*. The disjunction of the scalene and isosceles elements in fire and earth cannot be completely resolved in the way that 32 B suggests; but it does not matter, as this study has shown above, because the octaval *harmonia* is preserved anyway. The mediation of the primary solids cannot be achieved between the extremes of a visibility–tangibility duad as Miller has generated it because earth, alone, is tangible in Plato's account.[68]

Whatever may be the exact nuances of the "God-given method," it cannot be denied that a basic decadic pattern of generation repeats itself again and again throughout the *Timaeus*. The analysis above has sufficiently displayed how such a pattern links the macrocosmic level of soul sphere bands and the microcosmic level of Plato's genesis of geometric solids.

Yet another ostensibly decadic scheme governs the genesis of the four kinds of living creatures, heavenly, winged, water-dwelling, and earth-dwelling (39 E–40 A). The reader may work out this simpler case by himself or herself, if he or she is so inclined. The correspondence of the four categories of animals to the four elements, certainly implies a similar structure of genesis, though the demiurge himself makes directly only the creatures of the first class (40 A–B, 41 A–41 D). It appears, then, that all generated things participate in a harmonic circulation of the whole that proceeds in accordance with a decadic pattern.

Structural similarities at different levels of the *Timaean* universe should not be surprising. They only stand to reason since both the cosmic motions and the primary elements are encompassed within the same moving image seeking, in its totality, to express the one idea of "All Perfect Animal" (cf. 39 E and 69 C). Accordingly, in 56 C, Plato explicitly divulged that the demiurge adjusted the numbers, motions, and powers of the primary bodies in due proportion; and he made clear, in his reference to "the bands of analogy and symmetry" to which all sensible natures are connected as much as possible at 69 B, that this due proportion also relates to the cosmic motions:

> [69 B] ... Indeed, as we asserted towards the commencement of our discourse, when all sensible natures were in a disordered state of subsistence, Divinity rendered each commensurate with itself and all with one another *and connected them as much as possible with the bands of analogy and symmetry*. For then [at the time of the disordered state of subsistence] nothing participated of order except by accident; nor could any

68 The overriding point of Miller's paper, namely, that the "God-given method" governs generation in Plato's *Timaeus*, seems unobjectionable. The above analysis goes farther than his has, however, in suggesting that correct dialectic is itself governed by the Decad.

> thing with propriety be distinguished by the appellation which it receives at present, such for instance as fire, water, and the rest of this kind.[69] [Reference in brackets added; other bracketed material introduced for clarity; italics provided for emphasis.]

One might also expect, as a matter of course, that the influence of the condensing motions of the circuit of the whole, noted in 58 A–C, on the motions of the things in the Receptacle, would ensure analogous structures at different levels of the universe. Plato indicated that the cosmic motions work in cooperation with the agitation of elements in the Receptacle to ensure both a mingling that produces an infinite variety and a condensing that accounts for the coagulation of larger structures (56 C, 58 A–C). He even made the two interacting forces of agitation and circulation, intrinsic to the motion of the universe, responsible, in the end, for the whole gamut of human passions and disease (61C–80 E and 81 B–D). They are effects of that motion on and within the body, an especially tumultuous place just because it exists at a level within the universe subject to all of its varied motions (43 A–44 B); but even they reflect structures similar to other generated things in the universe.

Plato represented the passions of the body, for example, as being analogous in quality to the characteristics of the elements. One speaks of sharp pain, burning pain, by analogy with the sharpness of fire, because the element fire generates patterns of motion within a subject similar to the pattern of motion entering into its own constitution and giving it its peculiar character (61 D–62 C; 64 D, 65 B–68 D). In 90 D, however, he indicated man's antidote to the tumultuous motions intrinsic to the Receptacle by making clear that human happiness is subject to a proper regard for the cosmic motions. Man cannot win the fulfillment of the best life unless he harmonizes the circuits and revolutions of his own thought with the circuits and revolutions of the universe.

The circular motions of the universe, then, affect all that is part of it. The analysis above has shown that these motions are not arbitrary and chaotic. They, appear, on the contrary, to be intrinsically mathematically and harmonically ordered. Plato, in fact, went out of his way to characterize the form of the universe in mathematical and harmonic terms and, moreover, to root that form in a divine paradigm. For example, he stressed the spherical shape with which the father had endowed the universe as the form that made it most similar to the Demiurge.

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69 Plato *Tim.* (T. Taylor) 69 B.

> [33 A] … [H]e fabricated the universe one whole, composed from all wholes, perfect, undecaying, and without [33 B] disease. He likewise gave to it a figure becoming and allied to its nature. For to the animal which was destined to comprehend all animals in itself, that figure must be the most becoming which contains within its ambit all figures of every kind.[70] Hence he fashioned it of a spherical shape, in which all the radii from the middle are equally distant from the bounding extremities; as this is the most perfect of all figures, and the most similar to himself. For he considered that the similar was infinitely more beautiful than the dissimilar.[71] [References in brackets added; other bracketed material introduced for clarity.]

In addition, suggesting a link between musical and mathematical form, when describing the soul's infusion into the spherical form, as the means by which the universe was able to converse with itself in a friendly, harmonious way with itself, and so, be a happy god, Plato wrote:

> [33 C] … [T]he universe affords nutriment to itself through its own consumption; and being artificially fabricated, suffers and acts all things in itself, and from its own peculiar operations. For its composing artificer considered that it would be much more excellent if sufficient to itself, than if indigent [33 D] of foreign supplies. But he neither thought that hands were necessary to the world, as there was nothing for it either to receive or reject; nor [34 A] yet feet, nor any other members which are subservient to progression and rest. For from among the seven species of local motion [forward, backward, right, left, up, down, rotation] he selected one [rotation], which principally subsists about intellect and intelligence, and assigned it to the world as properly allied to its surrounding body. Hence, when he had led it round according to *same*, in *same*, and in itself, he caused it to move with a circular revolution. But he separated the other six motions from the world, and framed it void of their wandering progressions. Hence, as such a conversion was by no means indigent of feet, he generated the universe without legs and feet. When, therefore, that God who is a perpetually reasoning divinity [the father] cogitated about the God who was destined to subsist at some cer-

70 The dialogue is referring to the fact that any polygonal solid can be encompassed within a sphere, even one with infinitely many faces. The limit toward which the idea of the polygonal solid tends is the sphere, just as the limit toward which the idea of a polygon tends is the circle.

71 Plato *Tim.* (Taylor) 33 A–B.

> tain period of [34 B] time [the universe] he produced his body smooth and equable; and every way from the middle even and whole, and perfect from the composition of perfect bodies. But, placing soul in the middle of the world, he extended it through the whole: and besides this, he externally invested the body of the universe with soul; and causing circle to revolve in a circle, established the world one singular, solitary nature, able through virtue to converse with itself, indigent of nothing external, and sufficiently known and friendly to itself. And on all these accounts he rendered the universe a happy God ...[72] [Bracketed text added for clarity.]

Likewise, he attributed the capacity of the universe to be a living image of eternity to ordering numbers within the soul that contained it, as these are the generated and generative temporal expressions of the "eternal gods" (37 C). In other words, the soul's ordering of the universe clearly imitates some kind of mathematical paradigm on the divine plane.[73] As Plato stated:

> [37 C] ... But when the generating father understood that **this generated resemblance of the eternal Gods** moved and lived, he was delighted with his work, and in consequence of this delight considered how he might fabricate it still more similar to its exemplar. Hence, as that is an [37 D] eternal animal, he endeavoured to render this universe such, to the utmost of his ability. The nature indeed of the animal its paradigm is eternal, and this it is impossible to adapt perfectly to a generated effect. **Hence he determined by a dianoëtic energy to produce a certain movable image of eternity: and thus, while he was adorning and distributing the universe, he at the same time *formed an image flowing according to number, of eternity* abiding in one; and which receives from us the appellation of time.** But besides this, he fabricated [37 E] the generation of days and nights, and months and years, which had no subsistence prior to the universe, but which together with it rose into existence. And all these, indeed, are the proper parts of time. But the terms *it was* and *it will be*, which express the species of generated time, are transferred by us to

72 Ibid., 33 C–34 B; Plato *Tim.* [Bury] 34 A (n1), 43 B–C (&n1).

73 In the mathematical tradition of philosophizing, the distinction between generated and generative numbers of the temporal order and the numbers in the exemplar is expressed as the difference between scientific (mathematical) and divine (ideal) number. See, e.g., Nicomachus of Gerasa, *Introduction to Arithmetic* (trans. Martin Luther D'ooge, in *Nicomachus of Gerasa*, University of Michigan Studies Humanistic Series, vol. 16 [Ann Arbor: University of Michigan Press, 1938]) 1.6. The *Timaeus* may be among the first early texts to suggest the distinction by implication.

> an eternal essence, through oblivion of the truth. [38 A] For we assert of such an essence that it *was, is*, and *will be;* while according to truth the term *it is* is alone accommodated to its nature. But we should affirm, that *to have been* and *to be hereafter* are expressions alone accommodated to generation, proceeding according to the flux of time: **for these parts of time are certain motions**. But that which perpetually subsists the same and immovable, neither becomes at any time older or younger; neither has been generated in some period of the past, nor will be in some future circulation of time; nor receives any circumstance of being, which generation adapts to **natures hurried away by its impetuous whirl.** ***For all these are nothing more than species of time imitating eternity, and circularly rolling itself according to number***. Besides this, we likewise frequently assert that a thing [38 B] which was *generated*, IS *generated: that what subsists in BECOMING TO BE IS in generation;* that what WILL BE, IS TO BE; and that NON-BEING IS NOT: no one of which assertions is accurately true.[74] [References in brackets added; emboldening introduced for emphasis.]

Time, Plato informed his reader, in the passage quoted immediately above, is an image of eternity that flows according to number, such that its parts are certain motions. Surely, there is not a more fitting definition of music. The ordering numbers according to which time flows in music are, of course, all significant intervals in the musical scale; but it is clearly the harmonic pattern of cosmic motion that both ensures analogical structures at all levels within the sphere of the universe and constitutes the real bond among those structures proper to a unified image of eternity.[75] Accordingly, inasmuch as music is a moving expression of time, it is the perfect natural symbol for time imitating eternity.[76]

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74 Plato *Tim.* (T. Taylor) 37 C–38 B.

75 The *Timaeus* speaks largely of harmonic intervals. Augustine's *De musica*, an interesting text for comparison, primarily treats of rhythmic intervals. Analysis of *De musica*, in juxtaposition with this study's analysis of the *Timaeus*, reveals that the same grammar governs the generation of both kinds of interval. The passage quoted above from the *Timaeus*, in the obvious connection between the image of time flowing according to number and the notion of rhythm, shows that the *Timaean* conception of the cosmos is both harmonic and rhythmic. Time can be measured precisely because it flows according to number. Indeed, rhythm is the basis for the measure of time. A rhythmic notion of being emerges especially clearly in Augustine's *de musica*. Whether one interprets the *Timaean* cosmos as a rhythm or a *harmonia* will depend upon one's perspective. If one considers generated being, in its completion, and thus from the perspective of eternity, it will be a *harmonia*; but if one considers it in its process of being generated in time, it will be a *rhythm*.

76 The notion that Plato's account of the world soul involves constructing a universal *har-*

In view of this fittingness of a musical paradigm for time imitating eternity and the rootedness of the musical ratios in the Decad, the idea that "All Perfect Animal" or eternal animal is the Decad invites proof of plausibility. The analysis passes to that proof in a detailed examination of Plato's construction of the world soul.

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monia gains support from others of Plato's texts in the idea that *harmonia* is intrinsic to the realm of created being. It is needed to unite the like and unlike elements implicit in the commixture constituting it. In the *Philebus*, for example, the interlocutors explicitly recognize "commixture" as the appropriate locus for *harmonia.* (*Philebus* 31 C–D); further, in their search for the human good, they attempt, above all, to find a *harmonia* between pleasure and the life of intellect that achieves the happy life.

CHAPTER 2

Plato's Construction of the World Soul: the Text as a Number Generator from 35 A to a Conundrum in 36 B

Timaeus 35 A–36 D evidences Plato's profoundly harmonic conception of being. The intervals he defined as he described the Demiurge's creation of the world soul reflect a movement to a perfect disdiapason through the fourfold progression of reciprocally co-existent Dorian and Lydian transpositions of the fourth within the ambit of a decadic pattern.[1] The text thus expresses an "octaval *harmonia* of being" and appears to suggest an identification of "All Perfect Animal" with the Decad.

A simple derivation from the text of the tone numbers needed to articulate a perfect disdiapason may not itself be enough to convince a skeptical reader that an equation of "All Perfect Animal" with the Decad is plausible as a matter of Plato's intention. In fact, such a reader would point out that the harmonic expansion that the text seems to support goes beyond such a bound, ending, in the opinion of most commentators, at an apparently arbitrary limitation determined by the proper registers of the Greek male voice.[2] If "All Perfect Animal" were the Decad, he would argue, then the octave expansion would simply end with the articulation of a perfect disdiapason.

The present analysis will show that the numbers yielded by Plato's definition of intervals actually articulate a primary scale consisting in a decad of complete octaves, not the four octaves and a "major sixth" or four octaves, tone, and a fifth of standard *Timaeus* interpretation.[3] Two of those octaves are perfect, being uninvaded by chromatic elements. The other eight are complete but increas-

1 See the glossary to this study for the relationship between the Dorian and Lydian *tonoi* within the *Timaeus* scale and the arrangement of the scale from the perspective of each *tonos*.

2 Cornford, *Plato's Cosmology*, 67–68 (regarding the arbitrariness of such limitations).

3 Ibid., 66–72; Brisson, *Le même et l'autre*, 321–322; Brisson and Meyerstein, *Inventing the Universe*, 35; McClain, *Pythagorean Plato*, 61; Handschin, "Timaeus Scale," 15, 26–27; Proclus *in Timaeum* 3.2.192.1–14.; Calcidius *Platonis Timaeus* (Interprete Chalcidio cum eiusdem commentario, rescensuit Dr. Ioh. Wroel [Lipsiae: In Aedibus B.G. Teubneri, 1876]), 3.45 and 4.46; Theon of Smyrna *Mathematics Useful for Understanding Plato* (translated from the French edition by J. Dupuis and Robert and Deborah Lawlor, edited and annotated by Christos Toulis and others with an appendix of notes by Dupuis, a copious glossary, index of works, et cetera, Secret Doctrine Reference Series [San Diego: Wizards Bookshelf, 1979]) 13a.

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ingly invaded by chromatic elements. Octave order degenerates after ten complete octaves because chromatic interference by independent fourths deriving from Plato's insertion of means at 36 A destroys the integrity of octaval order beyond that point. Both the perfection of the first disdiapason, the first fruits, so to speak, of the fourfold expansion defined and governed by the Decad, and the "tenness" of the entire octave series lend plausibility to the idea that "All Perfect Animal" is, indeed, the Decad.

The standard limitation makes an unnecessary assumption about the significance of Plato's having terminated his primary sequence at 27. As readers of the *Timaeus* know, the primary sequence given at 35 B and C is 1, 2, 3, 4, 8, 9, and 27. The primary sequence by itself contains ratios indicating a triple octave (1:8). The insertion of means at 36 A yields 18, among other numbers, as the study will demonstrate below. This result extends the primary octave expansion beyond the quadruple octave ratio of 16. Scholars assume that, since 16, not in the primary sequence, would reflect a quadruple octave ratio, and 27 reflects the interval a sixth up from 16, while 32, also not in the primary sequence, reflects the quintuple octave ratio, Plato must have meant the progression of octaves to end at four octaves and a "major sixth" or, equivalently, at four octaves an independent tone and a fifth.[4] If one begins counting octaves at 384, the standard starting tone in ancient *Timaeus* commentaries, then 384 × 27 gives the number, 10368, corresponding exactly to the "major sixth" above the fourth complete octave, as the end of the expansion.[5]

Alternatively, one might multiply the tone numbers for "the model octave," namely, 384, 432, 486, 512, 576, 648, 729, 768, by 27 to get the final eight tone

4 The formula $16 \times (3/2 \times 9/8) = 27$ constitutes the mathematics for the interpretation that the scale is four octaves and a "major sixth." See Cornford, *Plato's Cosmology*, 68–71 for an approach like the one described. The formulas $16 \times 9/8 = 18$ and $18 \times 3/2 = 27$ constitute the mathematics for the interpretation of the scale as four octaves, an independent tone, and a fifth. See Brisson and Meyerstein, *Inventing the Universe*, 35 for an approach along these lines.

Note that the description of the limitation as four octaves and a "major sixth" is anachronistic. The "major sixth" was not a consonance of ancient Greek music. The limitation is more accurately expressed as four octaves, a fifth, and a tone. The corresponding ancient terminology would be two *disdiapasons*, a *diapente*, and a tone. Thomas Taylor, "Introduction to the *Timaeus*," 401 (for terminology only and not for the limit of the scale).

5 See Proclus *in Timaeum* 3.2.178.1–179.8 and 185.3–187.16 for an ancient example of the use of 384 as a starting tone. See, as well, the glossary to this study for Severus' difference of opinion. Luc Brisson, a modern commentator, has limited his extension at the 10368 consistent with the use of 384 as a starting tone and the mode of reasoning indicated in the text. Brisson, *Le même et l'autre*, 322. See, however, the immediately preceding note regarding the appropriateness of the term "major sixth."

numbers belonging to some octave series in the scale. These numbers are 10368, 11664, 13122, 13824, 15552, 17496, 19683, and 20736. This second method assumes that Plato simply wanted the model octave multiplied 27 times when he ended his primary sequence at 27.[6] The number 27, however, is a triple proportion and not a double proportion; so the series of numbers derived, while delineating octaves because of the other multipliers, is not the final complete octave of the scale, according to this line of thought, but overlaps with it.

The final complete octave in the scale naturally follows the double proportions. One finds it by multiplying the model octave by 16, since 16 marks the nearest starting tone number ("STN") below the 27 of the primary sequence.[7] The resulting series of numbers is 6144, 6912, 7776, 8192, 9216, 10368, 11664, 12288. The last tone number of the sequence proceeding from 27, i.e., 20736, is a "major sixth" or, if one prefers, an independent whole tone and a fifth above 12288, the number ending the sequence proceeding from 16.[8] Further analysis shows, however, that the viability of this interpretation entails that one start counting octaves from 768 ($768 \times 27 = 20736$), rather than 384 ($384 \times 27 = 10368$).[9] The octave expansion has the same magnitude found by the previous method, but the octaves counted are different.

Francis Cornford rightly noted that the limitation of the *Timaeus* scale to four octaves and a "major sixth" makes no musical sense at all.[10] This consideration caused him and, more recently, Luc Brisson, to question a harmonic interpretation of the *Timaeus* connoting a Pythagorean music of the spheres.[11] These scholars have supported the legitimate complaint about an arbitrary limitation of the scale with the further circumstance that Plato did not specifically

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6 See McClain, *Pythagorean Plato*, 52 and 60–62, for an approach along these lines. The explanation provided in the text is somewhat simplified for the sake of increasing its comprehensibility to the nonmusical reader.

7 The term "starting tone number" is employed above because "do" has implications that have nothing to do with ancient Greek music. Although the use of the latter term might seem to clarify the argument for the nonmusician, it would be objectionable from the viewpoint of the specialist. The ancient scale names for the steps of the diatonic octave scale, beginning with the tone of lowest pitch, were *hypate*, *parhypate*, *lichanos*, *mese*, *paramese*, *trite*, *paranete*, and *nete* (also *neate*). Mathiesen, *Apollo's Lyre*, 245. These names are used in this study as they become relevant and useful.

8 $12288 \times (9/8 \times 3/2) = 20736$. Also, $(12288 \times 9/8) \times 3/2 = 20736$.

9 Ernest McClain ended his extension at the 20736 consistently with this alternative mode of reasoning. McClain, *Pythagorean Plato*, 61. The ancient thinker Severus did so for other reasons. See the glossary to this work.

10 Cornford, *Plato's Cosmology*, 67–68 and 72. See note no. 4 to this chapter regarding the appropriateness of designating the limitation as four octaves and a "major sixth."

11 Ibid. 66–72, Brisson, *Le même et l'autre*, 321–332.

speak of any sound that the planets make.[12] The fact that even the ancient commentators adhered to the odd limitation of *circa* four octaves and a "major sixth" hardly has added to their comfort level with a musical interpretation.[13] Nonetheless, while agreeing that the limitation is arbitrary, Handschin maintained that it was more to Plato's purpose to articulate the principles of the musical scale than it was to elaborate a scale for its own sake.[14]

Indeed, the reciprocal character of the scale vis-à-vis the Lydian and Dorian *tonoi*, seen below, might be taken to support Handschin's point.[15] As long as the principles necessary for constructing a scale are properly in place, a musical interpretation is indicated, especially if it can elucidate the cosmological significance that the *Timaeus* had for the ancients. This result holds, particularly in view of Plato's reservation of the right to be somewhat inconsistent in the presentation of *Timaeus*' cosmology. The *Timaeus* is, after all, but a likely story (29 C). In view of this warning, the incompleteness of a musical metaphor cannot be taken as evidence of its invalidity.

The analysis offered by this study ascribes an entirely new significance to the limitation of the primary sequence at twenty-seven (27), emerging solely from the mathematics of its method and not from any musical or other presuppositions at the outset. It concludes that Plato meant for a Crantor-style matrix to be extended to the *twenty-seventh row* to capture the single number in it that his definition of intervals permits one to insert. The total set of numbers yielded by Plato's divisions, arranged from least to greatest, represents the slow rise of the octave scale from a triadic monad; articulates a perfect reciprocally Lydian/Dorian disdiapason; defines eight additional octaves displaying, in their succession, a slow but orderly creep of disorderly chromatic elements into the octave scale; and provides a set of numbers beyond the numbers delineating the ten complete octaves to display a replacement of octave periodicity by fifth periodicity as symmetrical as the scale's rise. The fifth periodicity itself finally terminates in a triadic monad.

The representation of the set of numbers comprising the *Timaeus* set in a Crantor-style matrix makes a symmetrical display yielding a fabric that one can "cut," per 36 B and C, to achieve the "chi" figure that Plato directed his reader

12 Cornford, *Plato's Cosmology*, 72; Brisson, *Le même et l'autre*, 329–330.

13 Brisson and Handschin each noted the ancient opinions on the construction of the *Timaeus* scale consistent with four octaves and a major sixth. Brisson, *Le même et l'autre*, 320–323; Handschin, "Timaeus Scale," 24–31. See note no. 4 to this chapter regarding the appropriateness of their terminology.

14 Handschin, "Timaeus Scale," 31–35.

15 See note no. 1 to this chapter for relevant musical points regarding the Lydian and Dorian *tonoi*.

to construct. The "chi" operation creates the conditions for the possibility of Plato's further definition of harmonic intervals at 36 C and D and supports the interpretation already given above for those segments of Plato's text, namely: they, too, evidence Plato's effort to articulate an octaval *harmonia* of being for the world soul.

It makes sense that Plato would have put some limitation on the octave extension that his intervals allowed. He was, after all, constructing a finite universe with a definite form in the *Timaeus*. Further, given the probable Pythagorean influences on the text, whether they came through Philolaus or Archytas or others of Plato's friends at Tarentum, it makes perfect sense that the limitation would be a decad of octaves. The character of that decad of octaves is certainly germane to other points that Plato made in the *Timaeus* text. The relation of order and disorder manifest in the stepwise genesis of chromatic elements within the scale as the octave repeats past the disdiapason is a fitting image of the manner in which order contains the chaos of a realm that moves.[16] In short, there could be no better way than the limitation that this study claims, for the *Timaeus*, to communicate the notion that "All Perfect Animal" is the Decad.[17]

The reason that the ancient commentators missed the analysis provided below is not as difficult a question as it may, at first, appear. They had neither the calculating devices nor the time to do the extensive factoring required for its discovery. Plato, genius that he was, may have been privy to a better, faster way of factoring than this study has devised. Notwithstanding the limitations of the ancient commentators, as has already been noted in the Introduction to this study, the most notable among them, e.g., Plutarch and Proclus, agreed that 384 begins the first complete scale; and Plutarch even assumed the Lydian transposition of the fourth, exactly the preference of this study.

16 Chromatic elements are numbers foreign to the primary diatonic octave scale that are possible on the basis of the number generation at 35 A–36 D of Plato's text. They belong to independent series of fourths, interfering with the octave order, deriving, *inter alia*, from Plato's division of the triple intervals at 36 A–36 B. Handschin, "Timaeus Scale," 14, 21. They add to the secondary diatonic and enharmonic possibilities of the ultimate number set, discussed in Chapter 6, and also make possible a different genus of octave scale known to the ancients as the chromatic.

17 Since one can also think of a decad of octaves as a pentad of disdiapasons, Plato might also have intended to provide a clue about the pentagonal shape of the faces of the dodecahedron comprising the body (rather than the soul) of the universe (55 C); but the latter point is purely speculative. Cornford, *Plato's Cosmology*, 218 (regarding the body of the universe).

The method of the study will be clear as it proceeds. Its beauty is its freedom from musical or other presuppositions. It just calculates all numbers that could possibly arise in the arithmetic that Plato asks his reader to perform to establish his intervals. Only when the calculations are complete is interpretation allowed. The *Philebus* has a strange aptness to the method selected. By simply concentrating on the math, as a primary matter, the method employed fits well with a straightforward understanding of the warnings that Plato issued in that text concerning the importance of discovering how many "something" actually "is" before one concludes that one knows it (*Philebus* 16 C–E). Through his recipe for the world soul, Plato established the precise limitation of the "Crantor" matrix necessary to articulate the parameters of the octaval *harmonia* of being described above and to reveal the Decad as "All Perfect Animal."

1 *Timaeus* 35 A

The present study begins at 35 A, as others do, though Plato made no intervals until 35 B. The end of *Timaeus* 34 C and the largest part of *Timaeus* 35 A read as follows:

> [34 C] … [T]he artificer of the world constituted soul both in generation and virtue prior to, and more ancient than, body, as being the proper lord and ruler of its servile nature; and that in the following manner:
>
> [35 A] From an essence impartible, and always subsisting according to sameness of being, and from a nature divisible about bodies, he mingled from both a third form of essence, having a middle subsistence between the two. And, again, between that which is impartible and that which is divisible about bodies, he placed the nature of *same* and *different*. And taking these, now they are three, he mingled them all into one idea. But as the nature of *different* could not without difficulty be mingled in *same*, he harmonized them together by employing force in their conjunction …[18] [References in brackets added; other bracketed material introduced for clarity.]

Timaeus 35 A, like the rest of the text, has been variously interpreted. Calcidius reported the view of some that the impartible essence was mind or intellect

18 Plato *Tim.* (T. Taylor) 34 C–35 A.

and the nature divisible about bodies was matter. Such a reading reflects the notion that the soul mediates between the incorporeal and corporeal.[19] Francis Cornford provided an analysis much more recently that seems less justified in some of its details and overly complicated; but some prefer it.[20]

Cornford posited two mixtures leading to the *mélange* that is the world soul. The first corresponds to the action of the demiurge described in the first sentence of 35 A, namely: "From an essence impartible, and always subsisting according to sameness of being, and from a nature divisible about bodies, he mingled from both a third form of essence, having a middle subsistence between the two."[21] Cornford opined that the indicated operation means that the demiurge created a third kind of existence intermediate between the impartible essence and the nature divisible about bodies.[22] According to him, the second mixture is described in the second sentence of 35 A and actually contemplates two operations, one involving "sameness" and one involving "difference." The demiurge took indivisible sameness and divisible sameness and mixed them to procure intermediate sameness. Likewise, he took indivisible difference and divisible difference and mixed them to obtain intermediate difference.[23] The three that the demiurge mixed to arrive at the soul stuff were the two products of the second operation and the one product of the first.[24]

Cornford's second mixture is highly problematic. Nothing in the sentence, "And, again, between that which is impartible and that which is divisible about bodies, he placed the nature of same and different," warrants positing the two kinds of sameness and two kinds of difference that Cornford suggests.[25] The text makes no suggestion that the natures of "same" and "different" are divided and offers no clue concerning what any such division might mean; further, it makes no claim that the "impartible" and "divisible about bodies" refer to any objects in the second sentence that they do not refer to in the first one. The labels are the same. It is much more likely that the second sentence of the passage in 35 A is a descriptive qualification and clarification concerning the operation of the first sentence.

19 Calcidius *In Timaeum* (commentary) (trans. John Magee [London: Harvard University Press, 2016])1.31.

20 Cornford, *Plato's Cosmology*, 59–62; Brisson and Meyerstein, *Inventing the Universe*, 30–31; Brisson, *Le même et l'autre*, 274–275.

21 Plato *Tim.* (T. Taylor) 35 A.

22 Cornford, *Plato's Cosmology*, 60–61.

23 Ibid.

24 Ibid.

25 Plato *Tim.* (Taylor) 35 A.

If Cornford's analysis is too speculative, one must offer an alternative interpretation that makes sense of the passage. Handschin's interpretation seems preferable because it is more faithful to the letter and spirit of the text.[26] He offered the following option:

> … Having mixed the two primary principles into a third, the Creator again mixed these three into one essence …[27]

The approach of this study is quite similar.

The first sentence of 35 A has the demiurge take the impartible essence that always subsists according to sameness of being and mix it with the nature that is divisible about bodies. The resulting essence has a nature intermediate between the two natures, having in it something impartible and the same and something divisible about bodies. The elements that go into the mix are not exhausted with the mixture because Plato referred to them again in the second sentence. In specifying, "[a]nd, again, between that which is impartible and that which is divisible about bodies, he placed the nature of same and different," he most plausibly referred to the very same elements that entered into the operation of the first sentence. Further, the nature of same and different placed between that which is impartible and that which is divisible about bodies is nothing other than the intermediate nature achieved through the operation in the first sentence. It seems, then, that these three—the impartible, that which is divisible about bodies, and the intermediate nature—enter into the final mix referred to in the third sentence: "And taking these, now they are three, he mingled them all into one idea."[28]

One might initially object and interpret the second sentence to mean that the demiurge created a new kind of mix from the same two original elements, calling it "the nature of the same and different." This could not really be, however, because it would mean that, after the operations of the first two sentences, there were four things: (a) the impartible essence that always subsists according to the same; (b) the nature divisible about bodies; (c) the intermediate nature; and (d) the nature of the same and the different. The third sentence specifies that there were only *three* kinds of things following the operations of the first two sentences. Accordingly, the second sentence must be a further specification of the operation described in the first.

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26 Handschin, "Timaeus Scale," 6.

27 Ibid.

28 Plato *Tim.* (T. Taylor) 35 A.

In particular, once he had arrived at the intermediate nature that has something both of the impartible essence that always subsists according to the same and the nature divisible about bodies, the demiurge had to define the blended nature that he had thus produced, as well as determine how to name it. In particular, he had to decide how to understand the nature that is divisible about bodies in relation to the impartible essence that always subsists according to the same. The simplest solution was to call it the "different." Accordingly, the whole name of the intermediate nature is "the nature of the same and the different." This naming completed the first step of the process toward the commixture that would become the World Soul. The operation of the third sentence completed the commixture. The three natures mingled in their entirety were: (a) the impartible essence that always subsists according to the same, *plus* (b) the intermediate nature, called the nature of the same and different, *plus* (c) the nature divisible about bodies. This commixture was the world soul stuff. Further, its three components indicate that it was a triadic monad. This one small fact is important for determining the originating moment of the musical scale in the analysis below.

Timaeus 35 A is not complete with the production of the commixture. Plato made an observation about the mixing process: "But as the nature of different could not without difficulty be mingled in same, he harmonized them together by employing force in their conjunction."[29] Even upon the creation of the intermediate nature, the impartible essence that ever subsists according to the same was incompatible with the nature that is divisible about bodies. Indeed, the ability of the two to co-exist in the intermediate nature is itself inexplicable. Plato could think of nothing other than "force" that could possibly join two so disparate natures together. This circumstance warrants a closer examination of Platonic mediation.

There is a question about the nature of the force. One might initially be tempted to say that the irreconcilability of opposites renders a violence holding them together an essential feature of the cosmos; but it appears that Plato had something else in mind. *Timaeus* 35 A indicates either that the force achieves a *harmonia*, or that the *harmonia* is itself the force that binds.[30] In either case, however, the *harmonia* is inextricably bound up with number, suggesting that number and the binding power are related.[31]

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29 Ibid.

30 Huffman, *Philolaus*, 123–124 and 129. Philolaus understood the *harmonia* as a force that "fits together" things that are unlike. Ibid.

31 Number appears to be the very power of the Divine Intellect in the *Philebus* (cf. *Philebus* 25 D & E, 26 A, 30 A–D).

2 End of *Timaeus* 35 A–Beginning of *Timaeus* 35 C

After his creation of the soul stuff, the demiurge took portions from it as specified from the end of *Timaeus* 35 A through the beginning of *Timaeus* 35 C. These sections read as follows:

> [35 A] … But after he had mingled these two with essence, and had [35 B] produced one from the three, he again divided this whole into becoming parts; at the same time mingling each part from *same, different*, and *essence*. But he began to divide as follows: In the first place, he received one part from the whole. Then he separated a second part, double of the first; afterwards a third, sesquialter of the second, but triple of the first; then a fourth, double of the second; in the next place a fifth, triple [35 C] of the third; a sixth, octuple of the first; and lastly a seventh, twenty-seven times more than the first …[32]

The text proceeds as though the artificer had been working with a lump of clay. He took parts from a single lump, bearing certain proportions to each other *via* their comparative magnitude. Magnitude is a difficult idea, however, since the soul stuff was incorporeal; so one might think of the demiurge, instead, as simply generating numbers from the soul stuff. They are incorporeal, yet capable of bearing comparative relation to each other in the mode that the passage suggests. One can find the numbers that the artificer made by separation from the soul stuff simply by following the directions that Plato supplied. The primary sequence of numbers, corresponding to the whole number ratios given in the text, is calculated as in Table 2, below.

Since Crantor (early third century B.C.), the scholarly tradition that developed around Plato's *Timaeus* has arranged this primary sequence of numbers in a *lambda* formation that effectively separates the principle of twice from thrice as in Figure 5, below.[33] The representation marks the *ordo* according to which the artificer chose his portions. He consistently selected first a dou-

32 Plato *Tim.* (Taylor) 35 A, B and C.

33 Crantor was a student of Xenocrates who belonged to the Academy of the early third century B.C. He is supposed to have written the very first commentary on the *Timaeus*. As McClain has explained, fragments of the Crantor commentary survived to Plutarch's time; and Plutarch preserved them for later generations in his own commentary. McClain, *Pythagorean Plato*, 60, 64–65, and 140; see, also, Huffman, *Philolaus*, 24, 377; Plutarch attributed the lambda arrangement presented in the text to Crantor's own *Timaeus* commentary. Plutarch, *De Procreatione* 1022 D and E and 1027 F.

TABLE 2 Plato's primary divisions

In the first place, he received one part from the whole.	1
Then he separated a second part, double of the first;	2
afterwards, a third sesquialter of the second (that is, wholly containing the second plus one-half the second) and triple of the first:	3
then a fourth double of the second;	4
in the next place a fifth, triple of the third;	9
a sixth, octuple of the first;	8
and lastly a seventh, twenty-seven times more than the first.	27

ble, then a triple, then another double, and then a triple, until he had arrived at the end of the sequence. Figure 5 also readily manifests otherwise virtually invisible relationships among the primary set of numbers.

Calcidius, among the ancient commentators, was especially insightful concerning Plato's possible rationale for the primary sequence. He noted that the sequence given revealed the principles requisite to the soul/body union. Since soul had to infuse both surface and solid areas of the world body with life, it had to be endowed with powers, implicit in number, resembling both surface and solid areas. Unity is that in reference to which both doubles and triples proceed but, lacking all parts, cannot account for dimension. The first double enables a line, the second a plane, and the third a cube, i.e., a body divisible by length, width, and breadth. The series of triples had affinity, according to Calcidius, with progression to the cube, as twenty-seven is the first cubic power among the odd numbers.[34]

Plato's primary sequence was limited to seven terms, according to Calcidius, because of the special properties of the number seven. Seven is the only number, he noted, within the limit of ten that neither generates from itself within that limit nor is generated from some other number within it. It is, thus, a virgin number, ever a maiden, and most noble, since many phenomena recurring

34 Calcidius *In Timaeum* 1.33.

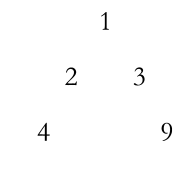


FIGURE 5
Primary Crantor matrix

according to the laws of nature emerge in connection with the number seven. He provided many examples of such laws.[35]

Calcidius observed further that the intervals on either side of unity are limited to three in the primary sequence because, having beginning, middle, and end, they mark the stages according to which bodies grow and progress. In addition, three intervals on both sides amount to six, another perfect number, as it has a special equality with its own parts ($2 + 2 + 2 = 6$ and $2 \times 3 = 6$).[36] In this equality, ostensibly reinforcing the appropriateness of the primary limitation of the number series to three intervals on each side, Calcidius noted that six brings to mind three other numbers that are sums of their parts somewhat differently: ten, comprising the sum of the first four numbers of the natural number series, i.e., one, two, three, and four; thirty-six, comprising the sum of the first four odd numbers (one, three, five, seven) with the first four even numbers (two, four, six, eight); and fifty-four, comprising the sum of the first four numbers of the series of doubles (1, 2, 4, 8) with the first four numbers of the series of triples (1, 3, 9, 27). He counted unity only once to avoid redundancy.[37]

Calcidius believed it especially appropriate that Plato's primary series of numbers be arranged in the triangular form of the lambda, with unity at the top. One was the beginning point, he noted, of both even and odd, containing within itself all shapes, e.g., plane, triangular, cubic, etc., associated with number, and so, it properly, held the place of a summit or pinnacle. It operated, he observed, as a kind of conduit, from which a bountiful river flowed, "as from the depths of a provident intelligence or the craftsman god himself."[38] As the origin of number, it provided from itself being for all things, embracing both the simple and manifold rational principles determining them. It stood alone, in its inviolate sovereignty, and persevered forever in a state of impassible felic-

35 Ibid., 1.36–37.
36 Ibid., 1.38.
37 Ibid.
38 Ibid., 1.39.

ity, Calcidius maintained, while other numbers underwent change and receded from their proper nature in reference to it.[39]

Calcidius' imagery of the fount, for unity, is particularly striking in view of properties of one, as unity, that Calcidius may or may not have had in mind. "One" is the geometric mean between Φ and 1/Φ, two infinities of great and small. The observation is bound up with Plato's analogy of the divided line in Republic 509 D–511 E.[40] In reference to one, as mean, Φ is expressible geometrically, but cannot be represented as a ratio of whole numbers due to its irrational value.[41] Φ is a very special number because of its ubiquity in nature.[42]

Although no interpretation will emerge as more probable than others until this analysis is complete, some of the ways in which modern commentators have interpreted the primary sequence of numbers are also worth noting. Thomas Taylor opined as follows:

> With respect to the first numbers, which are evidently those described by Plato, the first three of these, 1, 2, 3, as Syrianus beautifully observes, may be considered as representing the soul of the world, abiding in, proceeding from, and returning to herself, viz. abiding according to that first part, proceeding, through the second, and this without any passivity or imbecility, but returning according to the third: for that which is perfective accedes to beings through conversion …[43]

Taylor's explanation makes sense in view of the relations among 1, 2, and 3. Two is the double of one and so aptly symbolizes the soul proceeding from herself. Three and two are in sesquialter proportion. That is, three contains two plus an additional half of two beyond it, namely 1; so it returns 2 to 1 in a symbolic manner of speaking.

Taylor continued his explanation of the generation of numbers as follows:

> … But as the whole of the mundane soul is perfect, united with intelligibles, and eternally abiding in intellect, hence she providentially presides over secondary natures; in one respect indeed over those which are as it were proximately connected with herself, and in another over solid and

39 Ibid., 1.38–39.

40 Scott Olsen, *The Golden Section, Nature's Greatest Secret* (New York: Walker & Company, 2006), 4–7.

41 Ibid., 6.

42 Ibid., generally.

43 Thomas Taylor, "Introduction to the *Timaeus*," 402.

> compacted bulks. But her providence over each of these is twofold. For those which are connected with her essence in a following order, proceed from her according to the power of the fourth term (4), which possesses generative powers; but return to her according to the fifth (9), which reduces them to one. Again, solid natures, and all the species which are discerned in corporeal masses, proceed according to the octuple of the first part (i.e., according to 8), which number is produced by two, is solid, and possesses generative powers proceeding to all things; but they return according to the number 27, which is the regression of solids, proceeding as it were from the ternary, and existing of the same order according to nature: for such are all odd numbers.[44]

Taylor's explanation means essentially the following. The third row of numbers, containing 4 and 9, correspond to planar figures, being the squares of 2 and 3, respectively, while the fourth row of numbers, containing 8 and 27, correspond to solid figures, being the cubes of 2 and 3, respectively. Taylor indicates that the planar numbers proceed according to 4, the square of 2, and return to unity by 9, the square of 3. This analysis makes symbolic sense because 9 contains 4 twice with 1 remaining over, returning 2 to 1 in a manner reminiscent of the sesquialter proportion. That is, it contains 2, in a manner of speaking, with 1 remainder over, just as 3 contains 2 with 1 remainder over.

Solid figures similarly proceed according to 8, the cube of 2 and return to unity by 27, the cube of 3. The relations among these numbers work a bit differently than the relations among the planar numbers, yet in a way that amicably completes the symbolism. Twenty-seven contains 8 three times with 3 remainder over. In other words, the three eights in twenty-seven are the reflection of the primary progression 1, 2, 3 and the three remainder over is a reminder that 3 is the number of return. Reinforcing such symbolism is the circumstance that the ratio of 27 to 24 is the same as 9 to 8; and, as Taylor noted, 9 to 8, the sequioctave ratio contains 8 once with one remainder over, or, if one thinks in terms of corresponding planar numbers, it contains 4 twice with one remainder over, thus reflecting the primary sesquialter proportion whereby 3 symbolically returns two to one.

The symbolism goes farther, upon a consideration of the sesquioctave proportion, 9 to 8, to emphasize a link between the numbers by which planar and three dimensional solid objects proceed and return and a musical harmony inherent in those numbers. The sesquioctave proportion, as Taylor noted, "pro-

44 Ibid.

duces an entire tone, which is the principle of all symphony."[45] The "entire" tone allowed by this ratio brings one back, from the standpoint of the dialogue, to the idea of wholeness that is an image of unity.

If one considers the primary numbers of the soul, in Figure 5 above, with the intention of discovering musical consonances, then one easily finds them. As Taylor noted, the possibilities are five symphonies or consonances: the *diatessaron*, the *diapente*, the *diapason*, the *diapason diapente*, and the *disdiapason*.[46] The *diatessaron* exhibits a sesquitertian proportion attributable, according to *Timaeus* 36 A and B, to its two whole tones and *diesis* (9/8 × 9/8 × 256/243 = 4/3). A sesquitertian proportion occurs, in mathematical terms, when a greater number contains a lesser number plus one-third of the lesser number. The sesquitertian ratio is represented among the primary numbers as the proportion of 4 to 3; it is expressed in the *lambda* figure as the positional relation of the two numbers. In modern musical terminology, the interval it represents most closely is the fourth.

The *diapente* displays a sesquialter proportion due to its three whole tones and *diesis* (9/8 × 9/8 × 9/8 × 256/243 = 3/2). A sesquialter proportion exists, in mathematical terms, when a greater number contains a lesser number plus one-half of the lesser number. It is represented among the primary numbers as the proportion of 3 to 2 and is expressed in the *lambda* figure as the positional relation between those two numbers. In modern musical parlance the interval it represents most closely corresponds to a fifth.

The *diapason* is the double proportion (2/1, also 4/2 and 8/4) exhibited above in the relation of consecutive numbers along the left arm of the *lambda*. This corresponds to the octave. Taylor somewhat unaccountably described it, in approximate terms, as six tones (9/8 × 9/8 × 9/8 × 9/8 × 9/8 × 9/8, *circa*, 2.02728653).[47] Properly speaking, the *diapason* is two disjunct *diatessarons*, i.e., two sesquitertian parts separated by a whole tone, thus: {9/8 × 9/8 × 256/243} × 9/8 × {9/8 × 9/8 × 256/243} = 2 or, equivalently, an undivided *diatessaron* and *diapente* (sesquialter part): {9/8 × 9/8 × 256/243} × {9/8 × 9/8 × 9/8 × 256/243} = 2.

The *diapason diapente* is nine tones and a *diesis*, according to Taylor (9/8 × 9/8 × 9/8 × 9/8 ×9/8 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243, approximately,

45 Ibid., 401.

46 Ibid. Note that the four octaves, tone, and a fifth of standard interpretation regarding the extension of the *Timaeus* scale comprise two *disdiapasons*, a *diapente*, and a tone in the vocabulary of these ancient consonances.

47 Ibid.

3.040929794), who spoke, once more, in approximate terms.[48] More precisely speaking, it is an undivided *diapason* and *diapente*, containing eight whole tones and three *dieses* ({9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243} × {9/8 × 9/8 × 9/8 × 256/243} = 3). It is, therefore, properly represented by the triple ratio (3/1, also 9/3 and 27/9) exhibited in the relations of consecutive numbers along the right diagonal of the *lambda* figure. In modern parlance, the *diapason diapente* corresponds most closely to the musical twelfth.

The *disdiapason* or quadruple proportion 4/1 (also 8 to 2) is reflected in the relations between every other consecutive number along the left arm of the *lambda* figure. This is the double octave.[49] The whole tone itself (9/8) is represented by the positional relation of 9 and 8 in the *lambda* figure. Further, although 256/243 does not appear within the primary numbers, it is easily calculable from the relations germane to the *diatessaron* and *diapente* involving the primary numbers. Indeed, Plato hinted at such a derivation when he furnished his reader with this ratio in his recipe for the construction of the world soul. The analysis, below, will show how it naturally emerges from the calculations his directions permit.

It is clear from the above discussion that the primary numbers of the soul permit the expression of the musical harmonies that mattered most to the Greeks. The musical significance comes into even stronger relief upon the further progression of the *Timaean* soul numbers, once one has accepted the centrality to the dialogue of the diatonic scale.[50]

Brisson and Cornford have suggested that Plato was primarily engaged in constructing an armillary sphere, not a musical scale, in the *Timaeus*.[51] While such a project is not impossible, it is less likely than the suggestions that Thomas Taylor made. Taylor's interpretation has ties to legitimate themes relevant to the construction of a universe, i.e., how things become three dimen-

48 Ibid.

49 Ibid.

50 One finds that of three different possible kinds of harmonic scales, the diatonic, enharmonic, and chromatic, Plato's generation of numbers, together with their significant relations, assumes, as a primary matter, the diatonic progression. The diatonic scale moves continuously through two whole steps and a half-step with half-steps always being uniform. The enharmonic moves only through two equal halftones, while the chromatic ascends through two different kinds of halftones. A diatonic scale is the one assumed when one sings "do re mi fa so la ti do." This modern arrangement is most closely parallel to an ascending Lydian diatonic scale, among the ancient Greek scales discussed in this study; but one would be less than careful to assume that it had a precisely identical sound.

51 Brisson, *Le même et l'autre*, 38–44; Brisson and Meyerstein, *Inventing the Universe*, 35–40; Cornford, *Plato's Cosmology*, 74–78.

sional, for example. It is consistent with the notion that three-dimensionality arises harmonically, in line with the showing, above, that the primary bodies arise harmonically; so it fits neatly with Plato's harmonic divisions at 35 B and C, as well as at 36 C and D, aiding a unified interpretation of the text.

The armillary sphere interpretation seems to reduce the *Timaeus* to a diversionary intellectual toy, while neglecting the harmonic character of the further divisions that the artificer made, e.g., at 36 C and D. It adds little to an integrated interpretation of the *Timaeus*. One could not, for example, credibly link the armillary sphere theory to Plato's account of the primary elements; so it has serious drawbacks in comparison with the harmonic interpretation. The hermeneutical force of the harmonic interpretation is its explanatory power, as should be clear by the end of this analysis.

3 *Timaeus* 35 C and 36 A

Following the primary separations described above, the demiurge continued separating numbers from the soul stuff to "fill-up" the double and triple intervals:

> [35 C] … After this, he filled up the [36 A] double and triple intervals, again cutting off parts from the whole; and placed them so between the intervals, that there might be two mediums in every interval; and that one of these might by the same part exceed one of the extremes, and be exceeded by the other, and that the other part might by an equal number surpass one of the extremes, and by an equal number be surpassed by the other. But as from hence sesquialter, sesquitertian, and sesquioctave intervals were produced, from those bonds in the first spaces …[52]

He created two mathematical nodes, in other words, within each double and triple interval. The node corresponding to the exact middle between the end-terms of the interval, i.e., the one exceeding the lesser end-term by the same number whereby it is itself exceeded by the greater end-term, is the arithmetic mean. It is half the sum of the end terms [(a + b) / 2].[53] In the sequence 1, 2, and 3, the number 2 corresponds to the arithmetic mean between 1 and 3. Scholars

52 Plato *Tim.* (T. Taylor) 35 C and 36 A.

53 Brisson has used this formula as well; and Taylor's description of the arithmetic mean anticipated it. Brisson, *Le même et l'autre*, 316; Taylor, "Introduction to the *Timaeus*," 402–403.

TABLE 3 Double and triple intervals to be filled with arithmetic and harmonic means

Double intervals			**Triple intervals**		
1	to	2	1	to	3
2	to	4	3	to	9
4	to	8	9	to	27

agree that the other node to be inserted into every interval is the harmonic mean. The harmonic mean between two numbers exceeds the lesser number by the same part of the lesser as the part of the greater whereby the greater itself exceeds the mean.[54]

One might represent the idea of the harmonic mean as follows, mathematically, where "m" is the harmonic mean, "l" is the lesser number, "g" is the greater number, and "y" is the relevant ratio measuring the part of the lesser by which the mean exceeds the lesser and the part of the greater by which the greater exceeds the mean:

(a) $m > l$ by $y \cdot l$
(b) $g > m$ by $y \cdot g$

Therefore:

$l + yl = m$
$m + yg = g$

One simply solves the above system of equations algebraically to find the harmonic mean that Plato wanted his reader to insert into each double and triple interval. The intervals to be filled are those indicated in Table 3.

Filling the intervals with the harmonic and arithmetic means as Plato directed, one obtains the sequences of Table 4, arranged from least to greatest:[55]

54 See Taylor, "Introduction to the *Timaeus*," 402 (for this definition of the harmonic mean). There seems to be no dispute that Plato was asking the reader to insert the arithmetic and harmonic means into the intervals he defined. See, e.g., Cornford, *Plato's Cosmology*, 71; Brisson, *Le même et l'autre*, 316.

55 The designated intervals thus far exactly agree with the ones Brisson and Cornford have indicated. Cf., Cornford, *Plato's Cosmology*, 71; Brisson and Meyerstein, *Inventing the Universe*, 34; Brisson *Le même et l'autre*, 316.

TABLE 4 Double and triple intervals after insertion of arithmetic and harmonic means

Double intervals				**Triple intervals**			
1	4/3 (harm.)	3/2 (arith.)	2	1	3/2 (harm.)	2 (arith.)	3
2	8/3 (harm.)	3 (arith.)	4	3	9/2 (harm.)	6 (arith.)	9
4	16/3 (harm.)	6 (arith.)	8	9	27/2 (harm.)	18 (arith.)	27

Calculation of the arithmetic mean is intuitively straightforward, as mentioned above. Each number designated in Table 4 as an arithmetic mean is exactly half-way between the end-terms of the relevant sequence. One calculates the several harmonic means as is indicated in Table 5.[56]

It is convenient to represent in a Crantor type diagram the ratios that Plato generated with the divisions corresponding to the means operations at 36 A. A review of the table charting the results of that operation, in Table 6, is helpful to the task.

The Crantor diagram, begun with the first seven numbers, already represents all of the ratios in the above table, charting the means operation, except for 6, 18, and 16/3. A reexamination of the original lambda figure, in Figure 6, will make this fact evident.

Although Figure 6 does not represent fractions such as 4/3 directly, it does so indirectly by virtue of the positional relationship between 4 and 3. All numbers arising below 4 and 3 thereafter in the Crantor matrix, having a parallel positional relationship, also preserve a ratio of 4/3. One can test this feature of the matrix simply by extending it as far as one wishes and verifying the statement for oneself. One extends the matrix simply by filling in multiples of threes in the diagonals proceeding down toward the right or filling in multiples of 2 in the diagonals proceeding down toward the left. Performing either operation automatically performs the other, as well.

The same possibility of representation by a unique relational position obtains for 8/3, 3/2, 9/2 and 27/2, according to the same principle that a particular

56 There is, of course, a more concise way to find the harmonic mean. The mathematical formula, as Brisson has pointed out is 2ab/a+b. Brisson and Meyerstein, *Inventing the Universe*, 34; Brisson *Le même et l'autre*, 316. To find the harmonic mean between 1 and 2, one would simply compute $(2 \times 1 \times 2)/1 + 2$. This is 4/3. Likewise, for the harmonic means between 1 and 3, one computes $(2 \times 1 \times 3)/1 + 3$ to arrive at 6/4 or 3/2. For a nonmathematician, however, the relation represented by the harmonic mean is more evident if he does things the long way and just solves a system of equations.

TABLE 5 Calculation of harmonic means

Double Intervals			**Triple intervals**		
1	to	2	1	to	3
l + ly = m 1 + 1y = m 1 + y = m **y = m − 1**		m + yg = g m + 2**y** = 2 m + 2**(m − 1)** = 2 m + 2m − 2 = 2 m + 2m = 4 3m = 4 **m = 4/3**	1+ ly = m 1 + 1y = m 1 + y = m **y = m − 1**		m + yg = g m + 3**y** = 3 m + 3**(m − 1)** = 3 m + 3m − 3 = 3 4m − 3 = 3 4m = 6 m = 6/4 **m = 3/2**
2	to	4	3	to	9
l + ly = m 2 + 2y = m 2y = m − 2 **y = (m − 2)/2**		m + yg = g m + 4**y** = 4 m + 4**[(m − 2)/2]** = 4 m + 4 [m/2 − 1] = 4 m + 2m − 4 = 4 3m − 4 = 4 3m = 8 **m = 8/3**	l + ly = m 3 + 3y = m 3y = m − 3 **y = (m − 3)/3**		m + yg = g m + 9**y** = 9 m + 9**[(m − 3)/3]** = 9 m + 9[m/3 − 1] = 9 m + 9/3 m − 9 = 9 m + 3m − 9 = 9 4m − 9 = 9 4m = 18 m = 18/4 **m = 9/2**
4	to	8	9	to	27
l + ly = m 4 + 4y = m 4y = m − 4 **y = (m − 4)/4**		m + yg = g m + 8**y** = 8 m + 8**[(m − 4)/4]** = 8 m + 8[m/4–1] = 8 m + 2m − 8 = 8 3m − 8 = 8 3m = 16 **m = 16/3**	l + ly = m 9 + 9y = m 9y = m **y = (m − 9)/9**		m + yg = g m + 27**y** = 27 m + 27**[(m − 9)/9]** = 27 m = 27 [m/9 − 1] = 27 m = 3m − 27 = 27 4m − 27 = 27 4m = 54 m = 54/4 **m = 27/2**

TABLE 6 Double and triple intervals after insertion of arithmetic and harmonic means

Double intervals				**Triple intervals**			
1	4/3 (harm.)	3/2 (arith.)	2	1	3/2 (harm.)	2 (arith.)	3
2	8/3 (harm.)	3 (arith.)	4	3	9/2 (harm.)	6 (arith.)	9
4	16/3 (harm.)	6 (arith.)	8	9	27/2 (harm.)	18 (arith.)	27

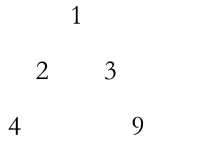


FIGURE 6
Crantor lambda with Plato's original seven numbers

kind of positional relation represents 4/3. Indeed, any two numbers in Figure 6 can be related as fractions on the principle articulated. It is irrelevant that one can represent some fractions to which Plato did not specifically direct his reader's attention, e.g., 27/4, 27/8. His divisions do produce these relations, after all, though it is not to his purpose to mention them all.

It is, in fact, highly significant that the Crantor matrix allows one to inventory all mathematical relations to which Plato's divisions give rise. The task, at this point, however, is simply to index the minimal set of numbers needed to express the relations that Plato did explicitly mention in every form that he permitted them to be named via the calculations one must do to produce his divisions. The significance of a redundancy of names will become clear as the analysis proceeds. No number that this study will include, below, in the Crantor matrix will be unwarranted from the standpoint of being a permitted name or from the standpoint of having an ultimate theoretical significance.

In sum, to represent any fraction that Plato's divisions generate, one simply inserts the whole numbers corresponding to the numerator and denominator into their proper places in the Crantor matrix. Since the original Crantor matrix does not allow one to represent 6, 18, and 16/3, numbers important in connection with the intervals that Plato asked his readers to construct, 6, 16, and 18 must be included as in Figure 7 below.

Plato stated at the very end of 36 A that the insertion of the harmonic and arithmetic means into each of the original double and triple intervals produced among them sesquialter, sesquitertian, and sesquioctave intervals. He based yet further divisions on the sesquitertian parts at 36 B. These divisions pro-

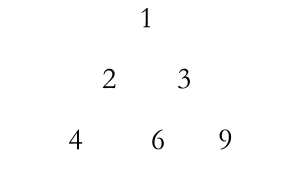


FIGURE 7
Crantor matrix following insertion of numbers needed to represent arithmetic and harmonic means

duce new ratios and, so, new numbers to be included within the Crantor set of this study. The divisions of the sesquitertian parts, in fact, generate a proliferation of different names for some key ratios. When one painstakingly and exhaustively follows the thread of this proliferation through 36 B to its final conclusion, one finds that it delineates a markedly non-arbitrary finite set of numbers. Plato's divisions limit the extension of the Crantor matrix in a way that makes it difficult to doubt the integrity, appropriateness, and significance of a musical metaphor for Plato's account of the construction of the world soul. To demonstrate these points, the study proceeds to the next set of Plato's divisions—the division of the sesquitertian parts.

4 *Timaeus* 36 A (con't) and 36 B

> [36 A] ... But as from hence sesquialter, sesquitertian, and sesquioctave intervals were produced, from those [36 B] bonds in the first spaces, he filled with a sesquioctave interval all the sesquitertian parts, at the same time leaving a part of each of these. And then again the interval of this part being assumed, a comparison is from thence obtained in terms of number to number, subsisting between 256 and 243 ...[57]

The directions that Plato provided for the division of the sesquitertian parts are deceptively simple. Note that any of the following means is a theoretical possibility for filling a sesquitertian part with sesquioctave intervals:

$4/3 = 9/8 \times 9/8 \times 256/243$
$4/3 = 9/8 \times 256/243 \times 9/8$
$4/3 = 256/243 \times 9/8 \times 9/8$

57 Plato *Tim.* (T. Taylor) 36 A–B.

TABLE 7 Brisson table of intervals resulting from the insertion of means at 36 A

1	4/3	3/2	**2**	8/3	3	**4**	16/3	6	**8**
<-4/3-><-9/8-><-4/3-><-4/3-><-9/8-><-4/3-><-4/3-><-9/8-><-4/3->									
1	3/2	2	**3**	9/2	6	**9**	27/2	18	**27**
<-3/2-><-4/3-><-3/2-><-3/2-><-4/3-><-3/2-><-3/2-><-4/3-><-3/2->									

Because there are several ways to fill a sesquitertian part, one of the issues raised by the text is the *ordo* of filling the parts.[58]

An even thornier issue exists. Brisson has produced a diagram of the relations among the bonds in the first spaces, after insertion of the means at 36 A. His table of the resulting sesquialter, sesquitertian, and sesquioctave intervals is represented, above, in Table 7.[59]

It is clear that Brisson has simply made a row for the original double intervals, showing the sesquialter, sesquitertian, and sesquioctave parts explicitly resulting within those whole number intervals from the insertion of means. He has produced a corresponding row for the original triple intervals. All of the sesquialter, sesquitertian, and sesquioctave intervals that Brisson has identified undoubtedly belong to the list of such intervals for which Plato's means operations at 36 A provided a foundation.

Commentators, including Brisson, have generally assumed that Plato wanted the reader to fill only the sesquitertian parts between the original doubles and triples that were obviously created by the insertion of means and marked as 4/3 intervals in Table 7. The assumption, however, is faulty, as a closer inspection of the table readily reveals.

A careful observer will note that nodes resulting from the means operation at 36 A define not only sesquitertian, sesquioctave, and sesquialter intervals but, also, multiple new double and triple intervals, intermixed along the double and triple arms of Table 7, to the extent permitted by "8" and "27," the limits set by the original intervals for the extension of the double and triple arms. One may view each arm as displaying either continuously overlap-

58 Haar also noted this issue. Haar, "Musica mundana," 16.

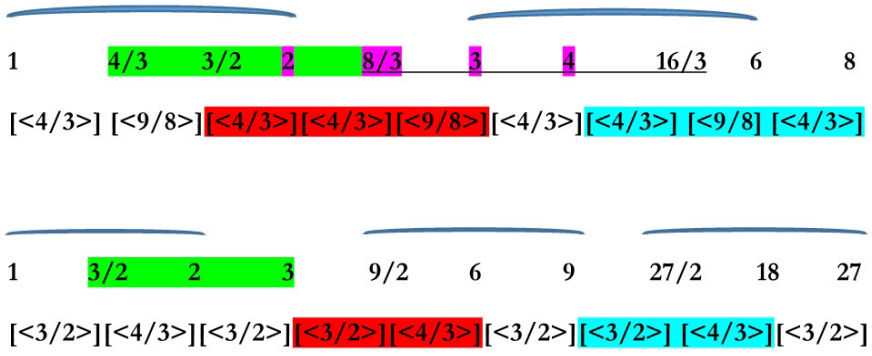
59 Brisson, *Le même et l'autre*, 316. Brisson's table is altered, in Table 7, only to the extent of emboldening the numbers of the primary sequence, so that the reader may more easily identify the first line as the line of doubles and the second as the line of triples. Also, Brisson has represented the intervals using a special type of parentheses not used here; this study compensates with its own way of setting off the same numbers.

TABLE 8 Doubles after the means operation at 36 A

(1:2)
(4/3:8/3 or 4:8)
3/2:3
(2:4)
8/3:16/3 or 8:16
3 to 6
9/2 to 9
9:18
27/2:27

TABLE 9 Table of intervals resulting from the insertion of means at 36 A

Doubles marked



ping double intervals or continuously overlapping triple intervals. In fact, each arm displays both simultaneously.

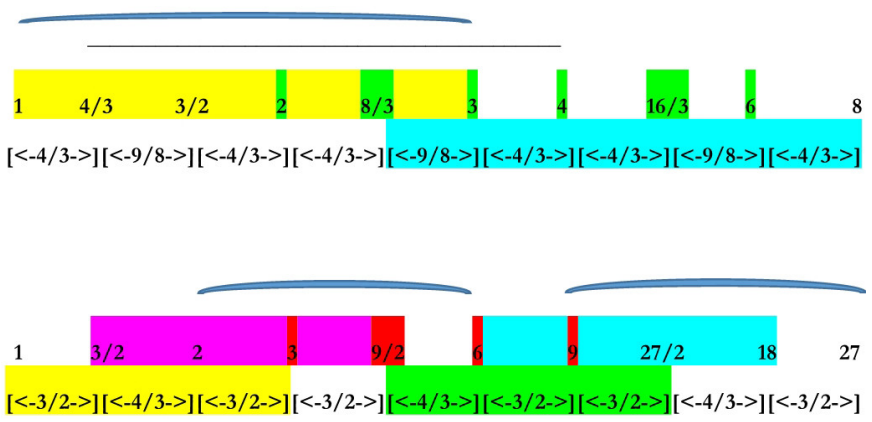
The original doubles, for example, are presented, in Table 8, emboldened in parentheses. The remainder are the new double intervals created by the nodes along both the double and triple arms of Table 7 after the means operation at 36 A. Note that they exist among both whole numbers and fractions. Inspection reveals the total set as a set of continuously overlapping doubles. Table 9, above, marked with the doubles, is useful to aid the reader's visualization. Note that limits bind the set. One does not, for example, find 16/3: 32/3 or 6 to 12 along the double arm because 32/3 and 12 exceed 8, the limit of the double extension. One does not find 18:36 along the triple arm because 36 exceeds 27.

TABLE 10 Triples after the means operation at 36 A

(1:3)
4/3:4
(3/2:9/2 or 3:9)
2:6
8/3:8
6:18
(9/2:27/2 or 9:27)

TABLE 11 Table of intervals resulting from the insertion of means at 36 A

Triples marked



Plato's nodes after the means operation at 36 A can also be read to define continuously overlapping triples along both the double and triple arms. The triples that the nodes define are marked in Table 10 with the original ones, emboldened, in parentheses. As in the case of the doubles, there are both new whole number triples and new triples among fractions. Inspection reveals the total set as a set of continuously overlapping triples. Table 11, above, marked with the triples, is useful to aid the reader's visualization.

Limits bind this set, too. Along the double arm: 4:12, 16/3:16, and 8:24 do not occur because the end terms of the proportions all exceed the limit of the double extension. Along the triple arm, 27/2:81/2 or 27:81 and 18:54 do not appear because the ending numbers all exceed the limit of the triple extension.

Sesquitertian parts are naturally immanent within all of the double and triple intervals along each arm. Double intervals consist in two sesquitertian parts and a sesquioctave interval. Triple intervals consist in three sesquitertian parts and a remainder over of 81/64. The latter remainder over is actually $9/8 \times 9/8$ or two sesquioctave intervals; so a triple is three sesquitertian parts and two sesquioctave intervals, when broken down into its simplest components. Note that the sesquitertian parts internal to the continuously overlapping triples comprise a different set of sesquitertian parts from those internal to the continuously overlapping doubles.

Because of the continuous and simultaneous overlap of double and triple intervals, respectively, along both the double and triple arms, one might well regard Plato's construction, resulting from 36 A, as two separate but simultaneously existing sets of periodic progressions of sesquitertian parts along the double and triple arms, which one is to find and fill at 36 B. It is not unreasonable to take 36 B as a direction to fill with sesquioctave intervals all of the sesquitertian parts explicitly defined or implicit within double and triple intervals *existing at the time of the command;* but no theoretical prospect could be more daunting, as the reader by now appreciates. One might, therefore, wish to seek some principle of selection.

There is a method to Plato's madness in his proliferation of doubles and triples through the insertion of means. He may have wanted, among other things, to educate his audience about the different ways in which doubles and triples can be filled with sesquitertian parts, so that they did not miss the less obvious possibilities. He probably did not intend that his reader ignore the sesquitertian parts immanent within the new double and triple intervals. He would surely have wanted to ensure that his reader was aware of those intervals and their internal sesquitertian parts; however, he probably did not want the reader to fill all of the sesquitertian parts it would be possible to find for all of the overlapping doubles and triples, whether original or new.

What Plato showed his reader concerning the possibilities for filling doubles and triples with sesquitertian parts, together with a few other considerations, is key to determining exactly what sesquitertian parts he wanted his reader to fill beyond those he obviously defined between the nodes. Plato was merely stimulating his reader to think by giving at least three possible alternatives for interpreting what sesquitertian parts are defined by the nodes, namely: (a) those obviously defined among them, as represented in Table 7; (b) all of those within the continuously overlapping triples; (c) all of those within the continuously overlapping doubles; or (d) some selective combination among (a)–(c).

The analyst must always bear in mind the warnings of the *Philebus* against either running on too quickly to infinity or cutting short the analysis prema-

turely (*Philebus* 16 C–E). If one, as a rule, always abides by the simplest defensible choices among options in the *Timaeus* derivation, then one eventually achieves a meaningful outcome for the charming game of hide and seek that Plato created. He made his reader work hard to find the treasure at the heart of the *Timaean* labyrinth. The text is a riddle with a correct solution; but it contains many possible byways to send a reader on hopeless and irrelevant quests. The analysis turns, at this point, then, to the first element of the riddle: how to identify the set of sesquitertian parts that Plato wanted his reader to fill with sesquioctave intervals, at 36 B.

CHAPTER 3

Solving the 36 B Conundrum: Deriving the Set of Sesquitertian Parts to Be Filled by Sesquioctave Intervals

The key to identifying the sesquitertian parts to be filled with sesquioctave intervals, at 36 B, is to start with the sesquitertian intervals obviously defined by the nodes of Table 7, and, then to recognize certain other possibilities for filling with sesquitertian parts the new whole number doubles and triples among the nodes. Concentrating only on the new whole number doubles and triples is justifiable in the quest for additional sesquitertian parts because of the special esteem in which the Pythagorean *mathematici* who influenced Plato held whole numbers.[1] They would have had no interest in filling doubles and triples among fractional parts just for the sake of filling them, although such parts come in handy for purposes of illustrating certain kinds of relations. If the *Timaeus* presents a Pythagorean cosmogony, as this study deems likely, then Pythagorean prejudices matter in the text's interpretation.

The new double and triple whole number intervals are the sole, new objects of interest also because they are the only ones that one might reasonably expect primarily interested Plato himself. Plato began his divisions at 35 B, after all, only with *such* double and triple intervals; further, it is those alone that he originally sought to fill at 35 C and 36 A. One can fairly speculate that he would want the sesquitertian parts within the new double and triple intervals filled with sesquioctave intervals; there is no apparent reason to ignore possibilities latent within some double and triple whole number intervals, after the means operation at 36 A, while exploring them for others. There is every reason, however, to be hesitant concerning doubles and triples among fractional parts. Plato performed no explicit operations on doubles and triples of the latter kind. It is reasonable and conservative, therefore, considering the options, to interpret 36 B as a direction to fill the sesquitertian parts that the nodes in Table 7 specifically define and, keeping the warnings of the *Philebus* in mind neither to run on too quickly to infinity nor to cut short the analysis prematurely (*Philebus* 16 C–E), to fill, as well, certain other sesquitertian parts immanent within the new double and triple whole number intervals.

1 Burkert referred to this special esteem. See Burkert, *Lore and Science*, 46.

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One can discern from inspecting Table 7, that the new whole number doubles and triples share some sesquitertian parts in common with the original double and triple intervals. However, the sesquitertian parts defined by the nodes in the table are not exhaustive of the sesquitertian parts implicit within either the new doubles and triples or even within the original doubles and triples. One may understand the means operation at 36 A as Plato's device for identifying and selecting the sesquitertian parts that he wanted to be filled with sesquioctave intervals, among the original whole number doubles and triples. One cannot make the same assumption for the new whole number doubles and triples because they come into existence only after the insertion of means into the original intervals; but, if one is to fill sesquitertian parts within them, beyond the sesquitertian parts that the table indicates they share in common with the original doubles and triples, then one must do so on the basis of some justifiable principle.

The necessary principle cannot be a strict equality principle, as between new and old doubles and triples, demanding the insertion of harmonic and arithmetic means into the new double and triple intervals. Plato did not direct his reader to insert additional harmonic and arithmetic means into any of the intervals existing after 36 A; so one cannot add or find sesquitertian parts to fill, for the new whole number doubles and triples, in that manner.

This chapter shows that Plato's designation of nodes at 36 A, selecting, *inter alia*, particular sesquitertian parts among the original doubles and triples, actually provides clues about the possibilities he wanted his reader to pursue for finding additional sesquitertian parts to fill within the new double and triple whole number intervals. It is best, then, to begin by concentrating on the sesquitertian parts explicitly defined by the nodes resulting from the insertion of means. One could attempt to identify the set merely by inspecting Table 7, but, to ensure completeness, a mathematically verifiable method is preferable.

1 Derivation of the Sesquitertian Parts

In the derivation that follows, a sesquitertian part explicitly defined by the set of nodes generated at 36 A (see Table 7) will be an interval $4/3 \times$ a node, such that the far limit of the interval, thus calculated, is also a node. Clearly, the sesquitertian parts thus discovered will be the entire set of such parts "visible" in Table 7 between the nodes. Upon testing each of the nodes produced by the operations of 36 A, mathematically, one obtains the set marked by checks in Table 12 below. The numbers in red are numbers arising in the verification process, not previously encountered by the reader in this study.

TABLE 12 Finding the sesquitertian parts explicitly defined by the "means" operations of 36 A

1 × 4/3 = 4/3	√
4/3 × 4/3 = 16/9	Not a node in Table 7 *
3/2 × 4/3 = 12/6 = 6/3 (or 4/2) = 2	√
2 × 4/3 = 8/3	√
8/3 × 4/3 = 32/9	Not a node in Table 7 *
3 × 4/3 = 12/3 = 4	√
4 × 4/3 = 16/3	√
16/3 × 4/3 = 64/9	Not a node in Table 7 *
6 × 4/3 = 24/3 = 8	√
8 × 4/3 = 32/3	Not a node in Table 7
9/2 × 4/3 = 36/6 = 18/3 (or 12/2) = 6	√
9 × 4/3 = 36/3 = 12	Not a node in Table 7 *
27/2 × 4/3 = 108/6 = 54/3 (or 36/2) = 18	√
18 × 4/3 = 72/3 = 24	Not a node in Table 7 *
27 × 4/3 = 108/3 =36	Not a node in Table 7

TABLE 13 Sesquitertian parts explicitly defined by the "means" operation of 36 A

Between 1 and 4/3
Between 3/2 and (12/6 or 6/3 or 4/2 or 2)
Between 2 and 8/3
Between 3 and (12/3 or 4)
Between 4 and 16/3
Between 6 and (24/3 or 8)
Between 9/2 and (36/6 or 18/3 or 12/2 or 6)
Between 27/2 and (108/6 or 54/3 or 36/2 or 18)

The sesquitertian parts explicitly defined by the nodes do not, as has been noted, comprise the set of all possible sesquitertian parts within the original double and triple intervals. For example, the parts marked with an asterisk, in Table 12, above, are also included in that set. The parts unmarked by a check or an asterisk in Table 12 are, in contrast, beyond the bounds of the double and triple extensions respectively marked by 8 and 27, as relevant. In any case, the set of sesquitertian parts explicitly defined by the nodes, based on Table 12 above, is designated in Table 13.

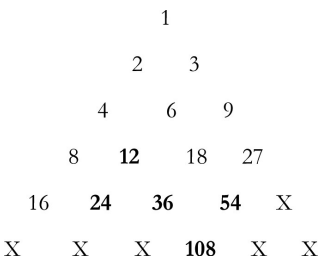

FIGURE 8
Crantor matrix expanded to include numbers representing sesquitertian intervals explicitly resulting from the "means" operation at 36 A

The sesquitertian parts yielded by the method of verification exemplified above are the same eight unique sesquitertian parts that Brisson found. However, the method of this study yields additional names for some intervals, not presented in Brisson's analysis. Those numbers are in red. The strictly mathematical approach of this study must recognize the new interval names. If one seeks to identify the mathematical structure of the world soul, then any redundancy of names produced by Plato's operations for the same interval is part of that structure and must be represented. The *Philebus* warns that one must find every number in something before one decides one knows it (*Philebus* 16 D). Fractions that one must reduce to intermediate forms before arriving at a final form are surely immanent within the thing being defined. The philosopher spends far too much time on things that "mediate" to warrant their summary dismissal.[2]

Assuming that Plato's directions are number generators, the above definition of sesquitertian intervals yields the following new numbers for the Crantor matrix, as set forth in Figure 8, according to principles already articulated above: 12, 24, 36, 54, and 108.

The derivation would be incomplete if it stopped with the definition of sesquitertian intervals, thus far achieved, for all of the reasons already given above. So far, it yields only the sesquitertian parts defined by the overlap among the multiple doubles and triples that Plato's nodes explicitly delineate (see Tables 7 and 13), ignoring all other possibilities immanent within those intervals. It has been noted, above, that, among these other immanent possibilities, the reader was probably called upon to pay particular attention to those involving the new double and triple whole number intervals and, further, that some principle must be found that would both allow the reader to fill those intervals with sesquitertian parts and provide him some direction for doing so that, minding the *Philebus*, would avoid his running off to infinity. The principle emerges through a closer study of Table 7. For the reader's convenience it appears, once more, below, as Table 14.

2 See, e.g., Plato *Philebus* 16 E, warning the analyst against allowing the intermediates to escape his notice.

TABLE 14 Brisson table of intervals resulting from the insertion of means at 36 A

1	4/3	3/2	**2**	8/3	3	**4**	16/3	6	**8**
<-4/3-><-9/8-><-4/3-><-4/3-><-9/8-><-4/3-><-4/3-><-9/8-><-4/3->									
1	3/2	2	**3**	9/2	6	**9**	27/2	18	27
<-3/2-><-4/3-><-3/2-><-3/2-><-4/3-><-3/2-><-3/2-><-4/3-><-3/2->									

Table 14 displays several options for filling double and triple intervals with sesquitertian parts, inspiring an attentive analyst to construct an exhaustive list of the conceivable possibilities. Table 15 is just such an inventory of the options presented by Table 14. Along with a few other indicators, it provides clues concerning how to approach the identification of sesquitertian parts to be filled within the new double and triple whole number intervals. The possibilities for filling double and triple intervals with sesquitertian parts are the following.

TABLE 15 Universe of possible patterns for filling double and triple intervals with sesquitertian parts

Double intervals: simple presentation	**Double intervals: ambiguous presentation**	**Triple intervals: simple presentation**	**Triple intervals: ambiguous presentation**
(1) 4/3 × 4/3 × 9/8 (+ + #)	(a) 3/2 × 4/3 Represents the following twofold ambiguity: (9/8 × 4/3) × 4/3 (# + +) (4/3 × 9/8) × 4/3 (+ # +)	(1) 4/3 × 4/3 × 4/3 × 9/8 × 9/8 (+ + + # #)	(a) 3/2 × 4/3 × 3/2 (-- + --) Represents a fourfold ambiguity among the following possibilities: (9/8 × 4/3) × 4/3 × (9/8 × 4/3) (# + + # +) (no. 9 left) (4/3 × 9/8) × 4/3 × (9/8 × 4/3) (+ # + # +) (no. 7 left) (9/8 × 4/3) × 4/3 × (4/3 × 9/8) (# + + + #) (no. 10 left) (4/3 × 9/8) × 4/3 × (4/3 × 9/8) (+ # + + #) (no. 8 left)

TABLE 15 Universe of possible patterns for filling intervals with sesquitertian parts (*cont.*)

Double intervals: simple presentation	**Double intervals: ambiguous presentation**	**Triple intervals: simple presentation**	**Triple intervals: ambiguous presentation**
(2) 4/3 × 9/8 × 4/3 (+ # +)	(b) 4/3 × 3/2 Represents the following twofold ambiguity: 4/3 × (9/8 × 4/3) (+ # +) 4/3 × (4/3 × 9/8) (+ + #)	(2) 4/3 × 4/3 × 9/8 × 4/3 × 9/8 (+ + # + #)	(b) 4/3 × 3/2 × 3/2 (+ -- --) Represents a fourfold ambiguity among the following possibilities: 4/3 × (4/3 × 9/8) × (4/3 × 9/8) (+ + # + #) (no. 2 left) 4/3 × (9/8 × 4/3) × (4/3 × 9/8) (+ # + + #) (no. 8 left) 4/3 × (9/8 × 4/3) × (9/8 × 4/3) (+ # + # +) (no. 7 left) 4/3 × (4/3 × 9/8) × (9/8 × 4/3) (+ + # # +) (no. 6 left)
(3) 9/8 × 4/3 × 4/3 (# + +)		(3) 4/3 × 9/8 × 9/8 × 4/3 × 4/3 (+ # # + +) (4) 9/8 × 4/3 × 9/8 × 4/3 × 4/3 (# + # + +) (5) 9/8 × 9/8 × 4/3 × 4/3 × 4/3 (# # + + +) (6) 4/3 × 4/3 × 9/8 × 9/8 × 4/3 (+ + # # +) (7) 4/3 × 9/8 × 4/3 × 9/8 × 4/3 (+ # + # +) (8) 4/3 × 9/8 × 4/3 × 4/3 × 9/8 (+ # + + #) (9) 9/8 × 4/3 × 4/3 × 9/8 × 4/3 (# + + # +) (10) 9/8 × 4/3 × 4/3 × 4/3 × 9/8 (# + + + #)	(c) 3/2 × 3/2 × 4/3 (-- -- +) Represents a fourfold ambiguity among the following possibilities: (4/3 × 9/8) × (4/3 × 9/8) × 4/3 (+ # + # +) (no. 7 left) (9/8 × 4/3) × (4/3 × 9/8) × 4/3 (# + + # +) (no. 9 left) (9/8 × 4/3) × (9/8 × 4/3) × 4/3 (# + # + +) (no. 4 left) (4/3 × 9/8) × (9/8 × 4/3) × 4/3 (+ # # + +) (no. 3 left)

The ratio 4/3, in Table 15, represents the sesquitertian parts. The 9/8 ratio, in the doubles, is the remainder over in every double interval after the insertion of two possible sesquitertian parts. Two 9/8 ratios occur in each triple interval upon the inclusion of three possible sesquitertian parts. The 3/2 ratio, in the "ambiguous presentation" column for triples and doubles, represents $4/3 \times 9/8$. Simple math verifies that the products of the doubles and triples columns are, respectively, doubles and triples. The parenthetical nonnumerical symbols in the table designate the ratio patterns represented.

To take stock of the patterns exhibited in Tables 7 and 14 for filling double and triple intervals with sesquitertian parts, one might start with the double intervals. These fall into three groups: (A) original double intervals; (B) new double whole number intervals; (C) new double intervals involving fractions. Ambiguous patterns represent all possibilities among which they are ambiguous. The inventory (referenced in Table 15) of the relevant patterns among the nodes defined at 36 A and the conclusions to be drawn on the basis of that inventory are set forth in Table 16.

TABLE 16 Inventory of patterns for filling double intervals with sesquitertian parts explicitly emerging after 36 A

A. Original double intervals		
(1) 1: 2	(1) 4/3 × 9/8 × 4/3 (+ # +) (no. 2) (2) 3/2 × 4/3 (9/8 × 4/3) × 4/3 (# + +) (no. 3) (4/3 × 9/8) × 4/3 (+ # +) (no. 2)	Two no. 2's (+ # +) One no. 3 (# + +)
(2) 2: 4	(1) 4/3 × 9/8 × 4/3 (+ # +) (no. 2)	One no. 2 (+ # +)
(3) 4:8	(1) 4/3 × 9/8 × 4/3 (+ # +) (no. 2)	One no. 2 (+ # +) One no. 3 (# + +)
(3*) 4/3:8/3 (alternate version of 4:8)	(2) 9/8 × 4/3 × 4/3 (# + +) (no. 3)	
Results for original intervals		Four no. 2's (+ # +) Two no. 3's (# + +)
B. New double whole number intervals		
(1) 3:6	(1) 4/3 × 4/3 × 9/8 (+ + #) (no. 1) (2) 3/2 × 4/3 (9/8 × 4/3) × 4/3 (# + +) (no. 3) (4/3 × 9/8) × 4/3 (+ # +) (no. 2)	One no. 1 (+ + #) One no. 2 (+ # +) One no. 3 (# + +)

TABLE 16 Patterns for filling doubles with sesquitertian parts after 36 A (*cont.*)

(2) 8:16	(1) 9/8 × 4/3 × 4/3 (# + +) (no. 3)	One no. 3 (# + +)
(2*) 8/3:16/3 (alternate version of 8:16)		
(3) 9:18	(1) 3/2 × 4/3 (9/8 × 4/3) × 4/3 (# + +) (no. 3) (4/3 × 9/8) × 4/3 (+ # +) (no. 2)	One no. 2 (+ # +) One no. 3 (# + +)
Results for new double whole number intervals		One no. 1 (+ + #) Two no. 2's (+ # +) Two no. 3's (# + +)
C. New double whole number intervals involving fractions		
(1) 3/2:3	(1) 4/3 × 4/3 × 9/8 (+ + #) (no. 1) (2) 4/3 × 3/2 4/3 × (4/3 × 9/8) (+ + #) (no. 1) 4/3 × (9/8 × 4/3) (+ # +) (no. 2)	Two no. 1's (+ + #) One no. 2 (+ # +)
(2) 9/2 to 9	(1) 4/3 × 3/2 4/3 × (4/3 × 9/8) (+ + #) (no. 1) 4/3 × (9/8 × 4/3) (+ # +) (no. 2)	One no. 1 (+ + #) One no. 2 (+ # +)
(3) 27/2: 27	(1) 4/3 × 3/2 4/3 × (4/3 × 9/8) (+ + #) (no. 1) 4/3 × (9/8 × 4/3) (+ # +) (no. 2)	One no. 1 (+ + #) One no. 2 (+ # +)
Results for new double intervals involving fractions		Four no. 1's (+ + #) Three no. 2's (+ # +)
Overall totals:		**Five no. 1's (+ + #)** **Nine no. 2's (+ # +)** **Four no. 3's (# + +)**
Original intervals and new double whole number intervals alone	**One no. 1 (+ + #)** **Six no. 2's (+ # +)** **Four no. 3's (# + +)**	
Original intervals and new double intervals involving fractions alone	**Four no. 1's (+ + #)** **Seven no. 2's (+ # +)** **Two no. 3's (# + +)**	

TABLE 16 Patterns for filling doubles with sesquitertian parts after 36 A (*cont.*)

New double whole number intervals and new double intervals involving fractions alone	**Five no. 1's (+ + #)** **Five no. 2's (+ # +)** **Two no. 3's (# + +)**

Summary: There is a roughly equal asymmetry around the second type of pattern (+ # +) in the overall result and a slight dearth of type no. 3 (# + +). An asymmetry also occurs around the second type (+ # +) when the original intervals are combined with either the new double whole number intervals, alone, or with the new double intervals involving fractions alone. The asymmetry in the former case is greater than the asymmetry in the latter case. There is a dearth of type no. 1 (+ + #) in the comparison of the original intervals with the new double whole number intervals. In the other case involving the comparison of original intervals with new double intervals involving fractions, there is a dearth of type no. 3 (# + +).

There is an inverse but equal asymmetry around the third type of pattern (# + +) when the new double whole number intervals are compared with the new double intervals involving fractions alone. The first type of pattern (+ + #), alone, is not the center of any kind of asymmetry. To make a symmetry among the asymmetries more patterns of the third type would be required.

Conclusions: The dearth of type no. 1 (+ + #), in the case of the comparison of new double whole number intervals with the originals is probably more important than the dearth of type no. 3 (# + +) as between new double whole number and new double intervals involving fractions alone just because it displays an underrepresented type among *the whole number intervals*. In the overall pattern it really cannot be said that there is underrepresentation of no. 3 (# + +) vs. no. 1 (+ + #) or no. 1 (+ + #) vs. no. 3 (# + +).

Plato may have sought to draw attention to asymmetries in representation among different types of patterns for filling the continuously overlapping doubles with sesquitertian parts; so one should pay attention to them. In this regard, underrepresentation of the first type of pattern (+ + #) stands out both because, unlike the other patterns, it is not the center of any asymmetry and because it is scarce among the whole number double intervals.

Indicated action: Add more patterns of the first type (+ + #) barring competing factors.

As Table 16 indicates, relevant asymmetries in the patterns for filling double intervals with sesquitertian parts seem to indicate that more parts of the first type, $4/3 \times 4/3 \times 9/8$ (+ + #) are required. A corresponding analysis of the triple intervals follows.

TABLE 17 Inventory of patterns for filling triple intervals with sesquitertian parts explicitly emerging after 36 A

A. Original triple intervals		
(1) 1:3	(1) 4/3 × 9/8 × 4/3 × 4/3 × 9/8 (+ # + + #) (no. 8)	One no. 7 (+ # + # +) Two no. 8's (+ # + + #) One no. 9 (# + + # +) One no. 10 (# + + + #)
	(2) 3/2 × 4/3 × 3/2	
	(9/8 × 4/3) × 4/3 × (9/8 × 4/3) (# + + # +) (no. 9)	
	(4/3 × 9/8) × 4/3 × (9/8 × 4/3) (+ # + # +) (no. 7)	
	(9/8 × 4/3) × 4/3 × (4/3 × 9/8) (# + + + #) (no. 10)	
	(4/3 × 9/8) × 4/3 × (4/3 × 9/8) (+ # + + #) (no. 8)	
(2) 3:9 (2*) 3/2:9/2 (alternate version of 3:9)	(1) 3/2 × 4/3 × 3/2	One no. 2 (+ + # + #) One no. 6 (+ + # # +) Two no. 7's (+ # + # +) Two no. 8's (+ # + + #) One no. 9 (# + + # +) One no. 10 (# + + + #)
	(9/8 × 4/3) × 4/3 × (9/8 × 4/3) (# + + # +) (no. 9)	
	(4/3 × 9/8) × 4/3 × (9/8 × 4/3) (+ # + # +) (no. 7)	
	(9/8 × 4/3) × 4/3 × (4/3 × 9/8) (# + + + #) (no. 10)	
	(4/3 × 9/8) × 4/3 × (4/3 × 9/8) (+ # + + #) (no. 8)	
	(2) 4/3 × 3/2 × 3/2	
	4/3 × (4/3 × 9/8) × (4/3 × 9/8) (+ + # + #) (no. 2) 4/3 × (9/8 × 4/3) × (4/3 × 9/8) (+ # + + #) (no. 8)	
	4/3 × (9/8 × 4/3) × (9/8 × 4/3) (+ # + # +) (no. 7)	
	4/3 × (4/3 × 9/8) × (9/8 × 4/3) (+ + # # +) (no. 6)	

TABLE 17 Patterns for filling triples with sesquitertian parts after 36 A (*cont.*)

(3) 9:27	(1) 3/2 × 4/3 × 3/2	One no. 2 (+ + # + #) One no. 6 (+ + # # +)
(3*) 9/2:27/2 (alternate version of 9:27)	(9/8 × 4/3) × 4/3 × (9/8 × 4/3) (# + + # +) (no. 9)	Two no. 7's (+ # + # +) Two no. 8's (+ # + + #) One no. 9 (# + + # +)
	(4/3 × 9/8) × 4/3 × (9/8 × 4/3) (+ # + # +) (no. 7)	One no. 10 (# + + + #)
	(9/8 × 4/3) × 4/3 × (4/3 × 9/8) (# + + + #) (no. 10)	
	(4/3 × 9/8) × 4/3 × (4/3 × 9/8) (+ # + + #) (no. 8)	
	(2) 4/3 × 3/2 × 3/2	
	4/3 × (4/3 × 9/8) × (4/3 × 9/8) (+ + # + #) (no. 2)	
	4/3 × (9/8 × 4/3) × (4/3 × 9/8) (+ # + + #) (no. 8)	
	4/3 × (9/8 × 4/3) × (9/8 × 4/3) (+ # + # +) (no. 7)	
	4/3 × (4/3 × 9/8) × (9/8 × 4/3) (+ + # # +) (no. 6)	
Results for original intervals		**Two no. 2's (+ + # + #)** **Two no. 6's (+ + # # +)** **Five no. 7's (+ # + # +)** **Six no. 8's (+ # + + #)** **Three no. 9's (# + + # +)** **Three no. 10's (# + + + #)** ***Missing*: Nos. 1, 3, 4, 5**
B. New triple whole number intervals		
(1) 2:6	(1) 4/ 3 × 9/8 × 4/3 × 4/3 × 9/8 (+ # + + #) (no. 8)	One no. 3 (+ # # + +) One no. 4 (# + # + +) One no. 7 (+ # + # +) One no. 8 (+ # + + #) One no. 9 (# + + # +)
	(2) 3/2 × 3/2 × 4/3	
	(4/3 × 9/8) × (4/3 × 9/8) × 4/3 (+ # + # +) (no. 7)	

TABLE 17 Patterns for filling triples with sesquitertian parts after 36 A (*cont.*)

	(9/8 × 4/3) × (4/3 × 9/8) × 4/3 (# + + # +) (no. 9)	
	(9/8 × 4/3) × (9/8 × 4/3) × 4/3 (# + # + +) (no. 4)	
	(4/3 × 9/8) × (9/8 × 4/3) × 4/3 (+ # # + +) (no. 3)	
(2) 6:18	(1) 3/2 × 3/2 × 4/3	One no. 3 (+ # # + +) One no. 4 (# + # + +) One no. 7 (+ # + + #) One no. 9 (# + + # +)
	(4/3 × 9/8) × (4/3 × 9/8) × 4/3 (+ # + # +) (no. 7)	
	(9/8 × 4/3) × (4/3 × 9/8) × 4/3 (# + + # +) (no. 9)	
	(9/8 × 4/3) × (9/8 × 4/3) × 4/3 (# + # + +) (no. 4)	
	(4/3 × 9/8) × (9/8 × 4/3) × 4/3 (+ # # + +) (no. 3)	
Results for new triple whole number intervals		**Two no. 3's (+ # # + +)** **Two no. 4's (# + # + +)** **Two no. 7's (+ # + # +)** **One no. 8 (+ # + + #)** **Two no. 9's (# + + # +)** ***Missing*: Nos. 1, 2, 5, 6, 10**
C. New triple whole number intervals involving fractions		
(1) 4/3:4	(1) 9/8 × 4/ 3 × 4/3 × 9/8 × 4/3 (# + + # +) (no. 9)	One no. 9 (# + + # +)
(2) 8/3:8	(1) 9/8 × 4/3 × 4/3 × 9/8 × 4/3 (# + + # +) (no. 9)	One no. 9 (# + + # +)
Results for new triple intervals involving fractions		**Two no. 9's (# + + # +)** ***Missing*: Nos. 1–8 and 10**

TABLE 17 Patterns for filling triples with sesquitertian parts after 36 A (*cont.*)

Overall totals:		**Two no. 2's (+ + # + #)** **Two no. 3's (+ # # + +)** **Two no. 4's (# + # + +)** **Two no. 6's (+ + # # +)** **Seven no. 7's (+ # + # +)** **Seven no. 8's (+ # + + #)** **Seven no. 9's (# + + # +)** **Three no. 10's (# + + + #)** **Missing: No. 1 (+ + + # #)** **and no. 5 (# # + + +)**
Original intervals and new triple whole number intervals alone	**Two no. 2's (+ + # + #)** **Two no. 3's (+ # # + +)** **Two no. 4's (# + # + +)** **Two no. 6's (+ + # # +)** **Seven no. 7's (+ # + # +)** **Seven no. 8's (+ # + + #)** **Five no. 9's (# + + # +)** **Three no. 10's (# + + + #)** **Missing: No. 1 (+ + + # #)** **and no. 5 (# # + + +)**	
Original intervals and new triple intervals involving fractions alone	**Two no. 2's (+ + # + #)** **Two no. 6's (+ + # # +)** **Five no. 7's (+ # + # +)** **Six no. 8's (+ # + + #)** **Five no. 9's (# + + # +)** **Three no. 10's (# + + + #)** ***Missing*: Nos. 1, 3, 4, 5**	
New Triple whole number intervals and new triple intervals involving fractions alone	**Two no. 3's (+ # # + +)** **Two no. 4's (# + # + +)** **Two no. 7's (+ # + # +)** **One no. 8 (+ # + + #)** **Four no. 9's (# + + # +)** ***Missing*: Nos. 1, 2, 5, 6, 10**	

TABLE 17 Patterns for filling triples with sesquitertian parts after 36 A (*cont.*)

Summary: In all of the comparisons of intervals above there is a roughly equal asymmetry of all patterns represented around the three most common patterns nos. 7 **(**+ # + # +**)**, 8 **(**+ # + + #**)**, and 9 **(**# + + # +**).** Aside from their separate occurrences in a fully articulated five element pattern, these three sequences result together from the 3/2 × 4/3 × 3/2 pattern, the most common ambiguous sequence (it occurs three times; the other two ambiguous sequences occur twice). No. 7 and No. 8 also result together from the 4/3 × 3/2 × 3/2 pattern; while No. 7 and No. 9 result separately from the 3/2 × 3/2 × 4/3 pattern. Aside from their separate occurrences in a fully articulated five element pattern, Nos. 3 and 4 result, otherwise, only from the 3/2 × 3/2 × 4/3 ambiguity, nos. 2 and 6, only from the 4/3 × 3/2 × 3/2 ambiguity, and no. 10 only from the 3/2 × 4/3 × 3/2 ambiguity. Patterns nos. 1 and 5 do not arise, at all, from any of the ambiguous sequences but, apparently, can only occur in fully expanded five element sequences. They are therefore, the rarest of all types.

Because it can result from all three ambiguous patterns, sequence no. 7 (+ # + # +) is a definite center among the triple intervals. It is followed closely by nos. 8 and 9, each of which can result from two of the three ambiguous sequences, as well as occur, separately, as expanded patterns. The most conspicuous feature about the examination of triples is the complete absence of patterns of the types represented by nos. 1 (+ + + # #) and 5 (# # + + +).

Conclusions: Plato may have been trying to draw attention to the reasons for frequency variance among the different options for arranging sesquitertian parts in triple intervals; further he may have wanted the reader to focus on the relative rarity of two particular types of patterns for filling triples, pattern no. 1 (+ + + # #) and pattern no. 5 (# # + + +).

Indicated action : Add more patterns of the types nos. 1 and 5 (+ + + # #) and (# # + + +), if also indicated by other factors.

The analysis of the triples indicates that two types of patterns for filling triples are extremely rare, namely, types nos. 1 and 5 (+ + + # #) and (# # + + +). As was previously observed, doubles of the type, 4/3 × 4/3 × 9/8 (+ + #) seem indicated. With these results in mind, one can identify which sesquitertian intervals, within the new double and triple intervals arising from the means operation at 36 A, are to be filled with sesquioctave intervals at 36 B. Once identified, these intervals, added to those explicitly exhibited among the nodes of Tables 7 and 14 are the complete set of sesquitertian parts to be filled.

Observe that one cannot simply divide the original double and triple intervals in a new way to obtain patterns of the seemingly required, rare types because Plato gave specific directions at 36 A concerning precisely how he wanted the original intervals divided. Therefore, to rectify the imbalances produced by the divisions of those original intervals, one must look to the new double and triple whole number intervals. They are presented, once more, in Table 18.

TABLE 18 New double and triple whole number intervals arising from the divisions of 36 A

New double whole number intervals resulting from the divisions of 36 A	**New triple whole number intervals resulting from the divisions of 36 A**
Between 3 and 6	Between 2 and 6
Between 9 and 18	Between 6 and 18
Between 8 and 16	

One cannot just divide the new double and triple intervals in Table 18 to rectify the pattern imbalances noted, without first considering every other factor that might provide a clue concerning what the reader is to do with them. One should be sure that all relevant factors point in the same direction and, also, be as conservative as possible about finding additional sesquitertian parts. Plato would not simply have given an analyst *carte blanche*.

There is at least one more relevant factor to consider; and it bears upon the division of the triples. Selecting sesquitertian parts from the triples in the pattern (# # + + +) would require one to make sesquioctave intervals before Plato's specific direction to do so. He did not tell the reader to insert sequioctave intervals until 36 B. All that the analyst should be trying to do at 36 A is to identify the sesquitertian parts that Plato wanted his reader to fill. He should not make calculations to find intervals that Plato did not let the reader make until later in the text.

While it is true that sesquioctave intervals, in fact, result from the insertion of harmonic and arithmetic means at 36 A, the analyst does not specifically calculate those intervals. They happen because of other explicit calculations that Plato had the reader perform. The only possibility of interest, then, regarding the selection of particular sesquitertian intervals from the new whole number triples has to be the pattern (+ + + # #). Proceeding according to this pattern allows one to find sesquitertian parts without inserting additional intervals not yet directed by Plato. One simply stops calculating after finding the three sesquitertian parts and the remainder is a ditone naturally divisible into two equal parts.

The same reasoning applies in the case of the doubles to rule out all possibilities except the first listed. The remainder in that case, after finding the sesquitertian parts, is a single 9/8 interval. Accordingly, the new double whole number intervals should be divided in the pattern (+ + #) and the new triple whole number intervals in the pattern (+ + + # #). Table 19 below finds the sesquitertian parts pertaining to the new double and triple intervals resulting upon the divisions of 36 A in line with the above considerations.

TABLE 19 Sesquitertian parts pertaining to the new double and triple intervals created by the divisions of 36 A

Sesquitertian parts pertaining to the new double intervals resulting from the divisions of 36 A	**Sesquitertian parts pertaining to the new triple intervals resulting from the divisions of 36 A**
3 and 6	2 and 6
(A) 3 × 4/3 = 12/3 = 4 (B) 12/3 × 4/3 = 48/9 = 16/3 and 4 × 4/3 = 16/3[3]	(A) 2 × 4/3 = 8/3 (B) 8/3 × 4/3 = 32/9 (C) 32/9 × 4/3 = 128/27
Further multiplication by 4/3 exceeds 6.	Further multiplication by 4/3 exceeds 6.
So there are sesquitertian intervals between (3 and 12/3) or (3 and 4) and between (12/3 and 48/9) or (12/3 and 16/3) or (4 and 16/3).	So there are sesquitertian parts between (2 and 8/3) and between (8/3 and 32/9) and between (32/9 and 128/27).
9 and 18	6 and 18
(A) 9 × 4/3 = 36/3 = 12 (B) 36/3 × 4/3 = 144/ 9 = 48/3 = 16 and 12 × 4/3 = 48/3 = 16	(A) 6 × 4/3 = 24/3 = 8 (B) 24/3 × 4/3 = 96/9 = 32/3 and 8 × 4/3 = 32/3 (C) 96/9 × 4/3 = 384/27 = 128/9 and 32/3 × 4/3 = 128/9
Further multiplication by 4/3 exceeds 18.	Further multiplication by 4/3 exceeds 18.
So there are sesquitertian intervals between (9 and 36/3) or (9 and 12) and between (36/3 and 144/9) or (36/3 and 48/3) or (36/3 and 16) and between (12 and 48/3) or (12 and 16).	So there are sesquitertian intervals between (6 and 24/3) or between (6 and 8) and between (24/3 and 96/9) or (24/3 and 32/3) and between (8 and 32/3) and between (96/9 and 384/27) or (96/9 and 128/9) and between (32/3 and 128/9).

3 Because redundancy in the names for mathematical ratios is important for an interpretation of the construction of the world soul as a number generator, the same mathematical oper-

TABLE 19 Sesquitertian part pertaining to new doubles and triples (*cont.*)

Sesquitertian parts pertaining to the new double intervals resulting from the divisions of 36 A	**Sesquitertian parts pertaining to the new triple intervals resulting from the divisions of 36 A**
8 and 16 (A) 8 × 4/3 = 32/3 (B) 32/3 × 4/3 = 128/9 Further multiplication by 4/3 exceeds 16. So there are sesquitertian parts between (8 and 32/3) and between (32/3 and 128/9).	

			1					
		2		3				
	4		6		9			
8		12		18		27		
	16		24		36		54	X
32		48		X		108	X	X
X	**96**		**144**		X	X	X	X
128	X	X		X	X	X	X	X
X	**384**	X	X		X	X	X	X

FIGURE 9 Crantor matrix expanded to include ratios representing relevant sesquitertian parts within the new double and triple intervals resulting from "means" operation at 36 A

ation is performed on both names for the endpoint of the first sesquitertian part to fix the endpoint of the second. The same principle governs all redundant multiplication in the table.

Following the methodological decision to regard Plato's text from 35 B to 36 B as a number generating machine, one must continue the inventory of all names for numbers arising in any calculations that Plato's text demands. The new numbers needed to represent the ratios in Table 19's derivations of sesquitertian parts from new double and triple intervals are 32, 48, 96, 128, 144, and 384, as indicated in Figure 9, above (X's in Figure 9 are empty spaces to keep the array symmetrical).

Figure 9, then, represents the Crantor extension, upon the discovery of all the sesquitertian parts in the new doubles and triples that one must fill with sesquioctave intervals per Plato's 36 B. Table 20 represents all relevant sesquitertian parts to be filled with such intervals, both those identified for the original intervals and those selected within the new octave and triple ratios.[4]

TABLE 20 Distinct sesquitertian parts to fill per 36 B

(1)	Between 1 and 4/3
(2)	Between (a) 3/2 and 12/6 **or** (b) 3/2 and 6/3 **or** (c) 3/2 and 4/2 **or** (d) 3/2 and 2
(3)	Between 2 and 8/3
(4)	Between (a) 3 and 12/3 **or** (b) 3 and 4
(5)	Between 4 and 16/3
(5 A)	Between (a) 12/3 and 48/9 **or** (b) 12/3 and 16/3 **or** (c) 4 and 48/9 **(See above no. 5 for 4:16/3)**
(6)	Between (a) 6 and 24/3 **or** (b) 6 and 8
(7)	Between (a) 9/2 and 36/6 **or** (b) 9/2 and 18/3 **or** (c) 9/2 and 12/2 **or** (d) 9/2 and 6
(8)	Between (a) 27/2 and 108/6 **or** (b) 27/2 and 54/3 **or** (c) 27/2 and 36/2 **or** (d) 27/2 and 18
(9)	Between (a) 9 and 36/3 **or** (b) 9 and 12 **(See above no. 4 (b) for 3 and 4)**
(10)	Between (a) 36/3 and 144/9 **or** (b) 36/3 and 48/3 **or** (c) 36/3 and 16 **or** (d) 12 and 144/9 **or** (e) Between 12 and 48/3 **or** (f) 12 and 16 **(See above no. 4 (b) for 3 and 4)**
(11)	Between 8 and 32/3
(11 A)	Between (a) 24/3 and 96/9 **or** (b) 24/3 and 32/3 **or** (c) 8 and 96/9

4 The table consolidates redundant intervals. The variations of names for given intervals sometimes reflect this consolidation. Variations of names are also derived from factoring larger fractions to smaller equivalent fractions. Factoring is done by 2's and 3's, so that all names for all intervals are taken into account.

TABLE 20 Distinct sesquitertian parts to fill per 36 B (*cont.*)

(12)	Between 32/3 and 128/9
(12 A)	Between (a) 96/9 and 384/27 **or** (b) 96/9 and 128/9 **or** (c) 32/3 and 384/27
(13)	Between 8/3 and 32/9
(14)	Between 32/9 and 128/27

Note that Table 20, the table of all sesquitertian parts to fill, lists all variations for names of these intervals that the calculations required for finding them allow one to identify. It is probably clear, by now, that alternate names derive from factoring fractions by two's and three's to reach the simplest forms and carefully identifying all such factoring possibilities. This same mode of proceeding will also apply at the next step of this derivation, the subject of Chapter 4, when sesquioctave intervals are inserted into sesquitertian parts and the remainders over verified.

CHAPTER 4

The Sesquioctave Operation within the Sesquitertian Parts

Plato's directions at 36 B indicate that sesquioctave intervals must be inserted into all fourteen of the sesquitertian parts identified in Chapter 3. Plato indicated that there would be a remainder over of 256/243 within each part after the operation. This is the ratio for the *leimma* in the musical interpretation that this study will show is justified for Plato's text. It is not immediately straightforward how one is to fill the sesquitertian parts with sesquioctave intervals, since there is more than one way of accomplishing the task. The possibilities appear in Table 21 below.

Upon performing the multiplications indicated in Table 21, one finds that the products all define sesquitertian parts, i.e., 4/3 intervals. Clearly, then, at 36 B, Plato gave his readers the ratios they need to fill the sesquitertian parts completely.

Determining how to fill the sesquitertian parts is similar to the former process of determining how to fill, with those parts, the new double and triple whole number intervals, produced by Plato's divisions at 36 A. Plato did not tell the reader to fill any interval with ratios of 256/243. The only ratios he wanted the analyst actively to insert were the sesquioctave portions. The remainder over has to be the passive result of specifically that operation.

The second of the three possibilities designated in Table 21 would require one to insert a ratio of 256/243 in the second step; or else, one would multiply the beginning of the sesquitertian interval by 9/8 and the end of the interval by 8/9 (working backward) to find the exact location of the 256/243 ratio in the sesquitertian part. Plato did not, however, draw the reader's attention to the latter "reciprocal" operation; and no one has, in fact, read the 36 B direction to require it. The latter circumstance is not determinative in itself, but it is persuasive when other factors are considered. Plato's text, while not definitively foreclosing the possibility of a 256/243 interval in the middle of a sesquitertian part, does not read as though the reader is supposed to get bogged down with finding that ratio. The simplest way of discovering it mathematically is simply to insert two consecutive sesquioctave intervals and then to calculate the remaining portion over of 256/243; further Plato's text reads as though he had just such a consecutive process in mind:

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TABLE 21 Universe of possibilities for inserting sesquioctave intervals into sesquitertian parts

9/8 × 9/8 × 256/243	# # +
9/8 × 256/243 × 9/8	# + #
256/243 × 9/8 × 9/8	+ # #

> [36 B] … he filled with a sesquioctave interval all the sesquitertian parts, at the same time leaving a part of each of these. And then again the interval of this part being assumed, a comparison is from thence obtained in terms of number to number, subsisting between 256 and 243 …[1]

Similar considerations apply to the third possibility of Table 21. In that case, one either actively inserts the 256/243 interval at the beginning of the sesquitertian part, something that Plato never instructed the reader to do, or performs a reciprocal operation requiring multiplication by 8/9 from the end of the sesquitertian part. When all is said and done, the most conservative approach is simply to do what Plato directed as straightforwardly as possible: fill the sesquitertian parts with sesquioctave intervals, automatically leaving a remainder over at the end.

Other commentators appear not to have addressed specifically the *ordo* in which to fill the sesquitertian parts with sesquioctave intervals. All have simply assumed that one inserts two 9/8 intervals, leaving the 256/243 remainder over.[2] Ernest McClain's text, however, provides a clue concerning why the indicated *ordo* might be particularly appropriate. He noted that the smallest integers that will produce the set of relations that Plato indicated are 192:216:243:256. He further noted that the progression appears in Plutarch's commentary on the *Timaeus*; so the ancients were aware of this sequence and its significance.[3]

The use of 384 as a multiplier for the diatonic scale, a practice going back at least as far as Crantor, assumes the same *ordo*, as this study demonstrates below. As McClain noted, the integers of the mathematically simplest set for Plato's intervals, 192: 216: 243: 256, have to be redoubled to get integer names

1 Plato *Tim.* (T. Taylor) 36 B.

2 Cf. Cornford, *Plato's Cosmology*, 72; Brisson, *Le même et l'autre*, 316–317; McClain, *Pythagorean Plato*, 60–62; and Handschin, "Timaeus Scale," 14.

3 McClain, *Pythagorean Plato*, 60.

for both fourths of a model octave.[4] In any case, for this simplest sequence, the ratio 216/192 is an interval of 9/8, as is 243/216. The relation of 256/243 is obvious for the last two numbers. The sequence thus assumes the *ordo* 9/8 × 9/8 × 256/243. For a mathematical operation, considerations of simplicity are always key; so, the derivation of this study will abide by the apparently most primitive *ordo* from the stance of integer relations. The insertion of sesquioctave intervals into sesquitertian parts, in this study, will follow the 9/8 × 9/8 × 256/243 pattern.

The operation of filling in the sesquitertian parts with sesquioctave intervals and verifying the remainder over generates a great proliferation of new numbers for the Crantor matrix. The method that produces this result will be described, below. An example of its operation on a single sample sesquitertian interval is relegated, because of its length, to Appendix 1 to this study. The numbers produced for the Crantor matrix for all fourteen cases, collectively, by the same method, are identified below; and tables are furnished, depicting the final results of the sesquioctave operation. The description of the method immediately below provides the reader all that he needs to verify the results summarized in this chapter, without burdening him with the pages and pages of calculations cumulatively producing Plato's final "Crantor set."

Consider the interval 2:8/3, sesquitertian part no. 3 in Table 20 of the last chapter, listing such parts. One finds the sesquioctave intervals within this sesquitertian part, as follows, by calculating two products, 2 × 9/8 and (2 × 9/8) × 9/8. The first product is 18/8; so the first sesquioctave interval is 2:18/8. However, this is not the simplest name for the interval because 18/8 can be further reduced. The fraction 18/8 can be expressed as 9:4; so another name for the first sesquioctave interval is 2:9/4. In calculating the next sesquioctave interval all names for the first one have to be taken into account to generate the possible names for the second. Accordingly, the second interval is 18/8 × 9/8 and is also calculable as 9/4 × 9/8. The product 18/8 × 9/8 is 162/64; so one name for the second sesquioctave interval is 18/8:162/64. However, 162/64 can be simplified in various ways. It is also 81/32; so a second name for the sesquioctave interval is 18/8:81/32. The product 9/4 × 9/8, the other way of generating the sesquioctave interval, of course, yields 81/32 immediately; so the possible names for the two sesquioctave intervals inserted into the sesquitertian part are (1) 2:18/8 or 2:9/4 and (2) 18/8:162/64, 18/8:81/32, or 9/4:81/32. One can summarize, thus far, as follows in Table 22.

4 Ibid., 60 and 140.

TABLE 22 Calculation of all sesquioctave interval names pertinent to the division of a sample sesquitertian part

2	× 9/8 yields 18/8 or 9/4 Names for first sesquioctave interval: 2:18/8 and 2:9/4	(18/8 or 9/4) × 9/8 yields 162/64 or 81/32 Names for the second sesquioctave interval are, therefore, 18/8: 162/64 and 18/8: 81/32 and 9/4:81/32.	**All names for all new sesquioctave intervals:** 2:18/8 2:9/4 18/8:162/64 18/8:81/32 9/4:81/32 New numbers needed in the Crantor table to represent these fractions: **81, 64, 162.** The others are already represented.

Two more calculations are needed to complete Plato's observations at 36 B. He specifically told his reader that the interval remaining after the insertion of the sesquioctave intervals into the sesquitertian parts is 256/243; so to be entirely consistent one must verify this result for every sesquitertian part. If one regards Plato's text from 35 B to 36 B as a number generator, then one must find all names of numbers that can arise during the verification process, as well. Since there are two possible ways to verify the ratio of the remainder over, the latter exigency requires taking both modes of verification into account.

The endpoint of sesquitertian interval no. 3 in Table 20 is 8/3, while the fractions 162/64 and 81/32 are alternative names for the endpoint of the second sesquioctave interval within it. One finds 8/3 from either one of the latter two numbers and so verifies a ratio of 256/243 between said numbers and 8/3 by performing the operations indicated in Table 23 on each. The number generating process progresses by the means of simplification alluded to in Table 23. Simplification is achieved by factoring numerators and denominators by 2's and 3's in as many ways as are possible, in imitation of Plato's original divisions by doubles and triples, and keeping track of the numbers that arise as a result of the reductions.

To see the remainder of the calculations for the sample interval 2:8/3, the reader should consult Appendix 1 to this study. When all is said and done, the new numbers arising for the Crantor matrix as a result of the verification pro-

TABLE 23 Method of verification of the *diesis* remaining after insertion of two sesquioctave intervals into a sesquitertian part for the sample sesquitertian part, 2:8/3

First mode of verification	**First mode of verification**	**Names for the ratio 256/243**
(1) 162/64 × n = 8/3	(2) 81/32 × n = 8/3	**512/486** and **256/243**
Note that 162/64 is one of the names for the endpoint of the second sesquioctave interval. n = 8/3 × 64/162 n = 512/486	Note that 81/32 is the remaining name for the endpoint of the second sesquioctave interval. n = 8/3 × 32/81 n = 256/243	
Simplifying, one verifies 256/243		New numbers needed in the Crantor table to represent these fractions: **243, 256, 486, 512**
Second mode of verification	**Second mode of verification**	**Second mode of verification**
(1) 162/64 × **256/243** = 41472/15552.	(2) 81/32 × **256/243** = 20736/7776	**New numbers needed in the Crantor matrix to represent fractions arising from calculations**
To get all of the names for this fraction, one must factor as far as one can go by 2's, then as far as one can go by 3's, then according to as many alternative ways of factoring by both 2's and 3's as possible until one reaches 8/3.	To get all of the names for this fraction, one must factor as far as one can go by 2's, then as far as one can go by 3's, then according to as many alternative ways of factoring by both 2's and 3's as possible until one reaches 8/3.	(1) **41472; 15552** (2) **20736; 7776**

cess for the interval 2:8/3 are 81, 64, 162, 243; 256; 486; 512; 41472; 15552; 20736; 7776; 10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72; 13824; 4608; 1728; 1536; 576; 192; 6912; 2304; 864; 768; 288; 3456; 1152; 432; 1728; 648; 324; 82944; 31104; 27648; 9216; 3072; and 1024.

A proliferation of numbers also results when operations parallel to those suggested above are performed for each of the thirteen remaining cases of

1

2 3

4 6 9

8 12 18 27

16 24 36 54 X

32 48 X 108 X X

X 96 144 X X X X

128 X X X X X X X

X 384 X X X X X X X

FIGURE 10 Reiteration of figure 9 Crantor matrix expanded to include ratios representing relevant sesquitertian parts within the new double and triple intervals resulting from the "means" operation at 36 A

sesquitertian parts. Some of these cases are much more complicated than the case of 2:8/3. Some sesquitertian parts have multiple names. For example, the parts listed as nos. 4, 6, and 9 in Table 20, the table of sesquitertian parts, have two names. The parts listed as nos. 5 A, 11 A, and 12 A have three names; and the parts listed as nos. 2, 7, and 8 have four different names. Part no. 10 has six different names. The calculations performed in the case of 2:8/3, above and in Appendix 1, have to be done for every single name of every sesquitertian part to be consistent with the position that the *Timaeus* text from 35 B–36 B is a number generator within parameters that Plato's divisions allow. In addition, because of the multiple names that arise for the remainder over of 256/243—some intervals produce variants much larger than 512/486 seen above—there will be several variants of the second mode of verifying that interval for many sesquitertian names. Many numbers generable, in this way, from one sesquitertian part can also be generated from various other sesquitertian parts.

The difference in the Crantor matrix after performing the calculations for 2:8/3 is dramatically evident in a "before and after" style comparison. As the reader may recall, the Crantor matrix appeared as in Figure 10, above, after the identification of sesquitertian parts to be filled in the new doubles and triples.

Following the calculations needed to fill the sesquitertian part 2:8/3 with sesquioctave intervals and to verify the 256/243 remainder over, the Crantor matrix appears as indicated in Figure 11, below.

1

2 3

4 6 9

8 12 18 27

16 24 36 54 81

32 48 72 108 162 243

64 96 144 216 324 486 X

128 192 288 432 648 972 X X

256 384 576 864 1296 1944 X X X

512 768 1152 1728 2592 3888 X X X X

1024 1536 2304 3456 5184 7776 X X X X X

X 3072 4608 6912 10368 15552 X X X X X X

X X 9216 13824 20736 31104 X X X X X X X

X X 27648 41472 X X X X X X X X X X

X X X 82944 X X X X X X X X X X X

FIGURE 11 Crantor matrix after filling the sesquitertian interval 2:8/3 with sesquioctave intervals and verifying the remainder over

Having seen this difference, the reader will want to examine the transformation of the Crantor matrix after the same kinds of calculations are complete for every name of every sesquitertian part. Figure 12 and Table 24 are required to present the results because the numbers become too large to exhibit conveniently in the Crantor format. Figure 12 is a "dot chart" showing the matrix pattern emerging from the number generation of Plato's divisions, with empty spaces indicated for a symmetrical presentation. Table 24 is a horizontal chart of the numbers that fill in the Crantor matrix, row by row. The last row of the Crantor extension is the twenty-seventh, corresponding to last number, 27, of the original set of numbers that Plato furnished for the world soul in the *Timaeus*.

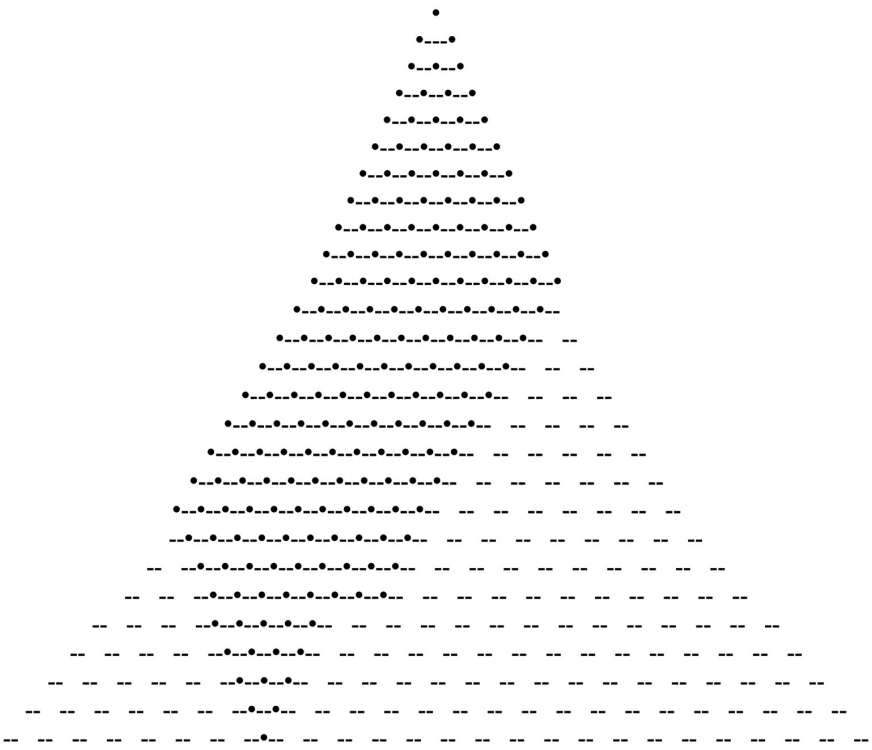

FIGURE 12 Dot chart showing pattern of Crantor matrix emerging from the *Timaeus*

In Figure 12, a fabric appears to cut "lengthwise" (along the diagonal) to make the χ ("chi") *per* 36 B and C. The numbers filling this fabric, indicated by the dots in the first through the twenty-seventh rows of the matrix, appear in Table 24, below.

The results in Table 24, delineating the *Timaeus* number set, identify every single number issuing from the analysis of the fourteen sesquitertian parts, in all of their various names, according to the method described, above, as exemplified in Appendix 1 for the sesquitertian part 2:8/3. No additional numbers are pertinent to these analyses; and no number constituting a factor in the analysis of any sesquitertian interval is outside of the set. There are 196 discrete terms, representing a perfect square with root fourteen.

Note that certain numbers are highlighted in Table 24. For now, simply observe that perfect squares are highlighted in green, perfect cubes are highlighted in yellow and numbers that are both perfect squares and perfect cubes are highlighted in purple. The red numbers are chromatic factors of 1719926784, a number, as the study will show, that has a very special significance. The term "chromatic" will be clarified apace. Just be aware, in this usage, that it

TABLE 24 Horizontal chart of numbers filling in the rows of the *Timaeus* Crantor matrix

(1) 1

(2) 2 3

(3) 4 6 9

(4) 8 12 18 27

(5) 16 24 36 54 81

(6) 32 48 72 108 162 243

(7) 64 96 144 216 324 486 729

(8) 128 192 288 432 648 972 1458 2187

(9) 256 384 576 864 1296 1944 2916 4374 6561

(10) 512 768 1152 1728 2592 3888 5832 8748 13122 19683

(11) 1024 1536 2304 3456 5184 7776 11664 17496 26244 39366 59049

(12) 2048 3072 4608 6912 10368 15552 23328 34992 52488 78732 118098

(13) 4096 6144 9216 13824 20736 31104 46656 69984 104976 157464 236196

(14) 8192 12288 18432 27648 41472 62208 93312 139968 209952 314928 472392

(15) 16384 24576 36864 55296 82944 124416 186624 279936 419904 629856 944784

(16) 32768 49152 73728 110592 165888 248832 373248 559872 839808 1259712 1889568

(17) 65536 98304 147456 221184 331776 497664 746496 1119744 1679616 2519424 3779136

(18) 131072 196608 294912 442368 663552 995328 1492992 2239488 3359232 5038848 7558272

(19) 262144 393216 589824 884736 1327104 1990656 2985984 4478976 6718464 10077696 15116544

(20) 786432 1179648 1769472 2654208 3981312 5971968 8957952 13436928 20155392 30233088

(21) 2359296 3538944 5308416 7962624 11943936 17915904 26873856 40310784 60466176

(22) 7077888 10616832 15925248 23887872 35831808 53747712 80621568 120932352

(23) 21233664 31850496 47775744 71663616 107495424

(24) 63700992 95551488 143327232 214990848

(25) 191102976 286654464 429981696

(26) 573308928 859963392

(27) 1719926784

refers to numbers not properly belonging to the sequence of a descending Dorian or ascending Lydian diatonic octave in a series of such octaves beginning from the number 384. There are actually two series of chromatic numbers on the chart. The second series consists of chromatic nonfactors of 1719926784. The numbers of that series are highlighted in olive and underlined. The first of the chromatic nonfactors, occurring at the end of the tenth row, is 19683. All numbers of the set beyond the chromatic nonfactors are factors of 1719926784.

Two tasks remain for this study before it can address the marvelous musical significance of the numbers identified above or illustrate Plato's χ ("chi") formation operation. The first is to give the reader some clues concerning how to derive the matrix numbers not generable from the 2:8/3 interval. The second is to point out some mathematically interesting features of the number set that correlate with the coming explanation of the set's musical and cosmological significance.

1 Deriving Matrix Numbers Not Generable from the 2:8/3 Interval

Illustrating the derivation of matrix numbers, not generable from the 2:8/3 interval, from the remaining sesquitertian parts, with their variants, *via* the operation demonstrated for the 2:8/3 interval, in the text above and Appendix 1, would require too much time and text. The calculations took days. It would take as many days again to make the kind of neat presentation, just completed for the 2:8/3 interval, for every sesquitertian part. Nonetheless, the reader should have at least one source interval for each number in the set that is not among those explicitly derived from the interval 2:8/3, for verification by his own iteration, on the relevant interval, of operations parallel to those performed, above and in Appendix 1, on the 2:8/3 interval. Such source intervals are indicated in the Table 25 below.

Table 25 presents fewer than all of the intervals listed in Table 20, as source intervals for the whole set of numbers not generable from the 2:8/3 ratio. The reason is that one can find various groups of numbers, in the set, beginning from any of several intervals. The numbers generable from the 2:8/3 interval, as demonstrated above in Tables 22, 23, and Appendix 1, for example, recur in the analysis of other sesquitertian parts too. A great deal of repetition among the intervals exists in the numbers that they yield; so, the reader should be aware that the intervals listed in Table 25 as sources for each number not generable from the 2:8/3 interval are typically not exclusive. While providing source intervals for particular numbers, Table 25 also does not list all other numbers similarly derivable from those intervals; the resulting repetition of various

TABLE 25 Sample source intervals for some numbers of the Crantor matrix derived upon filling all relevant sesquitertian intervals with sesquioctave parts and verifying remainder overs

Sesquitertian name	**Various numbers deriving from the analysis of the interval**
3/2 to 2 (variant of no. 2 in Table 20's list of sesquitertian parts)	62208
3/2 to 12/6 (variant of no. 2 in Table 20's list of sesquitertian parts)	729; 1458; 2916; 5832; 11664; 23328; 46656; 93312; 186824; 373248
3/2 to 4/2 (variant of no. 2 in Table 20's list of sesquitertian parts)	124416
4 to 16/3 (variant of no. 5 in Table 20's list of sesquitertian parts)	165888; 331776
12/3 to 48/9 (variant of no. 5 in Table 20's list of sesquitertian parts)	8748; 8957952; 4478976; 2239488
12/3 to 16/3 (variant of no. 5 in Table 20's list of sesquitertian parts)	2048; 4096; 6144; 12288; 18432; 36864; 110592; 55296; 497664; 995328; 746496; 1492992; 2985984
6 to 8 (variant of no. 6 in Table 20's list of sesquitertian parts)	248832
9/2 to 36/6 (variant of no. 7 in Table 20's list of sesquitertian parts)	2187; 4374; 13122; 26244; 34992; 52488; 69984; 104976; 139968; 209952; 279936; 419904; 559872; 839808; 1679616; 3359232
9/2 to 12/2 (variant of no. 7 in Table 20's list of sesquitertian parts)	1119744
27/2 to 108/6 (variant of no. 8 in Table 20's list of sesquitertian parts)	39366; 118098; 236196; 472392; 944784; 1889568; 3779136; 7558272; 15116544; 10077696; 30233088
27/2 to 54/3 (variant of no. 8 in Table 20's list of sesquitertian parts)	6561
27/2 to 18 (variant of no. 8 in Table 20's list of sesquitertian parts)	17496; 78732; 157464; 314928; 629856; 1259712; 2519424; 5038848
36/3 to 144/9 (variant of no. 10 in Table 20's list of sesquitertian parts)	80621568

TABLE 25 Sample source intervals for some numbers of the Crantor matrix (*cont.*)

Sesquitertian name	**Various numbers deriving from the analysis of the interval**
8/3 to 32/9 (No. 13 in Table 20's list of sesquitertian parts)	8192; 24576; 73728; 221184; 442368; 663552; 1327104; 1990656; 3981312
24/3 to 96/9 (variant of no. 11 in Table 20's list of sesquitertian parts)	5971968; 11943936; 17915904; 35831808
96/9 to 384/27 (variant of no. 12 in Table 20's list of sesquitertian parts)	19683; 59049; 13436928; 26873856; 53747712; 60466176; 107495424; 120932352; 214990848; 429981696; 859963392; 1719926784
96/9 to 128/9 (variant of no. 12 in Table 20's list of sesquitertian parts)	16384; 32768; 49152; 65536; 98304; 131072; 147456; 196608; 262144; 294912; 393216; 589824; 786432; 884736; 1179648; 1769472; 2359296; 2654208; 3538944; 5308416; 6718464; 7077888; 7962624; 10616832; 15925248; 20155392; 21233664; 23887872; 31850496; 40310784; 47775744; 63700992; 71663616; 95551488; 143327232; 191102976; 573308928; 2866654464

sequences of numbers among the intervals would serve no purpose. Plato may have included, within his intended 36 B set, sesquitertian parts that would produce such repetition, upon analysis, because he wanted to ensure that the reader actually would find all of the pertinent factors. Given the length and complexity of the factoring possibilities, a reader beginning merely from the intervals producing the most numbers could very easily miss half the set. He would need only miss one factoring possibility for one number in a lengthy sequence.

The full derivation of numbers given for the interval 2:8/3 in Table 22, Table 23, and Appendix 1 and the much abbreviated synopsis, in Table 25, of source intervals yielding additional numbers not generable from that interval, together account for all elements of Plato's number set. Chapter 5 will show that some of the set's mathematical features are profoundly significant to Plato's articulation of a musical cosmology. Accordingly, the study proceeds to the identification of those special features.

2 Special Mathematical Features of the Number Set Reflected in Table 24

The extension of numbers to the twenty-seventh row of a Crantor style matrix certainly challenges the standard musical interpretation of the *Timaeus*, limiting the number set to those numbers articulating an octave expansion of four octaves and a major sixth. However, the new extension fits well with an association made between the *Timaeus* and the twenty-eight stringed lyre in the later tradition of *Timaeus* interpretation. This association, though independent of the above analysis, is intriguing because the symbolic reasons justifying it would have been valid for Plato as a reason for extending the matrix to the twenty-seventh row.

In relation to the link between the *Timaeus* and the twenty-eight stringed lyre, Thomas Mathiesen has drawn upon fragments of anonymous *scholia* transmitted, in some manuscripts, with Nicomachus of Gerasa's *Manuale harmonicum* and attributed to Nicomachus because they appear to relate to his musical theory. Mathiesen has remarked that the *scholia* fragments do not form a coherent whole but do reveal "the general Pythagorean subject of the relationship between musical phenomena, number, and higher universal principles."[5]

The *scholia* present the expansion of strings on the lyre from eight to twenty-eight as allowing the lyre to sound all seven *tonoi* in all three genera of scale: diatonic, enharmonic, and chromatic. They further represent that the twenty-eight strings symbolize, in their expansion, the cosmic *harmonia* made by the sounds of the seven planets. The *scholia* purported to treat the manner in which the twenty-eight strings were arranged in fourths, as well as the relationship between those fourths and the cosmic *harmonia*.[6] Mathiesen has described the contents of the last three fragments of these *scholia* as follows:

> The twenty-eight strings of the lyre are likened to the seven numbers of the famous duple and triple multiples used by the demiurge in the psychogony of Plato's *Timaeus:*

5 Mathiesen, *Apollo's Lyre*, 406–407.

6 Ibid., 407.

1

2 3

4 9

8 27

> If 1, the principle of all things is added to 27, the number of strings, 28, results. These twenty-eight notes are arranged in five fourths, which represent beginning, middle, and end, a conjunctive cooperation, and a disjunctive cooperation.[7]

Mathiesen has noted that a similar analogy appears in Aristides Quintilianus *De musica* 3.14.

When one considers Plato's musical sophistication, together with the fact that the development of GPS was being hotly pursued in his day, it is not at all unlikely that he knew the common opinion that twenty-eight lyre strings would be required to sound all *tonoi* in all genera. The musical significance demonstrated later, in this study, for the numbers set forth in the *Horizontal Chart* (Table 24), is precisely that they are sufficient to account for the derivation of all *tonoi* in all genera. The musical significance of the twenty-eight lyre strings renders it even more unlikely that the numerical generation posited, in this study, for the *Timaeus* is some kind of weird coincidence.

Because of the complexity of the analysis required both to arrive at Plato's number set and to appreciate its full musical significance as set forth, below, in this study, one might speculate whether Plato left some clue in his original sequence of seven numbers that would operate as a quick key to identifying the numbers of the matrix in their entirety. Some very tentative suggestions along such lines will be offered here.

Plutarch's *De animae procreatione in Timaeo* provides some food for thought. In that work, Plutarch noted, *inter alia*, that Plato's initial set of numbers, i.e., 1, 2, 4, 8, 3, 9, and 27 yields remarkable numbers when they are added to one another and multiplied by each other.[8] He then proceeded for a number of pages to provide examples, none of which is especially striking for the present analysis.[9] His general observation is important, nonetheless, in view of the

7 Ibid., 410.

8 Plutarch *De Animae Procreatione in Timaeo* 1017 E.

9 Ibid., 1017 E–1019 B.

possibilities that he did not explore. The other crucial features in Plutarch's *Timaeus* commentary, for the purpose of finding some algorithm, are (a) the observation that, in designating the original set of seven world soul numbers, Plato paired plane numbers (2 and 3) off against square numbers (4 and 9) and paired both off against cubic numbers (8 and 27) and (b) the special status that he himself seemed to recognize for square and cubic numbers.[10] Again, beyond these general observations, Plutarch's comments have little bearing on the present analysis.[11]

Taking up, in the most straightforward way, Plutarch's hint that the multiplication and addition of the original *Timaeus* numbers result in significant numbers, one might observe that their total sum is 54 and their total product is 46656.[12] Although, Plutarch did not make such notations, they are intriguing for the present analysis because both numbers figure in Plato's final set. Both numbers appear to be important as indicators of the limit of extension of Plato's intended number set. The sum of the digits of 1719926784, the last number of the set, for example, is precisely 54, but, importantly from an entirely ancient standpoint, it is divisible by 54, adding to the circumstantial evidence that the last number of the set is a relevant limit. The product of the original seven *Timaeus* numbers, 46656, is significant, at the outset, because the sum of its digits is 27, but more importantly from an ancient standpoint, the number is divisible by 27, perhaps indicating that there is something significant about the twenty-seventh row.

Other considerations add to the significance of 46656 as an indicator of the limit to the extension of numbers, most importantly the special relationships of this number to 1, the beginning of the set and 1719926784, its last member and only member in the twenty-seventh row of the extended Crantor matrix. Like 1, the first number of the *Timaeus* sequence, 46656 is both a perfect square and a perfect cube; like it, 1719926784 is also a perfect square. The square of root of 1719926784, 41472, is also a number in the set and makes a 9/8 ratio with 46656.

---

10 Ibid., 1028 B. Plutarch observed that, in the interpretations of the *Timaeus* attempting to line up the planetary orbits with musical intervals, the tone number 729 is aligned with the sun. The sun is called a square and a cube, in such analyses, because 729 is, at the same time, a square and a cube. Presumably, such a unique numerical association would have been suited to the sun's importance.

11 Ibid., 1022 D–1027 F.

12 Note that addition of the digits of 54 yields 9 and addition of the digits of 46656 yields 27, the cube of nine; further 27 is half of 54. It is difficult to know what to make of such relationships, especially since the ancient Greeks did not use our system of digits; but they seem unlikely to be accidental.

This ratio defines a whole tone in the Pythagorean musical scale, as noted earlier in this work. String lengths related in such a ratio produce sounds separated by a whole tone. This special relationship may be Plato's hint that the whole tone and its indivisibility is one of the key preoccupations of his text. In any case, the relations noted above mean that one can find the last number in any *Timaeus* style extension (see Chapter 8 for others) by squaring the product of the first seven numbers and 8/9; for example, $[(1 \times 2 \times 4 \times 8 \times 3 \times 9 \times 27) \times 8/9]^2 = 1719926784$.

The further relationship of 46656 to the twenty-seventh row seems important to corroborating its function as a key to finding the limit of the *Timaeus* number set. If one works out the identities of the numbers corresponding to the *twenty-six* empty spaces of the *Horizontal Chart* (Table 24), in the twenty-seventh row, one observes that the row, completed, contains not one, but *fourteen* perfect squares, alternating with numbers not constituting perfect squares. An inspection of Table 24 shows that a similar pattern obtains for numbers of the *Timaeus* set in rows containing perfect squares. The perfect squares in the completed twenty-seventh row are 67108864 (root 8192), corresponding to the empty space in the first place; 150994944 (root 12288), corresponding to the empty space in the third place; 339738624 (root 18432), corresponding to the empty space in the fifth place; 764411904 (root 27648), corresponding to the empty space in the seventh place; 1719926784 (root 41472), the number belonging to the set in the ninth place; 3869835264 (root 62208), corresponding to the empty space in the eleventh place; 8707129344 (root 93312), corresponding to the empty space in the thirteenth place; 1.959104102^{10} (root 139968) corresponding to the empty space in the fifteenth place; 4.407984230^{10} (root 209952) corresponding to the empty space in the seventeenth place; 9.917964519^{10} (root 314928), corresponding to the empty space in the nineteenth place; 2.231542017^{11} (root 472392), corresponding to the empty space in the twenty-first place; 5.020969538^{11} (root 708588), corresponding to the empty space in the twenty-third place; 1.129718146^{12} (root 1062882) corresponding to the empty space in the twenty-fifth place; and 2.541865828^{12} (root 1594323) corresponding to the empty space in the twenty-seventh place. Note that the series of roots articulates a continuous succession of fifths. This relationship is unsurprising, as the same relation obtains among roots of numbers having square roots in rows of Table 24 containing perfect squares.

The square roots of all of the perfect squares identified in the previous paragraph make ratios with 46656 having significance for a musical interpretation of the *Timaeus*, some more obviously than others. The special relationship of 46656 to 41472 (the square root of 1719926784 in the ninth place in the twenty-seventh row), in forming the 9/8 ratio, has already been noted.

The other three obvious relationships of musical significance are the 4/3 ratio made by 46656 with 62208 (the square root of 3869835264 corresponding to the empty space in the eleventh place of the twenty-seventh row); the double ratio it makes with 93312 (the square root of 8707129344 corresponding to the empty space in the thirteenth place of the twenty-seventh row); and the triple ratio it makes with 139968 (the square root of 1.959104102^{10} corresponding to the empty space in the fifteenth place of the twenty-seventh row).

Relationships of less obvious significance are implicit in the remaining ratios. They are all remarkable in their expressibility as products of musical intervals. For example, the ratio of 46556 to 8192 (the square root of 67108864 corresponding to the empty space in the first place of the twenty-seventh row) is 5.6953125 (in our modern representation). This number is the product of $2 \times 2 \times 4/3 \times 2187/2048$, i.e., two octave ratios multiplied by the sesquitertian ratio multiplied further by the ratio of the *apotomē*.

The ratio of 46656 to 12288 (the square root of 150994944 corresponding to the empty space in the third place of the twenty-seventh row) is 3.796875 (in our modern representation). This number is the product of $2 \times 3/2 \times 9/8 \times 9/8$, i.e., the octave ratio multiplied by the sesquialter ratio multiplied further by two whole tone ratios.

The ratio of 46656 to 18432 (the square root of 339738624 corresponding to the empty space in the fifth place of the twenty-seventh row) is 2.53125 (in our modern representation). This number is the product of $2 \times 9/8 \times 9/8$, the octave ratio multiplied by two whole tone ratios.

The ratio of 46656 to 27648 (the square root of 764411904 corresponding to the empty space in the seventh place of the twenty-seventh row) is 1.6875 (in our modern representation). This number is the product of $9/8 \times 3/2$, the whole tone ratio multiplied by the sesquialter ratio.

The ratio of 209952 (the root of 4.407984230^{10} corresponding to the empty space in the seventeenth place) to 46656 is 4.5 (in our modern representation). This number is the product of $3/2 \times 3/2 \times 3/2 \times 9/8 \times 9/8 \times 256/243$, i.e., three sesquialter ratios multiplied by two whole tone ratios multiplied further by the ratio of the *leimma*.

The ratio of 314928 (square root of 9.917964519^{10} corresponding to the empty space in the nineteenth place of the twenty-seventh row) to 46656 is 6.75 (in our modern representation). This number is the product of $2 \times 2 \times 3/2 \times 9/8$, i.e., two octave ratios multiplied by the sesquialter ratio multiplied further by the whole tone ratio.

The ratio of 472392 (the square root of 2.231542017^{11} corresponding to the empty space in the twenty-first place of the twenty-seventh row) to 46656 is

10.125 (in our modern representation). This number is the product of 2 × 2 × 2 × 9/8 × 9/8, i.e., three octave ratios multiplied by two whole ratios.

The ratio of 708588 (the square root of 5.020969538[11] corresponding to the empty space in the twenty-third place of row twenty-seven) to 46656 is 15.1875 (in our modern representation). This number is the product of 2 × 2 × 2 × 3/2 × 9/8 × 9/8, i.e. three octave ratios multiplied by the sesquialter ratio multiplied further by two whole tone ratios.

The ratio of 1062882 (the square root of 1.129718146[12] corresponding to the empty space in the twenty-fifth place of the twenty-seventh row) to 46656 is 22.78125 (in our modern representation). This number is the product of 2 × 2 × 2 × 2 × 9/8 × 9/8 × 9/8, i.e., four octave ratios multiplied by three whole tone ratios.

The ratio of 1594323 (the square root of 2.541865828[12] corresponding to the empty space in the twenty-seventh place of the twenty-seventh row) to 46656 is 34.171875 (in our modern representation). This number is the product of 2 × 2 × 2 × 2 × 2 × 2187/2048, i.e., five octave ratios multiplied by the ratio of the *apotomē*. The relationships that give rise to 2187/2048 are especially interesting, since that ratio describes the *apotomē*, a second "semitone" interval that, together with the *leimma* (256/243) is relevant, as this study shows below, to building chromatic scales in Pythagorean analysis adhering strictly to Plato's divisions in the *Timaeus*.[13]

Nearly all of the square roots of perfect squares corresponding to empty spaces in the twenty-seventh row are elements of the *Timaeus* number set. The numbers 708588, 1062882, and 1594323, the roots of the numbers missing in the twenty-third, twenty-fifth, and twenty-seventh places of the twenty-seventh row, are the only ones excluded. Considering that only one actual number in the twenty-seventh row is itself part of the *Timaeus* set, this result is not surprising.

The four obvious ratios made by 46656 with the roots 41472, 62208, 93312, and 139968, namely: 9/8, 4/3, 2/1, 3/1, respectively, are primary ratios of impor-

13 Note that Archytas of Tarentum, a somewhat older contemporary of Plato within the Pythagorean school of thought, did not follow the *Timaean* scheme, constituting the object of this study, to reach his ratios for the diatonic, chromatic, and enharmonic fourths but constructed a genuine Pythagorean alternative. See Martin Luther D'ooge, "Studies in Greek Mathematics," in *Nicomachus of Gerasa*, trans. Martin Luther D'ooge, vol. 16, University of Michigan Studies, Humanistic Series (Ann Arbor: University of Michigan Press, 1938), 20 (Archytas as older contemporary of Plato). The reader interested in understanding the difference between Plato's and Archytas' approaches should consult Appendix 2 to this work, entitled "The Archytan Alternative in the Pythagorean School."

tance to Plato's original world soul divisions. Although none of the square roots of the fourteen perfect squares calculable for the twenty-seventh row makes a 3/2 ratio with 46656, the 3/2 ratio is found, laterally, along the whole consecutive line of square roots for perfect squares corresponding to empty spaces in the twenty-seventh row. Accordingly, the twenty-seventh row of an extended Crantor lambda originating from the original set of seven *Timaeus* numbers brings the world soul divisions back to themselves full circle in a special way.

The number 1719926784 acts like a monad in the generation of the sequence 9/8, 4/3, 2/1, 3/1 and 3/2; for the perfect squares grounding these ratios, that are in relation to 46656, by their roots, and with each other, by the 3/2 ratio, are consecutive perfect squares, beginning with 1719926784. The number 1719926784, alone, belongs to the *Timaeus* set because, as a new monad, 1719926784 is like 1, the very first number of the *Timaeus* set. It is, therefore, the proper ending number in the twenty-seventh row.

The special numbers, 54 and 46656, hidden within the original sequence of seven numbers as their sum and product, respectively, may allow further clues concerning the short identification of the set. If one were to guess how one might simply describe the other numbers belonging to the set, after having derived it in its entirety and after having noted the special relationships among 54, 46656, and 1719926784, one might reason that other elements of the set should be like 54 and 46656 in some way.

In the search for relevant similarities, one might observe that both 54 and 46656 divide 1719926784 evenly; further the numbers resulting from the respective divisions, 31850496 and 36864, respectively, also belong to the set. In addition, 46656 bears a special relationship to its corresponding factor, 36864. Both numbers are perfect squares (46656, as noted above, is also a perfect cube). The square root of 46656, 216, multiplied by the square root of 36864, 192, is equal to 41472 the square root of 1719926784. As noted previously, 46656 makes a 9/8 ratio with 41472 as 46656/41472; 41472 also, however, makes a 9/8 ratio with 36864 as 41472/36864. In addition, the square root of the ratio (46656/36864) of the two factors is 9/8.

One might venture that, whenever one deals with *perfect squares* of the *Timaeus* set that are, *together*, paired factors of 1719926784, the product of their square roots will yield 41472; this is, in fact, the case. For example, 1719926784/4 = 429981698. Both factors are in the *Timaeus* set. The square root of 4 is 2; and the square root of 429981698 is 20736. The product of 2 and 20736 is 41472. Likewise, 1719926784/144 = 11943936. Again, both factors are in the *Timaeus* set. The square root of 144 is 12; and the square root of 11943936 is 3456. The product of 12 and 3456 is 41472.

It is never the case, however, except for the paired factors 36864 and 46656, that 41472 yields the 9/8 ratio with both factors and further that the square root of the ratio of the paired factors themselves is 9/8. If, therefore, the intractable problem of the division of the whole tone is at the heart of Timaeus 35 A through 36 D, then this special set of relations may be the key to finding the last number of the set. One must find the number that divided by 46656 (product of the original set of numbers) produces this very special set of relations.

One could further hypothesize that other numbers of the set should also divide 1719926784 evenly and produce factors likewise, belonging to the set. It is, in fact, possible to describe all but twenty-five very special *Timaeus* numbers in just such a fashion. The exceptional twenty-five members of the set (recall that the number of elements of the set is a perfect square) are nonfactors of 1719926784. They are the following: 19683, 39366, 59049, 78732, 118098, 157464, 236196, 314928, 472392, 629856, 944784, 1259712, 1889568, 2519424, 3779136, 5038848, 7558272, 10077696, 15116544, 20155392, 30233088, 40310784, 60466176, 80621568, and 120932352. They appear together on the *Horizontal Chart* (Table 24), underlined in olive. One might initially think that their lack of a factorial relationship to 1719926784 indicates the inappropriateness of including them in the *Timaeus* set. It is, on the contrary, impossible to interpret *Timaeus* 36 C and D accurately without them, as Chapter 7 will make clear. Among other considerations, particular numbers of this subset create the "tail" important to the χ-making operation at 36 B.

The set of twenty-five peculiar numbers not constituting factors of 1719926784, in fact, bear their own set of internal references to the final number, when they are considered as a distinct subset. They consist in two strings of doubles, one descending from 19683, a perfect cube with root 27, and the other descending from 59049, a perfect square with root 243. Note that 19683 makes no string of triples, just a single triple with 59049. Other triples from 19683 are outside the number fabric. The single interpretational possibility contrasts with the usual circumstance that each number of the *Timaeus* set is simultaneously involved in distinct series of double and triple relations.

The relationship between the originating elements of the two strings of doubles as a relation of perfect cube to perfect square underscores Plato's specific concentration throughout the *Timaeus* on doubles and triples and the sesquialter ratio. The very first element, 19683 or 27^3 is an especially appropriate starting point for a *bona fide* subset among the *Timaeus* numbers, given the position of 27 as the end both of Plato's original series and one complete decad of numbers in the *Timaeus* set.

Relationship to 41472 is prominent in the subset of twenty-five nonfactors of 1719926784. The doubles column descending from 19683 progresses to

the fourth number (80621568) evenly divisible by 41472 (the other three are 10077696, 20155392, and 40310784), while the doubles column descending from 59049 (again, there is no triples column descending from 19683) extends to the third number (120932352) evenly divisible by 41472 (the other two are 60466176 and 30233088). There is a 4/3 proportion between such numbers in the doubles column descending from 19683 to such numbers in the doubles column descending from 59049 (again, there is no triples column descending from 19683). The sesquitertian proportion is, of course, one of the significant proportions of the *Timaeus*; the association of 41472 with the ratio among the set of nonfactors of 1719926784 indicates the importance of 41472. It is the square root, clearly, of 1719926784, the last element of the *Timaeus* set.

The set of nonfactors of 1719926784 also suggest the significance of the relationship between 46656 and 41472. The doubles column descending from 19683 proceeds to a seventh term (80621568) evenly divisible by 46656 (the other six are 1259712, 2519424, 5038848, 10077696, 20155392, and 40310784), while the doubles column descending from 59049 (again, there is no triples column descending from 19683) progresses to a sixth term (120932352) evenly divisible by 46656. The other five are 3779136, 7558272, 15116544, 30233088, 60466176. Clearly only four terms in the doubles column descending from 19683 (10077696, 20155392, 40310784, and 80621568) and three terms in the doubles column descending from 59049 (again, there is no triples column descending from 19683) (30233088, 60466176, and 120932352) are evenly divisible by both 46656 and 41472. The circumstance invites a comparison of the two numbers and an observation of their 9/8 ratio. Their product (1934917632), in relation to the last element of the *Timaeus* set (1719926784), also forms a 9/8 ratio.

A further clue that 1719926784 is especially fitting as a limit may be its capacity to be the basis for various whole number ratios closely approximating irrational values of possible musical interest to ancient Greek thinkers. These include but are not limited to the sixth root of 2 and the square root of the sixth root of two, the size of a true whole tone and true semitone, respectively. The demonstration of the usefulness of 1719926784, in this regard, occurs below, in this study, after the discussion of enharmonic scales completes the identification of relevant intervals and differences among them.

It is surely not coincidental that 1719926784 is a foundation for discovering a number series, the terms of which are in a continuous geometric proportion (the middle term between two others has the same ratio to the first as the last has to it) measured by a stable value very closely approximating Φ. It is different from the famous Fibonacci sequence the terms of which are

known to converge, asymptotically on Φ as a limit, in the proportions that they make with each other.[14] Φ, the golden section, is a very special, irrational proportion ubiquitous in nature, the cosmos, and plane and solid geometry. Plato knew it as a geometrical construction.[15]

Calculation shows that the terms of the following whole number sequence are united by a stable proportion of approximately 1.618033988, accurate to the ninth or tenth digit as a representation of Φ:

656953574 1062973211 1719926784 2782899994 4502826775
7285726764 1.178855353^{10}, $\mathbf{1.907428028^{10}}$ ...[16]

Following the first term, the series appears to be indefinite. However, that tentative conclusion would have to subjected to a rigorous mathematical proof, beyond the scope of this study, for its confirmation and also to a verification that the Fibonacci series continues to manifest as terms of the series articulated above proceed (see the algorithm below for the genesis of terms), preserving the constant ratio hovering around 1.618033988, closely approximating Φ, between the terms.

The algorithm for the series follows, where "n_1" is 656953574 and the succeeding terms are n_2, n_3, etc.:

$$\underset{}{n_1}, \underset{n_2}{(n_1 \times 1.618033988)} - 0, \underset{n_3}{(n_1 + n_2)} - 1, \underset{n_4}{(n_2 + n_3)} - 1, \underset{n_5}{(n_3 + n_4)} - 3,$$
$$\underset{n_6}{(n_4 + n_5)} - 5, \underset{n_7}{(n_5 + n_6)} - 8, \underset{n_8}{(n_6 + n_7)} - 13 \ldots$$

In other words, one derives the first term via multiplication by the Φ approximation and then derives all subsequent terms thus: one adds the two previous terms and subtracts, for each successive term, a series that appears to converge

14 Olsen, *The Golden Section*, 10.

15 See Plato *Republic* 509 D–511 E; Olsen, *The Golden Section*, 2–6 (and, more, generally, the whole book for the ubiquitous manifestation of the proportion in nature and the cosmos).

16 Olsen, *The Golden Section*, 52 (for the measure of Φ to the twenty-first digit). The term 406019638 may be the real beginning of the series, instead of 656953574, but the mathematics is not always consistent starting from that term, in yielding a whole number as the next term, though when one works backward by division from 656953574, one consistently arrives at 406019638. The case is a close one. It is certain that there is no number smaller than 406019638 that could be the first term.

with the Fibonacci sequence (each number in the Fibonacci sequence is the sum of the previous two: 0, 1, 1, 2, 3, 5, 8, 13 ...) from the n_5 term going forward.[17]

None of the above observations of mathematical relationships based upon the *Timaeus* set yields an algorithm for the set, although they may all be important in the quest for one. As they stand, they merely allow one to describe the set as consisting of all numbers, in rows one through twenty-seven of an expanded Crantor style matrix bounded by the original seven *Timaeus* numbers, that are factors of 1719926784, plus an additional subset of twenty-five numbers consisting of a column of doubles and a column of triples, originating from 27^3 (19683) and extending as far as is necessary for the achievement of a 4/3 ratio between the two columns, as to numbers evenly divisible by both 41472 and 46656. Although, standing alone, the description offered might appear to be a device somewhat arbitrarily contrived to fit the set, especially for minds entertaining suspicion about the subset of twenty-five, the analysis in succeeding chapters of the musical significance of the numbers underscores their necessity for clarifying the *Timaeus* at 36 C.

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17 Ibid., 10–11, 53. If one counts 406019638 as the first term, instead of 656953574, then the algorithm is the following:

$$n_1, (n_1 \times 1.6180339\overset{n_2}{88}) - 0, \overset{n_3}{(n_1 + n_2)} - 1, \overset{n_4}{(n_2 + n_3)} - 1, \overset{n_5}{(n_3 + n_4)} - 1, \overset{n_6}{(n_4 + n_5)} - 3,$$
$$\overset{n_7}{(n_5 + n_6)} - 5, \overset{n_8}{(n_6 + n_7)} - 8, \overset{n_9}{(n_7 + n_8)} - 13$$

In this case, the series subtracted converges with the Fibonacci series beginning with the sixth term.

CHAPTER 5

The Musical Significance of Plato's Number Matrix: the Primary *Timaeus* Scale

It is time to see whether all the effort spent simply generating numbers, on a principled basis, in previous chapters, has yielded any fruit. The numbers are, in fact, no ordinary set. This chapter demonstrates that, on the most elementary examination, they exhibit (1) the *rise* of the octave scale in the diatonic genus and Lydian or Dorian *tonos*; (2) one perfect, ascending, diatonic, Lydian or descending diatonic Dorian disdiapason having no chromatic elements, i.e., elements foreign to the characteristic diatonic pattern; (3) eight more complete octaves in the original *tonoi*, displaying chromatic elements, for a *decad* of a.Lydian/d.Dorian diatonic octaves in all; (4) multiple instances within the bounds of that decad of a fully articulated standard perfect system in all three octave genera; (5) multiple instances of a fully articulated perfect system particularly proper to the *Timaeus* within those same bounds; (6) a primary *Timaeus* scale running from 384 to 393216 of the *Timaeus* number set, originating as a descending Dorian or ascending Lydian diatonic octave and acquiring enharmonic and chromatic variants that are more complete as the scale progresses; (7) a slow but orderly entrance of chromatic elements into the octave cycles beyond the first a.Lydian/d.Dorian diatonic disdiapason; and (8) the slow but orderly wane of the a.Lydian/d.Dorian diatonic octave scale, after the tenth complete one of that kind, in a decad of incomplete a.Lydian/d.Dorian sequences witnessing the emergence and disappearance of a new order of fifth periodicity within a disintegrating a.Lydian/d.Dorian framework.

The multiple repetition of the decadic structure, discernible from the observations indicated in the above paragraph, suggests that the Decad truly may be at the heart of Plato's labyrinthine *Timaeus*, just as Speusippus had claimed.[1] Furthermore, the octave *harmonia* is the *harmonia* of being not only because Plato's numerical extension defines a decad of a.Lydian/d.Dorian diatonic octaves, but also because the decad of octaves, interpreted as a higher order monad, stands, in relation to the decad of incomplete a.Lydian/d.Dorian sequences delineating the rise and fall of the new order of fifth periodicity as "same" to "different": 1:2.

1 Dillon, "*Timaeus* in the Old Academy," 82.

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One might observe that the slow but orderly entrance of chromatic elements, into the original diatonic octave scale, after the first disdiapason, and the slow wane of that original diatonic octave scale to fifth periodicity and, then, to nothing, results from the interference of fourths generated by the means operation at 36 A. There are actually two series of chromatic numbers, making strings of overlapping doubles and triples, *intra se*, peculiar to themselves. The distinction between the two series rests in whether or not the numbers in each are factors of 1719926784, the final *Timaeus* number. The set of twenty-five peculiar numbers identified in the previous chapter is, in its entirety, one subset of chromatic numbers, the one not comprising factors of 1719926784. The elements of the other set will be evident below.

On a secondary examination of the *Timaeus* number set, as this chapter also shows, one sees that the number set raises the specter of various alternative nonstandard possibilities for the structure of UPS (GPS together with the Lesser Perfect System ["LPS"]), beyond *Timaeus* UPS, and interesting possibilities for musical modulation.[2] One also begins to discern the ambiguities attending the interpretation of the primary *Timaeus* scale, exclusively as an a.Lydian/d.Dorian diatonic octave, beyond the first disdiapason, because of new possibilities allowed by the invasion of the chromatic elements. One additionally anticipates, for further verification in Chapter 6, that the *Timaeus* number set articulates, within a consistent Pythagorean framework, all three octave genera recognized in the ancient Greek music system (diatonic, enharmonic, and chromatic) in each of the seven predominant octave species of Plato's day (Dorian, Lydian, Mixolydian, Phrygian, Hypophrygian, Hypolydian, and Hypodorian).[3]

1 Numerical Arrangement of the *Timaeus* Numbers with Key

It is easiest to exhibit the many interesting musical features of the *Timaeus* numbers if one abstracts them from the Crantor format and arranges them in increasing numerical order from least to greatest, as in Table 26 below.

2 For the development of UPS, including GPS and LPS in the fourth century, see the "Preface" to this work.

3 Ibid.

TABLE 26 Fully annotated table of *Timaeus* numbers arranged in numerical order from least to greatest

Key

1. Blue Superscripts ^{PC} and ^{PS} indicate the perfect cubes and perfect squares in the *Timaeus* Number Set.

2. Letters in red between numbers throughout the chart indicate intervals as follows. O indicates an octave interval. DP signifies diapente, the ancient fifth (denoted as a "fifth" from time to time in this study). DT indicates diatessaron, the ancient fourth (denoted as a "fourth" from time to time in this study). T indicates the ancient whole tone. TS indicates an undivided tone plus a *leimma* of size 256/243. S indicates a *leimma* of size 256/243. S′ indicates a *diesis* of size 2187/2048. This *diesis*, slightly larger than the *leimma* of size 256/243 emerges when chromatic numbers split a whole tone between elements belonging to the repeating descending Dorian or ascending Lydian diatonic octave scale, originally arising with the *Timaeus* numbers as the primary *Timaeus* scale.

3. Numbers in (A) highlighted in bright green correspond to the very first cognizable fourth of any kind, following the pattern of a descending Mixolydian chromatic fourth. The pattern is T (TS) from 24: 9/8 × (9/8 × 256/243).

4. Numbers in (B) highlighted in light gray are emphasized to indicate the very first cognizable diatonic fourth, that of an ascending Lydian or descending Dorian variety. The pattern is T T S from 192: 9/8 × 9/8 × 256/243. The sequence also lays the base for an ascending Phrygian or descending Dorian or Hypodorian enharmonic fourth from 192 in the pattern (TT) S, where S indicates an interval into which two quarter tones might be calculated.

5. Numbers highlighted or otherwise indicated in turquoise, in plain brackets, e.g., [384] are starting tone numbers (STN) of the first and nine more complete octave repetitions of the descending Dorian or ascending Lydian diatonic octave arising, as the primary scale, in the *Timaeus* number set. All such octaves follow the pattern T T S T T T S calculated from any STN: 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243. Note that this diatonic sequence forms the base for an ascending Phrygian enharmonic octave from 384 in the pattern (TT) S (TT) T S or descending Dorian enharmonic octave in the pattern (TT) S T (TT) S where S indicates an interval into which two quarter tones might be calculated.

TABLE 26 Fully annotated table of *Timaeus* numbers arranged in numerical order (*cont.*)

6. Underlined, emboldened numbers in and subsequent to (D) are chromatic elements entering the scale. Those highlighted in gray are factors of 1719926784. Those highlighted in olive are nonfactors of 1719926785. The latter number, 1719926784, is the last number in the *Timaeus* number set. All numbers in the *Timaeus* number set except the numbers highlighted in olive are factors of 1719926784.

7. Numbers affected by red marking in (D) correspond to the very first cognizable chromatic octave sequence among the *Timaeus* numbers, following the pattern of a descending Dorian chromatic octave from 3456 as follows: [(TS) S′ S] T [(TS) S′ S].

8. *Italicized* numbers in (E) are scale numbers belonging to incomplete descending Dorian or ascending Lydian diatonic octaves. The parenthesized, emboldened, italicized elements beginning in (E) e.g., (*first*), identify a given number with a specific step in the incomplete scale, according to the *ordo* of a descending Dorian or ascending Lydian diatonic octave.

9. Unitalicized, underlined numbers in (E) represent nonscalar elements.

10. Highlighted, enlarged, fancy, bold brackets { } in (E), of the same color, enclose sequences of numbers containing scalar elements of the same incomplete descending Dorian or ascending Lydian diatonic octave sequence.

11. The degeneration of the octaves in the set is marked by their increasing departure from the d.Dorian/a.Lydian diatonic pattern T T S T T T S because of an increasing nonoccurrence of numbers necessary to articulate the same.

12. Numbers highlighted in dark green in and just prior to (E) are STNs marking the beginnings of the first and second incomplete octave sequences.

Number Analysis

(*A*) *Rising to the Primary Timaeus Scale; first chromatic fourth* (*in green*)*: d.Mixolydian variety:* 1 PS PC O 2 DP 3 DT 4 PS DP 6 DT 8 PC T 9 PS DT 12 DT 16 PS T 18 DT 24 T 27 PC TS 32 T 36 PS DT 48 T 54 TS 64 PS PC T 72 T 81 PS TS 96 T 108 TS 128 T 144 PS T 162 TS (*B*) *First diatonic fourth* (*in gray*)*: d.Dorian/a.Lydian variety; also constitutes* (*at least*) *the base for the first enharmonic fourth: d.Dorian/d.Hypodorian/a.Phrygian variety:* 192 T 216 PC T 243 S 256 PS T 288 T 324 PS TS (*C*) *Perfect diatonic octaves and disdiapason between blue markers* (*1*) *and* (*2 ends*)*, ending at* (*D*)*: d.Dorian/a.Lydian type; also first perfect enharmonic*

TABLE 26 Fully annotated table of *Timaeus* numbers arranged in numerical order (*cont.*)

octave and disdiapason: a.Phrygian/d.Dorian: (*1*) [384] T 432 T 486 S 512^{PC} T 576^{PS} T 648 T 729$^{PS\ PC}$ S [768] (*1 ends*) (*2 begins*) T 864 T 972 S 1024^{PS} T 1152 T 1296^{PS} T 1458 S [1536] (*D*) (*2 ends*) (*3 begins*) *Eight d.Dorian/a.Lydian diatonic octaves (also at least the base for as many d.Dorian/a.Phrygian enharmonic octaves) increasingly invaded by chromatic elements; d.Mixolydian/a.Hypolydian diatonic scale possible with third octave* T 1728^{PC} T 1944 S 2048 S′ 2187 S 2304^{PS} T 2592 T 2916^{PS} S [3072] (*3 ends*) (*4 begins*) T 3456 T 3888 S 4096$^{PS\ PC}$ S′ 4374 S 4608 T 5184^{PS} T 5832^{PC} S [6144] (*4 ends*) (*5 begins*) S′ 6561^{PS} S 6912 T 7776 S 8192 S′ 8748 S 9216^{PS} T 10,368 T 11,664^{PS} S [12,288] (*5 ends*) (*6 begins*) *Unaccustomed pattern TTTTSTS first possible* S′ 13,122 S 13,824^{PC} T 15,552 S 16,384^{PS} S′ 17,496 S 18,432 S′ 19,683^{PC} S 20,736^{PS} T 23,328 S [24,576] (*6 ends*) (*7 begins*) S′ 26,244^{PS} S 27,648 T 31,104 S 32,768^{PC} S′ 34,992 S 36,864^{PS} S′ 39,366 S 41,472 T 46,656$^{PS\ PC}$ S [49,152] (*7 ends*) (*8 begins*) S′ 52,488 S 55,296 S′ 59,049^{PS} S 62,208 S 65,536^{PS} S′ 69,984 S 73,728 S′ 78,732 S 82,944^{PS} T 93,312 S [98,304] (*8 ends*) (*9 begins*) S′ 104,976^{PS} S 110,592^{PC} S′ 118,098 S 124,416 S 131,072 S′ 139,968 S 147,456^{PS} S′ 157,464^{PC} S 165,888 T 186,624^{PS} S [196,608] (*9 ends*) (*10 begins*) S′ 209,952 S 221,184 S′ 236,196^{PS} S 248,832 S 262,144$^{PS\ PC}$ S′ 279,936 S 294,912 S′ 314,928 S 331,776^{PS} T 373,248^{PC} S (*10th complete Dorian/Lydian octave ends here* ⇒ {[*393,216*])

(*first*) ⇐(*E*) *Here, also begins First Incomplete d.Dorian/a.Lydian diatonic sequence; d.Dorian/a.Lydian diatonic sequence wanes (d.Dorian/a.Phrygian enharmonic octave possibility degenerates with diatonic pattern):* S′ 419,904^{PS} S *442,368* (*second*) S′ 472,392 S *497,664* (*third*) T 559,872 S *589,824*^{PS} (*fifth*) S′ 629,856 S *663,552* (*sixth*) T *746,496*^{PS} (*seventh*) S {[*786,432*] (*eighth; first*)} (*End of 1st incomplete d.Dorian/a.Lydian octave–misses fourth element of pattern TTSTTTS; full d.Mixolydian/a.Hypolydian diatonic octave still intact in pattern TTTSTTS; unaccustomed sequence TTTTSTS also still intact*) (*2nd incomplete d.Dorian/a.Lydian sequence begins.*) S′ 839,808 S *884,736*^{PC} (*second*) S′ 944,784^{PS} S *995,328* (*third*) T 1,119,744 S *1,179,648* (*fifth*) S′ 1,259,712^{PC} S *1,327,104*^{PS} (*sixth*) T {*1,492,992* (*seventh*)} (*End of 2nd incomplete d.Dorian/a.Lydian sequence ends–misses fourth and eighth elements; not full octave; no other pattern*)(*Third incomplete d.Dorian/a.Lydian sequence begins.*) T 1,679,616^{PS} S *1,769,472* (*second*) S′ 1,889,568 S *1,990,656* (*third*) T 2,239,488 S *2,359,296*^{PS} (*fifth*) S′ 2,519,424 S *2,654,208* (*sixth*) T {*2,985,984*$^{PS\ PC}$ (*seventh*)} (*End of Third incomplete d.Dorian/a.Lydian sequence–misses first, fourth, and eighth elements of original pattern; but full d.Hypophrygian/a.Hypodorian diatonic octave in pattern TSTTSTT; d.Hypodorian/a.Hypophrygian diatonic octave in pattern TTSTTST; d.Phrygian/a.Phrygian diatonic octave in pattern TSTTTST; and one unaccustomed octave in pattern, TTSTSTT calculable from last occurrent element of original d.Dorian/a.Lydian pattern*) (*Fourth incomplete d.Dorian/a.Lydian sequence begins.*) T 3,359,232 S *3,538,944* (*second*) S′ 3,779,136^{PS} S *3,981,312* (*third*) T 4,478,976 T 5,038,848 S {*5,308,416*^{PS} (*sixth*) T *5,971,968*(*seventh*)} (*End of Fourth incomplete d.Dorian/a.Lydian*

TABLE 26 Fully annotated table of *Timaeus* numbers arranged in numerical order (*cont.*)

sequence–misses first, fourth, fifth, and eighth elements); but full d.Hypodorian/a.Hypophrygian diatonic octave in the pattern TTSTTST and full d.Phrygian/a.Phrygian diatonic octave in the pattern TSTTTST calculable from last occurrent element of original d.Dorian/a.Lydian octave pattern) (Fifth incomplete d.Dorian/a.Lydian sequence begins.) T 6,718,464^{PS} S *7,077,888*^{PC} (*second*) S′ 7,558,272 S *7,962,624* (*third*) T 8,957,952 T 10,077,696^{PC} S *10,616,832* (*sixth*) T {*11,943,936*^{PS}(*seventh*)} *(End of Fifth incomplete d.Dorian/a.Lydian sequence–misses first, fourth, fifth, and eighth elements); but full d.Hypodorian/a.Hypophrygian diatonic octave in the pattern TTSTTST and full d.Phrygian/a.Phrygian diatonic octave in the pattern TSTTTST calculable from last occurrent element of original d.Dorian/a.Lydian octave pattern) (Sixth incomplete d.Dorian/a.Lydian sequence begins.)* T 13,436,928 T 15,116,544^{PS} S *15,925,248* (*third*) T 17,915,904 T 20,155,392 S *21,233,664*^{PS} (*sixth*) T {*23,887,872*^{PC} (*seventh*)} *(End of Sixth incomplete d.Dorian/a.Lydian sequence–missesfirst, second, fourth, fifth, and eighth elements); but full d.Hypodorian/a.Hypophrygian diatonic octave in the pattern TTSTTST calculable from last occurrent element of original d.Dorian/a.Lydian octave pattern (Seventh incomplete d.Dorian/a.Lydian sequence begins.)* T 26,873,856^{PS} T 30,233,088 S *31,850,496* (*third*) T 35,831,808 T 40,310,784 {TS *47,775,744*^{PS} (*seventh*)} *(End of Seventh incomplete sequence d.Dorian/a.Lydian–misses first, second, fourth, fifth, sixth, and eighth elements; no remaining fully articulated octave pattern; if last interval were divided, there would be a new d.Dorian/a.Lydian sequence calculable from the last occurrent element of the original d.Dorian/a.Lydian pattern; this octave is structured like the one that precedes the first complete sequence of its type in the table) (Eighth incomplete d.Dorian/a.Lydian sequence begins.)* T 53,747,712 T 60,466,176^{PS} S *63,700,992* (*third*) T 71,663,616 T 80,621,568^{PC} {TS *95,551,488* (*seventh*)}*(End of Eighth incomplete d.Dorian/a.Lydian sequence–misses first, second, fourth, fifth, sixth, and eighth elements; if last interval were divided, there would be a new d.Dorian/a.Lydian sequence calculable from that last occurrent element of the original d.Dorian/a.Lydian pattern; this octave is structured like the one that precedes the first complete sequence of its type in the table) (Ninth incomplete d.Dorian/a.Lydian sequence begins.)* T 107,495,424^{PS} T 120,932,352 TS 143,327,232 DT {*191,102,976*$^{PS\ PC}$ (*seventh*)} *(End of Ninth incomplete d.Dorian/a.Lydian sequence–misses first, second, third, fourth, fifth, sixth, and eighth elements) (Tenth incomplete d.Dorian/a.Lydian sequence begins.)* T 214,990,848 DT 286,654,464 DP 429,981,696^{PS} DT 573,308,928 DP 859,963,392 O 1,719,926,784^{PS}} *(End of Tenth incomplete d.Dorian/a.Lydian sequence; missing all original scalar elements; not even one octave but, rather, a tone and three octaves or two fifths and two octaves).*

Clearly, Table 26 shows the numerically ordered numbers and the intervals between them. The notations in the table regarding the degeneration of d.Dorian/a.Lydian octave periodicity are specifically tied to the octave sequences continuous with the first perfect disdiapason in the *Timaeus* number set, identified in the text below.

2 The First Cognizable Fourth of Any Kind

The first cognizable tetrachord of any kind in the numerical array of Table 26 is the sequence 24 T 27^{PC} TS 32. It follows a descending Mixolydian chromatic pattern. The primacy of the chromatic fourth is interesting from the standpoint of Stephen Hagel's observation, in his careful study of the development of ancient Greek musical notation, that chromatic music came into its own as "the new music," in the late fifth century, replacing an earlier preference for enharmonic music.[4] Chromatic phenomena are certainly important in the *Timaeus* set; and it is possible that Plato ordered his recipe for constructing the world soul deliberately to allow possibilities for chromatic music to arise within it. No full chromatic octave scale arises in the number sequence until long after the first diatonic and enharmonic octave sequences arise together, as the text below makes clear, however; so the primary *Timaeus* scale could not be chromatic.[5] A full chromatic octave scale runs, first, from 3456, in a descending Dorian pattern [(TS) S′ S] T [(TS) S′ S]: 3456 TS 4096 S′ 4374 S 4608 T 5184 TS 6144 S′ 6561 S 6912.[6]

3 The First Diatonic and Enharmonic Fourth

The first complete diatonic fourth of Table 26 is the sequence 192 T 216^{PC} T 243 S 256^{PS} highlighted in gray. The pattern of tones and semitones indicates four possibilities for diatonic interpretation: (a) an ascending Lydian or (b) ascending Hypophrygian diatonic fourth, if the numbers resulting from Plato's division of the world soul stuff are *indices* to string vibrations upon impact

4 Hagel, "Ancient Greek Music," 44, 52, 110–111.

5 The ancient commentator Macrobius (*circa* 400 A.D.) apparently believed that Plato would have disapproved of chromatic music for its "voluptuousness." Haar, "Musica mundana," 16 (n. 41). A chromatic scale would surely not have been primary for Plato, in the *Timaeus*, if he, in fact, held such an opinion.

6 Reese, *Music*, 31 (for descending Dorian chromatic scale pattern).

(without being an actual measure of the number of impacts a string makes on the air per unit time); alternatively, (c) a descending Dorian or (d) descending Hypodorian diatonic fourth, if the numbers represent string lengths.[7] On the level of the initial fourth, all octave scales referenced have the same tone/*diesis* pattern.[8]

One might interpret the sequence from 192 to 256, entirely differently, as a descending Dorian, descending Hypodorian, or ascending Phrygian enharmonic tetrachord in the pattern TT S, from 192: 192 TT 243 S 256.[9] Hagel has opined that the late fifth century evolution reflecting the preference, noted above, for chromatic music, constituted a shift from an ancient enharmonic that did not split quarter tones; thus arise the indicated possibilities.[10] He has also observed, however, that the QT enharmonic actually had its beginnings in the second half of the fifth century.[11] It was not in great use, according to Hagel, until Aristoxenus popularized it, even though, by Aristoxenus' time the semitone enharmonic was exceptional. Hagel himself finds it difficult to deny that the QT enharmonic that Aristoxenus presented was rooted in ancient musical practice.[12] He has noted, further, that instrumental evidence for a QT enharmonic exists only for the *aulos* and, also, that the tuning process for it on a seven-stringed lyre would have been difficult. The technique of half-covering a hole on the *aulos* could have produced sufficiently small intervals for a QT enharmonic on that instrument.[13]

Plato's life extended from the last quarter of the fifth century to the mid fourth century; so the *Timaeus* arose during the general period in which the evolution from a semitone to a QT enharmonic occurred by Hagel's account. It cannot be ruled out that the enharmonic of the *Timaeus*, while providing a definite basis for a semitone enharmonic, may also have contemplated a QT enharmonic. Although the text appears to be concerned with strings, rather than *auloi*, in its primary concentration on diatonic heptatony, as we shall see, difficulties of lyre tuning would not have been Plato's concern.[14] Ideal structures relevant to articulating a cosmic paradigm were his main concentration.

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7 See Creese, *Monochord*, 83, 100–102, 156, 164–166, 170, 219, 243 (regarding relationship between pitch and impacts for ancient Greek harmonic theorists and ancient modes of achieving pitch variation).

8 Reese, *Music*, 30.

9 Cf., ibid., 32.

10 Hagel, *Ancient Greek Music*, 413–414 and 417.

11 Ibid., 417.

12 Ibid., 413–419.

13 Ibid. 135, 418.

14 Ibid., 135, 436, 443 (for the strong association between strings and diatonic heptatony).

Accordingly, this study, as it continues, demonstrates the *Timaeus*' capacity to accommodate a QT enharmonic, making clear, as a matter of course, that it plainly allows a semitone enharmonic.

Since the sequence from 192 to 256 legitimately gives rise to both diatonic and enharmonic interpretations for fourth articulation, one might well be puzzled concerning which of these genera has precedence in the *Timaeus*. Even the meticulous Hagel has assumed that Plato was concerned with the diatonic in the *Timaeus* text.[15] This study gives the diatonic precedence over the enharmonic because, *inter alia*, only the diatonic pattern accounts for all elements in the sequence 192 to 256, corresponding to the first diatonic/enharmonic fourth. The enharmonic rides on top of the diatonic articulation, so to speak. One can calculate the undivided ditone and semitone and reach the numbers appropriate to enharmonic intervals, but the sequence 192 TT 243 S 256 does not occur by itself in the number series without the number 216 that splits the undivided TT. Had it thus appeared, then one might well have to consider the enharmonic as a candidate for primacy in the *Timaeus*. The study shows, below, that enharmonic possibilities disappear in the number series immediately upon the first signs of degeneration in the diatonic octave pattern. The enharmonic possibilities in the number set, therefore, appear to be dependent upon the diatonic possibilities.

An additional consideration favoring the diatonic as the predominant focus of the *Timaeus* is its precedence in age among the genera.[16] As Hagel has noted, Near Eastern music displayed a complete cyclic system of seven diatonic tunings, forming the basis of lyre music, by the second millennium B.C. or possibly earlier.[17] It is difficult to imagine that an ancient text incorporating a musical cosmology would relegate the diatonic genus to a position of secondary importance.

The tradition of the *Timaeus* text is still another reason for attributing a diatonic concentration to it. As Donald Creese has noted, in his insightful study of the monochord, Plato's text in the *Timaeus*, describing the harmonics of the

15 Ibid., 163.

16 Ibid., 10 (n. 35) for Aristoxenus' acknowledgement that the diatonic genus was older than the other two genera). Macrobius had opined, as Haar points out, that the enharmonic genus fell into disfavor because of its difficulty. Haar, "Musica mundana," 16 (n. 41). Such a motivation for the ancient Greek departure from the enharmonic in the late fifth century B.C. would have militated against the enharmonic as a primary choice of concentration for Plato. Macrobius further believed that Plato was inclined toward diatonic music because it was simple, noble, and more natural. Ibid.

17 Ibid., 10 (n. 35), 106, 436, 442–443.

world soul, made the diatonic division of the octave normative for the great bulk of later harmonic theory. He has also observed that the diatonic division remained the foundation for harmonic theory and, further, that its predominance on a practical plane gave it universal recognition and centrality in Greek music theory.[18]

4 The "Model" Octave and the Perfect Disdiapason

The first complete octave scale of any kind, and so, the "model octave" for purposes of this study, begins at 384 and ends at 768, with the following diatonic pattern, where the letters in red between the numbers indicate the sizes of intervals: [384] T 432 T 486 S 512^{PC} T 576^{PS} T 648 T 729$^{PS\ PC}$ S [768]. Another perfect octave of this kind ends at 1536. The two-octave sequence from 384 to 1536 is a disdiapason. It is undistorted, and so, perfect, because it contains only elements of the indicated octave pattern. The sequence from 384 to 768 also supports two enharmonic possibilities, one in the a.Phrygian pattern (TT) S (TT) T (S): 384 TT 486 S 512 TT 648 T 729 S 768 and the other in the d.Dorian pattern (TT) S T (TT) S: 384 TT 486 S 512 T 576 TT 729 S 768.[19] Because each of these scales rests upon the diatonic sequence, neither qualifies as the primary pattern of the model octave.

The TTS TTTS pattern indicated above for the model octave designates a descending Dorian or ascending Lydian diatonic sequence. These two kinds of scales are exact reciprocals of each other.[20] This special relationship, as well as the importance of both string vibration and string lengths in ancient Greek musical theory suggest that Plato may have intended the ambiguity for interpretational purposes.[21] His construction of the world soul had not, after all, been free of ambiguity from the outset, as this study has already demonstrated. Recall that the intervals created by the insertion of harmonic and arithmetic means into the original double and triple intervals of the soul stuff could be simultaneously interpreted as continuously overlapping doubles along both the double and triple arms of the original extension and continuously overlapping triples along both double and triple arms.

18 Creese, *Monochord*, 158–159.

19 Cf. Reese, *Music*, 32.

20 Ibid., 30.

21 Handschin reported that Pseudo-Timaeus, Proclus, and Michael Psellus left the question open as between the Dorian and Lydian scales, just as does this study. Handschin, "Timaeus Scale," 21.

Clearly, the exact same scalar numbers (and the exact same chromatic numbers following the perfect disdiapason) are involved in exactly the same relative *ordo*, whether one ultimately interprets the primary *Timaeus* scale as an ascending Lydian or descending Dorian scale. A possible argument for Dorian primacy may lie in the consideration that the d.Dorian enharmonic is one of the two possibilities for the first enharmonic octave and the d.Dorian chromatic is the only possibility for the first chromatic octave. The Dorian has prominence in the number set, in other words, for all three genera of octave.

A descending, rather than ascending interpretation for the diatonic sequence may make the most sense, as well. There appears to be no solid evidence that the ancients could make accurate measurements of string tension (as an index to a rate of string vibration or number of string impacts on the air), at all, much less in a way that correlates easily with the scale numbers.[22] The *Timaeus* scale is not, however, ultimately, about practical music making, but, rather a *harmonia* among numbers relating to the macrocosmic order and comprising, in some manner, a standard for all other harmonious relations.[23] The Lydian description cannot, therefore, be ruled out. For ease in presentation, just because it is more customary for most readers, today, to think in terms of rising scales, this study will sometimes favor Plutarch in assuming the Lydian option, although most modern commentators take the Dorian route.[24] At other times, it will prefer the Dorian interpretation.

Note that sequence prior to the model octave from 192 to 384 is almost complete, as a diatonic pattern, failing only to divide the final TS sequence: 192 T 216^{PC}T 243 S 256^{PS} T 288 T 324^{PS} TS 384. This sequence is, perhaps, a candidate for the Philolaic octave scale that proceeded in six, rather than seven steps, prior to Plato's writing career, in the first half of the fifth century.

It is, of course, difficult to arrive at an exact determination of the Philolaic octave.[25] Andrew Barker might be especially interested in the possibility suggested, however, because of some of the unit differences between the numbers. They correlate to some of Philolaus' characterizations of ratios and numbers, as reported by Boethius, and suggest that Philolaus' primary interest may have been a specific attunement of the seven-stringed lyre.

For example, Barker notes Boethius' claim that Philolaus identified the whole tone with a specific number of "units," namely twenty-seven and, further,

22 Creese, *Monochord*, 83, 164–165.

23 Ibid., 160.

24 Handschin, "Timaeus Scale," 24 and 26.

25 See Hagel, *Ancient Greek Music*, 112–113, 143–145, and 448 and Barker, *Science of Harmonics*, 264–286.

divided this into two unequal parts, a *diesis* of thirteen units and an *apotomē* of 14.[26] Barker attests to some puzzlement at these assignments, especially when they are considered from the viewpoint that the *diesis* was to be measured by the 256/243 remainder within a perfect fourth, after two whole tones. He posits that Philolaus was actually using two different systems of measurement for the *harmonia* attested by his fragment 6A.[27]

One need not draw Barker's conclusion. Note the following "unit" differences among the numbers of the sequence.

192 T 216 T 243 S 256 T 288 T 324 TS 384.
24 27 13 32 36 60

Assume, hypothetically, that Philolaus was working specifically with the above sequence in consideration of the string lengths needed for a specific lyre attunement, starting from a string 192 units long. He would have experimented with string lengths to arrive by ear at an interval that sounded like a tone to him and would have noted that the first tone after the string of a 192 unit length could be reached only with a string 24 units longer, namely one that was 216 units long. He would have observed the ratio of 9/8 between 216 and 192. By ear, he would have experimented with string lengths to arrive at the next string length that would produce a tone with the string under it and would have concluded that this next string had to be 243 units long. He would have observed the ratio of 9/8 between 243 and 216, too.

Philolaus would then have measured the string needed to produce the concordance of the fourth and would have found that only a string of length 256 would produce that concordance, together with the string of length 192. Observing the unit difference between 256 and 243, he would have concluded that the *diesis* in this particular attunement could only be reached from 243 with a string thirteen units longer than 243. He would have continued his tuning by ear to reach the string lengths necessary to fill out the rest of the octave, verifying in parity with his ear, that every increase of string length needed for a next tone arrived at a number producing a 9/8 interval with the number before it.

Philolaus would have run into a problem after 324 because there is no whole number that will split that interval to produce the next tone; and he might, with his very good ear, have verified that he could not find it by ear. He

26 Barker, *Science of Harmonics*, 272. Twenty-seven rows evoke twenty-seven units.
27 Ibid.

would have found that the next attunement he could reach by ear from 324 sounded like the interval between 256 and 216, and he would have observed that that string length needed to reach it was sixty units longer than 324. In other words, Boethius' report that Philolaus assigned a size of thirteen units to the *diesis* and twenty-seven to the tone were never sizes that Philolaus took to be absolute, but, rather, sizes he assigned to specific steps in a very particular lyre attunement. If it was the most common lyre attunement, he may never have had any practical need to go beyond it. It was certainly less cumbersome to string a lyre with shorter than with longer strings; and such attunement may have been a pleasing register for the musical ears of his day. He was simply instructing others how to achieve it. This account, is, as noted hypothetical, but it may be worth some interested scholar's further investigation.

Regardless of what Philolaus' project may have been, the sequence 192 T 216^{PC}T 243 S 256^{PS} T 288 T 324^{PS} TS 384 is not primary for this *Timaeus* study just because the better defined diatonic pattern exemplified from 384 to 768 follows upon it and receives special emphasis through orderly repetition among the numbers beyond 768.

5 Rise to the Perfect Disdiapason

Regardless of one's hermeneutical preference, close examination of Table 26 reveals that the model octave scale, beginning with 384 and perfect disdiapason ending at 1536 arise in an orderly way. One arrives at the 384 STN for the model octave (and all that follows it in Table 26) by taking 3 (representing the primary triad) as the very first STN and calculating the scale according to the TTSTTTS pattern of an ascending Lydian (numbers are indices to string vibrations or impacts on air) or descending Dorian diatonic scale (numbers represent string lengths), over and over again, until one finally arrives at the complete diatonic sequence. Table 27, below, illustrates the rise of the perfect disdiapason.[28]

28 As a matter of interest, Barker has opined that Philolaus intended to trace the musical numbers to an origin in the number three. Barker, *Science of Harmonics*, 281. The fact that the *Timaeus* points in the same direction, influenced as it was by Pythagorean thought, should not be too surprising.

TABLE 27 Rise of the diatonic scale in the set of *Timaeus* numbers

(1) Taking "3" as first STN, one calculates model scale numbers in the pattern TTSTTTS until one finds whole numbers corresponding to an entire octave.

(2) 3 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 yields the following numbers: 3.375; 3.796875; **4**; 4.5; 5.0625; 5.6953125; **6**. Clearly most of these numbers are off the chart and, accordingly belong to no scale; **but one does arrive at 4 as the end of the first fourth and at 6 as the second STN.**

(3) Taking 6 as the second STN, one calculates: 6 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243. The calculation yields the following sequence: **6**; 6.75; 7.59375; **8**; **9**; 10.125; 11.390625; **12**. Again, some numbers are off the chart and do not belong to a scale; **but now, one arrives at 8, marking the end of the first fourth; 9, marking the end of the first fifth; and 12 marking the third STN.**

(4) Taking 12 as the third STN, one calculates: 12 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243. The calculation yields the following sequence: **12**; 13.5; 15.1875; **16**; **18**; 20.25; 22.78125; **24. Similarly to case (3) above, the calculation finds the fourth, the fifth, and the next STN.**

(5) Taking 24 as the fourth STN, one calculates: 24 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243. And the calculation yields: **24; 27**; 30.375; **32; 36**; 40.5; 45.5625; **48. This sequence yields the "second" for the first time and, again, the fourth, the fifth, and the next STN.**

(6) Taking 48 as the fifth STN, one calculates: 48 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243. And the calculation yields: **48; 54**; 60.75; **64; 72; 81**; 91.125; **96. This sequence yields the second, fourth, fifth, the sixth for the first time, and the next STN.**

(7) Taking 96 as the sixth STN, one calculates: 96 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243. The calculation yields: **96, 108**, 121.5; **128; 144; 162**; 182.25; **192. Like case (6) above, this sequence yields the second, fourth, fifth, sixth, and next STN.**

(8) Taking 192 as the seventh STN, one calculates: 192 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243, yielding: **192; 216; 243; 256; 288; 324**; 364.5; **384. This time one arrives at the first complete fourth with the introduction of the third and every scale element except the seventh.**

(9) Taking 384 as the eighth STN, one calculates: 384 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243, yielding: **384; 432; 486; 512; 576; 648; 729; 768. One arrives at the first complete octave**. As the reader can see by consulting the sequential table of *Timaeus* numbers, no numbers lying between those listed occur in the Crantor matrix. They form a continuous sequence. This octave, in other words, contains no chromatic elements.

TABLE 27 Rise of the diatonic scale in the set of *Timaeus* numbers (*cont.*)

(10)	Taking 768 as the ninth STN, one calculates: 768 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243, yielding: 768; 864; 972; 1024; 1152; 1296; 1458; 1536. Like the octave before it, this octave, too, is a continuous sequence in the numerically ordered table of *Timaeus* numbers. This means the octave contains no chromatic elements.
(11)	1536 is the tenth STN. It ends the perfect disdiapason, the model two octave sequence.

6 First Octave of the Model Diatonic Octave Chain Containing Chromatic Elements

The diatonic octave from 1536 to 3072 is calculated thus: 1536 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243, yielding: 1536; 1728; 1944; 2048; 2304; 2592; 2916; 3072. When the *Timaeus* numbers are arranged in order from least to greatest as in Table 26, 2187 intervenes between 2048 and 2304. Musicologist Ernest McClain identified all such intervening numbers as chromatic elements entering the octave scale.[29] The octave from 1536 to 3072 is the first sequence in the octave chain continuous with 384 displaying a chromatic element. McClain explained that the chromatic elements are members of fourths foreign to the model octave chain that, nonetheless, overlap its fourths and make their influence felt *via* the intervening elements.[30] Recall that, according to Handschin, the interfering fourths came into being when Plato divided not only the double intervals, but also the musical twelfths (the triples).[31] Recall, as well, that the independent, interfering fourths include fourths within doubles and triples, beyond the original ones, that were newly generated with that same division. Those new doubles and triples continuously overlap both each other and the original order of doubles and triples.

One can easily discern one way in which the entrance of the chromatic elements into the model diatonic octave chain relates to patterns of overlapping doubles and triples interfering with the original octave periodicity (and each other). One need only inspect Table 26, concentrating on the relations between

29 McClain, *Pythagorean Plato*, 61–62.

30 Ibid.

31 Handschin, "Timaeus Scale," 19–24.

and among the two sets of chromatic numbers it indicates, namely those that do and do not constitute factors of 1719926784.

Upon considering each subset of chromatic numbers, in turn, and recalling that a double interval of the diatonic genus can be articulated as a fourth plus a fifth, in the pattern 3/2 × 4/3 or 4/3 × 3/2, or as the joinder of two disjunct fourths separated by a sesquioctave interval, in the pattern 4/3 × 9/8 × 4/3 or 9/8 × 4/3 × 4/3 or 4/3 × 4/3 × 9/8, one finds overlapping diatonic doubles entirely defined by elements of the factor or nonfactor subset of chromatic numbers, respectively, as set forth in Appendix 5, "Two Overlapping Sequences of Doubles, Including Coincident Diatonic Octaves Within Each, Bounded Entirely by Chromatic Factors of 1719926784" and Appendix 6, "Two Overlapping Sequences of Doubles, Including Coincident Diatonic Octaves Within Each, Bounded Entirely by Chromatic Nonfactors of 12719926784." The constituents of these doubles appear as foreign elements within the primary ascending Lydian or descending Dorian diatonic scale of the *Timaeus*. Other instances of overlapping doubles are provided in Appendix 7, "Continuously Overlapping and Contiguous Chains of Doubles, Including Coincident Diatonic Octaves Within Each, Bounded Entirely by Model Scale Numbers and Their Multiples." One could similarly catalogue all instances of overlapping triples. Because they are not the focus of this study, it is unnecessary and so, unwarranted, from the standpoint of length to include such catalogues among the apparatus.

7 First Instances of Standard GPS, LPS, Diatonic UPS, and UPS in All Genera

Because the *Timaeus* numbers initially rise to an ascending Lydian or descending Dorian disdiapason unadulterated by chromatic elements, they may present an early species of GPS. GPS is a simple reference to the disdiapason, described by Reese as "the basis of the Greek tonal fabric at the height of its development."[32]

According to Reese, "at least in the earlier centuries in which they [Greeks] discuss series of tones arranged in order of pitch," GPS assumes descending pitch values, so that the numbers used to represent intervals indicate string lengths proceeding from shorter lengths and higher pitch to longer lengths and lower pitch. If the numbers, instead, represent string impacts vibrating slowly

32 Reese, *Music*, 21; see also, this study's "Glossary" and Appendix 3, entitled "Greater and Lesser Perfect Systems and Associated Questions."

and so, making sounds lower in pitch and progress to those vibrating faster and, thus, making sounds higher in pitch, then they would indicate an ascending order.[33] Scholars cannot be absolutely sure that GPS always assumes descending pitch values; so one must register the possibility that the ancients could present ascending pitch values within the GPS structure.[34] In general, however, this study will follow the accustomed opinion that descending values were the GPS norm.

As noted in the "Preface" to this study, the GPS system was probably not yet definitively developed in Plato's time, though Eratocles (422 B.C.) had made significant strides toward something like it; and Aristoxenus (late fourth century) had definitely achieved a perfect system with a consistent rationale.[35] Stephen Hagel has assigned GPS to the end of the fourth century, at the latest; opined that, in all probability, Plato already knew it; and observed that it was of some practical importance for the boring of a single-mode aulos.[36] Donald Creese has concluded that both GPS and LPS were constructs of the fourth century.[37]

LPS was an eleven note system, sharing eight notes with GPS, from *proslambanomenos* of GPS to *mese* of GPS.[38] There it departed from GPS, adding a conjunct fourth, called *Synemmenon* (meaning "hooked"), beyond *mese*.[39] Modulation could occur between these two systems because of the eight notes that they had in common.[40] Modulation was strictly limited within the parameters set by the combined systems; so they are known together as the Unmodulating (Perfect) System ("UPS").[41]

West maintained that there were various articulators of perfect systems before Aristoxenus, but little is known of them.[42] The point was to demonstrate all seven possible Greek octave scales as transpositions of each other within one unified scheme in all three octave genera, so that melodies could be easily transposed into different vocal ranges for instrumental accompani-

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33 Reese, *Music*, 21–22 (material in brackets added for clarity in quoted material).

34 See ibid., 21, indicating that, in earlier centuries, the descending pitch order was usual.

35 West, *Ancient Greek Music*, 228–229; Mathiesen, *Apollo's Lyre*, 294 (for date).

36 Stephen Hagel, *Ancient Greek Music, A New Technical History* (Cambridge: Cambridge University Press, 2010), 387–388. Donald Creese has concluded that both GPS and LPS were constructs of the fourth century B.C.

37 Donald Creese, *The Monochord in Ancient Greek Harmonic Science* (Cambridge: Cambridge University Press, 2010), 21. (GPS was a fourth century construct).

38 Reese, *Music*, 23.

39 Ibid.

40 Ibid.

41 See Hagel, *Ancient Greek Music*, 83–84 and 159 (for reference to GPS and LPS, together, as the "Unmodulating System").

42 West, *Ancient Greek Music*, 228.

ment. The disdiapason was considered the relevant range of transposition for most voices.[43]

According to Reese, GPS, in its full standard development, comprised a system of four fourths with an independent whole tone appearing in the middle of the system and an independent whole tone appearing at the end.[44] It displayed the Dorian octave as normative (assuming, it seems, by "normative" the position of the Dorian octave at the center of the GPS system) because of the ease with which Dorian melodies could be played on a lyre. He explained:

> To a Greek, the *tonos* corresponding to a key without sharps or flats would, in low tuning, have been the one requiring no stoppings on a lyre except for the missing C and F. The *tonos* of which this was true was, as I have stated, called the Dorian in the earlier days and obviously, it is the *tonos* that produced the Dorian series within the characteristic octave.[45]

He observed that octave species of the other modes can certainly be understood within such a GPS as redistributed Dorian octaves.[46] Chapter 6 of this study shows that Plato's system is capable of accommodating such an understanding, too, despite an apparent departure from the standard version of GPS.

For the sake of exploring Plato's departure from the standard GPS model later in this chapter, it is important to do some preliminary work with what came to be accepted as the standard model of the perfect system. The discussion and comparison of the way in which the *Timaeus* numbers accommodate both the standard model and a specific alternative model arising in the number set will be key to determining the scale properly identifiable as the primary *Timaeus* scale. Readers interested in exploring the general range of other experimental possibilities suggested by the *Timaeus* number set should consult Appendix 3 hereto, "Greater and Lesser Perfect Systems and Associated Questions" and Appendix 4 hereto, "Alternative Perfect Systems."

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43 West, *Ancient Greek Music*, 275; Reese, *Music*, 30–31; Dupuis, "Note XII, On the Perfect Musical System formed of Two Octaves," note to Theon of Smyrna, *Mathematics Useful for Understanding Plato*, ed. by Christos Toulis et al., with an appendix of notes by Dupuis, Secret Doctrine Reference Series, translated from the 1892 Greek/French edition of J. Dupuis by Robert and Deborah Lawler (San Diego, CA: Wizard's Bookshelf, 1979), 148.

44 Reese, *Music*, 21; see also, notes 18–23 to this study's "Glossary" and Appendix 3, entitled "Greater and Lesser Perfect Systems and Associated Questions."

45 Reese, *Music*, 30.

46 Ibid.

7.1 *Standard GPS*

As noted, above, GPS, in what came to be its standard presentation put the descending Dorian octave (could also be an ascending Lydian) after the first fourth of a system constructed, essentially, from a descending Hypodorian (or ascending Hypophrygian) disdiapason, but conceived as four TTS fourths with one disjunction at the center of the system and one at the bottom of the system, thus: [(TTS) (TTS) T (disjunct) (TTS) (TTS) T (disjunct)].[47] In the preceding sentence, the parentheses indicate the fourths, the red brackets the disdiapason, and the yellow highlighting the central octave. The fourths had distinctive names. In the order given above, they were tetrachord Hyperbolaion (extra fourth); tetrachord Diezeugmenon (fourth of the disjunction because it preceded the disjunction preceding the next fourth); tetrachord Meson (fourth of the middle because it followed the tone of disjunction constituting the middle of the system) and tetrachord Hypaton (highest fourth, presumably from the standpoint of a player's position holding the lyre, since the pitches in this fourth were lower than those of the fourths preceding it).[48]

The first instance of the standard presentation of GPS in the *Timaeus* table is the following sequence, in which the central descending Dorian (or ascending Lydian) octave sequence appears emboldened in brackets:

> 576 T 648 T 729 S [**768 T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536**] T 1728 T 1944 S 2048 T 2304

The undivided TS interval between 324 and 384 in the octave preceding 576 prevents the sequence from 288 through 1152, in Table 26, from being the first occurrence of GPS.

7.2 *Standard LPS*

LPS, in what came to be its standard presentation was conceived as two conjunct TTS tetrachords separated from a TST tetrachord by a disjunct tone, such that the sequence beginning with the second TTS tetrachord through the TST tetrachord coincided with standard GPS from the middle note of that system (eighth note from either end). The tonal structure was the following: [(TTS) {(TTS) T] (TST)}.[49] In the preceding sequence the fourths appear in paren-

47 See Reese, *Music*, 22 and 30 (for a comparison of octave patterns and for the presentation of standard GPS).

48 Ibid., 22.

49 Ibid., 22–23.

theses. The tonal sequences in bold black brackets and bold red braces are each, respectively, descending Hypodorian (or ascending Hypophrygian) diatonic octave sequences.[50] The fourths in this system also had names. In the order given, they were tetrachord Synemmenon (meaning "hooked"); tetrachord Meson (same as that of GPS); and tetrachord Hypaton (same as that of GPS).

The first instance of the standard LPS tone sequence among the *Timaeus* numbers begins with 1728 and ends with 4608, the tone a fourth below the octave ending with 3456 (or above if one is working in ascending order), as follows:

1728 T 1944 T 2187 S 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608

Note that the occurrence of standard diatonic LPS depends upon the emergence of 2187, the first chromatic number in the *Timaeus* set, highlighted above in gray. Observe that this first articulation of LPS does not relate to the first articulation of standard GPS; it relates, rather, to the second articulation of standard GPS possible on the basis of the *Timaeus* numbers as follows:

1152 T 1296 T 1458 S 1536 T 1728 T 1944 S 2048 T 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608

Observe that 2304 is the center of this second instance of standard GPS among the *Timaeus* numbers. It is in the eighth position from either end of the sequence. This second GPS articulation shares the eight notes of the standard model that one would expect with standard LPS, beginning with 2304 and ending with 4608; and standard LPS, as identified, likewise, departs from standard GPS as one would expect at 2304 by positing a conjunct fourth beyond that number of the type TTS. In standard GPS, there is a disjunction from *mese* by a tone, at that point, and then a fourth above the disjunction of the TTS type.

7.3 *Standard Diatonic UPS*

Clearly modulation could occur between standard LPS and standard GPS at the number 2304 within a combined diatonic UPS. The combined first articulation of standard UPS in the diatonic genus among the *Timaeus* numbers is

50 Ibid., 30.

the following sequence. *Mese*, the middle of the system, is highlighted in yellow. Underlined numbers are part of LPS. Notes that belong only to LPS are further indicated by an asterisk following the number. Underlined notes without asterisks are also part of GPS.

> 1152 T 1296 T 1458 S 1536 T 1728 T 1944 S 2048 S′ 2187* S 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608

7.4 *Unacceptable Modulation*

The *Timaeus* numbers suggest a nonstandard kind of modulation between the first articulation of standard LPS and the first articulation of standard GPS, involving the last fourth of the latter system. Comparing the first articulation of standard GPS to the first articulation of standard LPS, one sees that modulation is possible at 1728, the second number of the last fourth of GPS. From this point, standard GPS posits TS of its last fourth and the disjunct T, and LPS makes a TTS fourth between 1728 and 2304, allowing modulation between the two systems.

> *1st standard GPS*: 576 T 648 T 729 S [**768 T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536**] T 1728 T 1944 S 2048 T 2304

> *1st standard LPS*: 1728 T 1944 T 2187 S 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608

Perhaps this kind of modulation was thought to be aesthetically inappropriate because it actually split a fourth of standard GPS. It did not, in any case, become the basis of a standard model.

7.5 *Standard Combined UPS in All Three Octave Genera*

The *Timaeus* numbers not only allow articulation of standard UPS in the diatonic genus; but they also permit multiple iterations of a comprehensive standard UPS, contemplating chromatic and enharmonic sequences and modulation. According to modern scholarly opinion, the range for standard UPS was a two octave system starting [a^2] to [a], comprising, as noted above, a Hypodorian disdiapason, with a Dorian diapason at the center, following the first fourth and running from [e^2] to [e^1]. Note Appendix 3 regarding the possibly artificial character of assigning note values familiar to modern music to particular numbers. It is safer, because it assumes less, simply to associate ancient note names, e.g., "nete hyperbolaion" (note name followed by the name of fourth in the system to which it belonged) with numbers.

Table 28 below articulates standard UPS, encompassing both GPS and LPS and modulation between them in all three *genera*, in three complete iterations supported by the *Timaeus* numbers, within a particular series starting from 20736. It also displays a fourth sequence, prior to and continuous with the same series, that almost but does not quite succeed. This fourth sequence includes Thrasyllus' values for diatonic and chromatic GPS.[51] Table 28 assumes descending order (highest pitch, shortest string length), for purposes of presentation, and relies upon Mathiesen's list of note designations and intervals for standard UPS.[52] Be aware that the iterative series in the table is probably not exhaustive of the standard UPS systems articulable on the basis of the *Timaeus* number set. An interested reader may find that he can discover and construct additional standard UPS iterations by consulting Appendices 5–7 to this study, after identifying and analyzing all possible Hypodorian descending series originating from any number in the set, as each of these would position a descending Dorian octave at the center of the system. Appendices 5–7 should also inspire the reader to seek and consider the possible nonstandard UPS systems articulated by the numbers.

TABLE 28 Some completely articulated standard UPS systems within the *Timaeus* set numbers

Descending Hypodorian disdiapason arranged thus: [(TTS) (TTS) (T)] [(TTS) T (TST)] for GPS in Dorian [(TTS) T (TTS)] tonos.

Name of note with reference to appropriate fourth	**GPS or LPS**	**Complete UPS No. 1**	**Complete UPS No. 2**	**Complete UPS No. 3**	**Nearly complete UPS**
[a^2] *nete hyperbolaion*	GPS only	20736	41472	82944	10368
T = 9/8					
[g^2] diatonic *paranete hyperbolaion*	GPS only	23328	46656	93312	11664
S = 256/243					
[$f^{\#2}$] chromatic *paranete hyperbolaion*	GPS only	24576	49152	98304	12288
S′ = 2187/2048 *The two semitones together make a 9/8 (T) interval.*					

51 See Mathiesen, "Ancient Greek Music," 118.

52 Ibid., 118 and 122.

TABLE 28 Some completely articulated standard UPS systems (*cont.*)

Name of note with reference to appropriate fourth	GPS or LPS	Complete UPS No. 1	Complete UPS No. 2	Complete UPS No. 3	Nearly complete UPS
[f^2] *trite hyperbolaion*;	GPS only	26244	52488	104976	13122
also [f^2]enharmonic *paranete hyperbolaion*	GPS only				
S = *256/243* Ends first fourth of the descending Hypodorian disdiapason.					
[e^2] *nete diezeugmenon*	GPS only	27648	55296	110592	13824
[e*^{2}] (lower than [e^2] *trite hyperbolaion* when enharmonic in GPS)	GPS only	26946	53892	107784	13473
T = *9/8 from* [e^2] *Independent whole tone between the first and second fourths of the descending Hypodorian disdiapason.*					
[d^2] diatonic *paranete diezeugmenon* in GPS	GPS and LPS	31104	62208	124416	15552
[d^2] *nete synemmenon* in LPS					
S = 256/243					
[c#^{2}] chromatic *paranete diezeugmenon*	GPS only	32768	65536	131072	16384
S′ = 2187/2048 *Makes 9/8 (T) interval with preceding S interval*					
[c^2] *trite diezeugmenon* in GPS; also	GPS and LPS	34992	69984	139968	17496
[c^2] enharmonic *paranete diezeugmenon* in GPS; and					
[c^2] diatonic *paranete synemmenon* in LPS					
[b*^{1}] *trite diezeugmenon* when enharmonic in GPS	GPS only	33880	67760	135520	16490

TABLE 28 Some completely articulated standard UPS systems (*cont.*)

Name of note with reference to appropriate fourth	**GPS or LPS**	**Complete UPS No. 1**	**Complete UPS No. 2**	**Complete UPS No. 3**	**Nearly complete UPS**
S = 256/243 (from [c²])					
[b¹] *paramese* in GPS; and [b¹] chromatic *paranete synemmenon* in LPS	GPS and LPS	36864	73728	147456	18432
S′ = 2187/2048					
[b♭¹] *trite synemmenon* in LPS; also [b♭¹] enharmonic *paranete synemmenon* in LPS	LPS only	39366	78732	157464	19683
[a*¹] *trite synemmenon* when enharmonic in LPS	LPS only	38115	76230	152460	Incalculable
S = 256/243 *Makes 9/8 interval with preceding S′ ([b♭¹]). Ends second fourth (disjunct) of the descending Hypodorian disdiapason*					
[a¹] *mese* in both GPS and LPS	GPS and LPS	41472	82944	165888	20736
T = 9/8 *Begins second descending Hypodorian diapason in the two-octave GPS system with first fourth (conjunct)*					
[g¹] diatonic *lichanos meson* in both GPS and LPS	GPS and LPS	46656	93312	186624	23328
S = 256/243					
[f*¹] chromatic *lichanos meson* in both GPS and LPS	GPS and LPS	49152	98304	196608	24576
S′ = 2187/2048 *Makes a 9/8 (T) interval with preceding S*					

TABLE 28 Some completely articulated standard UPS systems (*cont.*)

Name of note with reference to appropriate fourth	GPS or LPS	Complete UPS No. 1	Complete UPS No. 2	Complete UPS No. 3	Nearly complete UPS
[f^{1}] enharmonic *lichanos meson* in both GPS and LPS; also [f^{1}] *parhypate meson* in both GPS and LPS	GPS and LPS	52488	104976	209952	26244
[e^{*1}] enharmonic *parhypate meson* in both GPS and LPS	GPS and LPS	50820	101640	203280	25410
S = 256/243 (from [f^{1}]) *Ends third fourth of the descending Hypodorian disdiapason* (*first in the second octave*).					
[e^{1}] *hypate meson* in both GPS and LPS	GPS and LPS	55296	110592	221184	27648
T = *9/8* *Independent whole tone of the second octave in the descending Hypodorian disdiapason*					
[d^{1}] diatonic *lichanos hypaton* in both GPS and LPS	GPS and LPS	62208	124416	248832	31104
S = 256/243					
[c$^{\#1}$] chromatic *lichanos hypaton* in both GPS and LPS	GPS and LPS	65536	131072	262144	32768
S′ = 2187/2048 *Makes a 9/8* (*T*) *interval with preceding S*					
[c^{1}] enharmonic *lichanos hypaton* in both GPS and LPS; also [c^{1}] *parhypate hypaton* in both GPS and LPS	GPS and LPS	69984	139968	279936	34992
[b*] enharmonic *parhypate meson*	GPS and LPS	67760	135520	271040	33880
S = *256/243*					
[b] *hypate hypaton* in both GPS and LPS	GPS and LPS	73728	147456	294912	36864

TABLE 28 Some completely articulated standard UPS systems (*cont.*)

Name of note with reference to appropriate fourth	GPS or LPS	Complete UPS No. 1	Complete UPS No. 2	Complete UPS No. 3	Nearly complete UPS
T = 9/8 *Ends last fourth* (*disjunct*) *of the descending Hypodorian disdiapason.*					
[a] *Proslambanomenos* in both GPS and LPS	GPS and LPS	82944	165888	331776	41472

All of the numbers in Table 28 are *Timaeus* numbers except for certain quarter tone values, necessary for articulating a QT enharmonic. Those quarter tone values are highlighted in red in Table 28. They are calculated in simple fashion from the *Timaeus* numbers, upon the occurrence of relevant S (256/243) intervals, according to the formula (x + y)/2 = z, where x and y are the numbers above and below the quarter tone split and z is the value corresponding to the bottom of one the quarter tone intervals and the top of the other one.

When one splits an interval of the size 256/243 to create two quarter tones, then the respective sizes of those quarter tones are the following: z/x≈ 1.026748971 and y/z≈1.026052104. These are the only kinds of intervals actually split in the standard GPS of the *Timaeus* to achieve quarter tones. Theoretically, in some other version of GPS, one could try to split intervals of the size 2187/2048 to create two quarter tone intervals. In that case the respective sizes of the quarter tones would be the following: z/x≈1.033935547 and y/z≈1.032821724. In the case of either kind of semitone split, the method illustrated yields two quarter tones of slightly different size because neither semitone can be split exactly in half to permit the quarter tones to be expressed as ratios of whole numbers. Perhaps the slight disparity in the sizes of the quarter tones made no noticeable difference to the ear.

Persons who take the *Timaeus* number set as supporting, really, only a semitone enharmonic can ignore the quarter tone values in the table. They are included in Table 28 only because it is theoretically possible that a QT enharmonic existed in Plato's day and it can, in fact, be calculated from numbers in the *Timaeus* set.

Although the *Timaeus* numbers allow the presentation of standard UPS, that system does not appear to be the primary focus of the set. The first perfect disdiapason is not the descending Hypodorian (or ascending Hypophrygian) two-octave sequence corresponding to standard UPS, but, rather a descending

Dorian (or ascending Lydian) two-octave array that would provide a foundation for an alternative UPS. That primary alternative is explored below.

8 First Instances of Properly *Timaean* GPS, LPS, Diatonic UPS, and UPS in All Genera

8.1 *The* Timaeus *GPS*

With sensitivity to the circumstance that scholars have no evidence for GPS in any other presentation than the standard model, one might observe that GPS, theoretically, should be capable of alternative expressions in other *tonoi*. As noted, 384 is the beginning of the first completely articulable octave in the *Timaeus* set; it is also, then, the beginning of the first completely articulable disdiapason. If one thought that the first perfect disdiapason of the set should have some special significance because it is first and perfect, then one could justifiably propose that the standard GPS model was not a primary focus for Plato. One might, accordingly, redefine GPS, as follows, conceiving it analogously to the standard model as a system of four fourths with two independent whole tones:

[(TTS) **T (disjunct) (TTS) (TTS)** T (disjunct) (TTS)].

The four fourths are in parentheses, above, the disdiapason in red brackets, and the central octave highlighted in **blue**. The disjunctions in this system occur after the first and third fourths, in contrast to their placement in the standard system.

The first subset of numbers in the *Timaeus* number set expressing a descending or ascending sequence according to the redefined pattern is the following:

384 T 432 T 486 S [**512 T 576 T 648 T 729 S 768 T 864 T 972 S 1024**] T 1152 T 1296 T 1458 S 1536

The sequence marks a descending diatonic *Dorian* disdiapason or an ascending diatonic *Lydian* disdiapason, running from 384 to 1536. The octave sequence in the middle is a descending diatonic *Mixolydian* sequence or an ascending diatonic *Hypolydian* sequence running from 512 to 1024.[53] The argument for such an alternative GPS for Plato is its appearance first among theoretical possibil-

53 See Reese, *Music*, 30 (for comparison of all octave patterns).

ities for GPS systems among the *Timaeus* numbers, as well as its emphasis on Dorian and Lydian-type diatonic octaves that also occur first in the number set among octave patterns.

8.2 *The* Timaeus *LPS*

In the standard model, LPS shares with GPS the notes from prolambanomenos (the note of lowest pitch) to *mese* (the middle note of the system) and then adds a conjunct fourth above *mese*, in contrast to the disjunct tone occurring in GPS calculated from the same place. This study assumes that some such contrast, in terms of conjunction and disjunction, is important to preserve between any GPS and LPS, however redefined.

The first subset of *Timaeus* numbers fitted to the right kind of relationship with the *Timaeus* GPS clarifies that the *Timaeus* UPS, if a descending system like standard GPS and standard LPS, has a GPS and LPS sharing, not the notes from *mese* to *proslambanomenos*, but the notes from *nete hyperbolaion* to *mese*. *Timaeus* LPS adheres to the following pattern: [(TTS) (T) TTS (T) (TTS)]. In this presentation, *nete hyperbolaion* precedes the very first interval of the sequence; the interval above *mese* is highlighted in green and is a semitone, rather than the tone of the standard system; the tone in red is a tone of disjunction occurring directly below *mese*, in contrast to the conjunction of fourths occurring at the same place in *Timaeus* GPS; and the fourth highlighted in yellow is the tetrachord Synemmenon of *Timaeus* LPS.

Observe that *Timaeus* LPS and standard LPS would be exactly parallel, in terms of shared notes from *proslambanomenos* to *mese* with their respective GPS systems, only if *Timaeus* LPS and GPS were an ascending Lydian, rather than descending Dorian sequence. In that case, the *Timaeus* perfect systems and standard perfect systems would run in opposite directions. For the sake of simplicity, comparison, and the previously mentioned scholarly preference for descending interpretations of GPS, this study will assume instead, that *Timaeus* GPS is also a descending system (shortest string length indicates highest pitch).

The first set of numbers fitting the proposed *Timaeus* LPS pattern is the following:

768 T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536 T 1728 T 1944 T 2187 S 2304

Note that the occurrence of LPS depends upon the emergence of 2187, the first chromatic number in the *Timaeus* set, highlighted above in gray. As in the case of standard GPS, the first articulation of *Timaeus* LPS, relates not to the first articulation of *Timaeus* GPS but, rather, to the second one, like so:

2nd Articulation of Timaeus *GPS*: (768 T 864 T 972 S 1024) (T 1152) (T 1296 T 1458 S **1536**) (T 1728 T 1944 S 2048) (T 2304) (T 2592 T 2916 S 3072)

1st Articulation of Timaeus *LPS*: (768 T 864 T 972 S 1024) (T 1152) (T 1296 T 1458 S **1536**) (T 1728) (T 1944 T 2187 S 2304)

Above, the fourths and independent tones of each system are in parentheses. Note the two conjunct fourths highlighted in yellow in GPS. Observe the position of the number 1728 in GPS. It is the first element of the second of these conjunct fourths. Note that, in LPS, 1728 is an independent, disjunct tone, highlighted in green. ***Mese***, corresponding to the middle number of the GPS system is in bold.

The proper contrast is clearly preserved between the proposed *Timaeus* GPS and LPS. Where conjunction occurs in *Timaean* GPS, the tetrachord Synemmenon of LPS can only be added after a disjunction. In contrast, where disjunction occurs in standard GPS, the tetrachord Synemmenon of LPS must be conjunct.

8.3 *Diatonic* Timaeus *UPS*

Clearly modulation could occur between *Timaeus* LPS and *Timaeus* GPS from 1536 within a combined diatonic UPS. This is the proper point of modulation because a conjunct fourth follows 1536 in the case of GPS and a tone of disjunction in LPS. The combined first articulation among the *Timaeus* numbers of Diatonic *Timaeus* UPS is the following sequence.

768 T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536 T 1728 T 1944 S 2048 S′ 2187* S 2304 T 2592 T 2916 S 3072

Mese, the middle of the system, is highlighted in yellow. Underlined numbers are part of LPS. Notes belonging only to LPS are further indicated by an asterisk following the number. Underlined notes without asterisks are also part of GPS.

8.4 Timaeus *Combined UPS in All Genera*

The *Timaeus* numbers not only allow articulation of the *Timaeus* UPS in the diatonic genus; but they also permit several iterations of a comprehensive *Timaeus* UPS, contemplating chromatic and enharmonic sequences and modulation. Some effort is required to construct *Timaeus* UPS.

As an initial approach to finding the first sequence among the *Timaeus* numbers accounting for all diatonic, chromatic, and enharmonic possibilities within a *Timaeus* GPS, one must search Table 26 for a descending Dorian diapa-

son continuous with the series beginning from 384 having the most complete articulation of semitones possible within the table. This fullest semitone structure sets the broad parameters for the chromatic and enharmonic possibilities of *Timaeus* GPS, even if they can be realized earlier in the series. This most complete articulation of semitones occurs only after the entrance of all chromatic numbers into the series descending from 384. This occurs at the disdiapason beginning with the number 49152 and ending with 196,608. The full tone structure of *Timaeus* GPS, based upon the intervals in the latter two-octave span is the following.

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S]

Note that the semitones designated S′ are of the 2187/2048, rather than 256/243 variety. Not all elements internal to the structure of "T" actually manifest as notes in *Timaeus* GPS, as the study will clarify below.

As an initial approach to finding the first sequence among the *Timaeus* numbers accounting for all diatonic, chromatic, and enharmonic possibilities within *Timaeus* LPS, one searches Table 26 for the sequence continuous with the series originating from 384 containing the most complete articulation of semitones within the structure of a descending Dorian disdiapason and locates LPS [(TTS) T (TTS) T (TTS)] within it. In other words, one consults the same span from 49152 to 196608 in Table 26 and maps *Timaeus* LPS onto it. The mapping reveals the full LPS tone structure. The full tone structure of *Timaeus* LPS is the following:

[(T= S'S)(T= S'S)S] [(T= S'S)] [(T= S'S)(T)S] [(T= S'S)] [(T= S'S)(T= SS′)S]

Not all elements internal to the structure of "T," below, manifest as notes in LPS, as the study will clarify apace.

When one lays *Timaeus* LPS over *Timaeus* GPS in the *Timaeus* UPS system, it has certain intervals in common with *Timaeus* GPS, as indicated in green highlighting below. It departs from *Timaeus* GPS by making the tone in blue highlighting independent and then including a TTS fourth, the three elements of which are marked below in yellow, lavender, and gray, respectively.

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S]

It is necessary to elucidate why the GPS and LPS diatonic tone structures provided above allow for all chromatic and enharmonic possibilities for *Timaeus* GPS and LPS, in addition to the diatonic possibilities. Note that the general structure of a descending Dorian chromatic disdiapason (the chromatic GPS), embedding a central descending Mixolydian chromatic octave (see portion in red) is the following:

[(TS) SS] [T] [(TS) SS] [(TS) SS] [T] [(TS) (SS)][54]

A mapping of the chromatic disdiapason onto the *Timaeus* diatonic GPS system manifests that its first possible articulation arises shortly after the diatonic sequence is underway with the following infrastructure:

[(TS=SS'S) SS'] [T= SS'] [(TS=ST) SS'] [(TS=SS'S) SS'] [T= SS'] [(TS=ST) SS']

In the mapping below, onto two articulations of the *Timaeus* GPS system, elements belonging to the same chromatic fourth share the same color. Independent whole tones are in blue highlighting.

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S]

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S]

As one can see, an extra semitone spilling over into the next *Timaeus* GPS disdiapason is required for the full mapping of chromatic GPS. Clearly such an articulation of chromatic GPS does not fit the parameters of a genuine perfect system because it exceeds the two-octave bound; so one must look for other infrastructure possibilities for chromatic GPS.

A second possibility, in fact, brings the chromatic GPS within the bounds of a perfect system, though it does so somewhat clumsily. An alternative structure for a descending Dorian chromatic disdiapason (the chromatic GPS), embedding a central descending Mixolydian chromatic octave (see portion in red) is the following:

54 Reese, *Music*, 31.

[TS S'S] [T] [(TS=SS'S) S'S][TS S'S] [T] [(TS=SS'S) S'S]

A mapping of the chromatic disdiapason, thus redefined, onto the GPS infrastructure shows, again, that a repetition of the *Timaeus* GPS system is actually required for the full mapping. In the representation that follows, elements belonging to the same chromatic fourth share the same color and independent whole tones are in blue highlighting.

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T = S'S)S] [(T=S'S)] [(T = S'S)(T)S]

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S]

The nature of the spilling over, in this case, however, makes the excess unproblematic. Note that the spillover into the first octave of the second GPS system maps exactly back onto the first octave, of the first system, so that one may interpret the spillover notes as those spilling over from a system preceding the first disdiapason, above, representing GPS. Practically speaking, this means that the last roughly one and one-half fourths and the second independent tone of chromatic GPS occur above its first roughly two and one-half and first independent tone in every two-octave sequence articulating *Timaeus* UPS belonging to the same octave chain as the perfect disdiapason beginning with 384.

The suggested mapping for chromatic GPS plainly succeeds in including the whole of it within the bounds of a two-octave descending Dorian system, although it may still strike one as less than ideal. One might well prefer a scenario in which chromatic GPS began with its Hyperbolaion tetrachord in synchronization with diatonic GPS. As the study shows further below, descending Dorian sequences satisfying this elegant condition, within the bounds of a disdiapason, exist among the *Timaeus* numbers, though they are not continuous with the primary series emanating from 384. Because they allow a neater, more harmonious arrangement, they are better candidates for a standard presentation of *Timaeus* UPS.

The enharmonic genus for *Timaeus* GPS and both the chromatic and enharmonic genera for *Timaeus* LPS remain to be considered before a definitive determination of *Timaeus* UPS is possible. The structure of a descending Dorian enharmonic disdiapason (the enharmonic GPS), embedding a central Mixolydian enharmonic octave (see portion in red below) is the following:

[(TT) QQ] [T] [(TT) QQ] [(TT) QQ] [T] [(TT) QQ][55]

This sequence maps onto the Timaeus GPS infrastructure as follows, where green highlighting represents undivided ditones; S highlighted in blue represent an interval to be split for a QQ sequence; underlining indicates elements belonging to the same fourth; and yellow highlighting represents independent whole tones:

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S]

Clearly, the enharmonic GPS fits very neatly into the overall system.

The structures of chromatic descending *Timaeus* LPS and enharmonic descending *Timaeus* LPS must also be accommodated within the system. They are not exactly parallel with the chromatic descending *Timaeus* GPS and descending enharmonic *Timaeus* GPS. The basic structure of chromatic *Timaeus* LPS is [(TS) SS] [T] [(TS) SS] [T] [(TS) SS]. It mirrors the diatonic structure in comprising a descending Dorian octave with a repetition of the final fifth.

The infrastructure of chromatic *Timaeus* LPS, imbedding a Mixolydian chromatic octave sequence highlighted in yellow and green is the following:

[(TS= S'SS) S'S] [T] [TS S'S] [T] [TS= S S'S S'S]

The chromatic *Timaeus* LPS maps onto the *Timaeus* GPS infrastructure as follows:

[(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S] [(T= S'S)(T= S'S)S] [(T=S'S)] [(T= S'S)(T)S].

The enharmonic structure of descending *Timaeus* LPS is the following.

([TT] QQ) T ([TT] QQ) T ([TT] QQ)

This sequence maps onto the *Timaeus* GPS infrastructure as follows, where green highlighting represents undivided ditones; yellow highlighting represents independent whole tones; S highlighted in blue represents intervals to be split for QT intervals; and underlining indicates elements belonging to the same fourth:

55 Ibid., 32.

[(T= S′S)(T= S′S)S] [(T=S′S)] [(T= S′S)(T)S] [(T= S′S)(T= S′S)S] [(T=S′S)]
[(T= S′S)(T)S]

Clearly enharmonic *Timaeus* LPS fits neatly within the two-octave span of *Timaeus* GPS.

The discussion above has demonstrated that all diatonic, chromatic and enharmonic possibilities for *Timaeus* GPS and LPS can be mapped onto a descending Dorian disdiapason in the same octave chain as the perfect disdiapason beginning from 384. One can use the fully articulated semitone sequence of *Timaeus* GPS as a template for calculating the intervals comprising all possibilities within *Timaeus* UPS.

Table 31 is the completed template for a continuous series of full iterations of *Timaeus* UPS in the same octave chain as the perfect disdiapason. It includes the disdiapason between 49152 and 196608, but actually begins earlier with 12,288, since the full semitone structure of the disdiapason between 49152 and 196608 actually provides broader parameters than are necessary for the full articulation of the *Timaeus* UPS system.

The note names assigned to values in Table 31 will seem strange to the reader without a preliminary explanation. They fit the tetrachord structure of the *Timaeus* UPS, rather than standard UPS, as *Timaeus* UPS is not exactly parallel to standard UPS. For example *Timaeus* LPS exhibits the *Synemmenon* tetrachord as the last fourth in descending order toward the bottom of the system, rather than the first, as in the standard system; so it is appropriate to speak of *hyperbolaion, diezeugmenon*, and *synemmenon* tetrachords in *Timaeus* LPS, rather than *synemmenon, meson*, and *hypaton* tetrachords as in standard LPS.

Be aware that none of the nomenclature constructed in this study for the *Timaeus* UPS was actually used in any integral working system, since the evidence scholars have relates only to standard GPS. It is useful to provide names for notes in *Timaeus* UPS for purposes of comparison with standard GPS. Table 29, below, paves the way for Table 30 by laying out the consistent, descriptive, hypothetical nomenclature adopted by this study for note values in *Timaeus* UPS.

TABLE 29 Nomenclature utilized for the *Timaeus* UPS

Diatonic GPS	**Chromatic GPS**	**Enharmonic GPS**	**Diatonic LPS**	**Chromatic LPS**	**Enharmonic LPS**
Nete Hyperbolaion	*Lichanos Meson* **Previous c. GPS**	*Nete Hyperbolaion*	*Nete Hyperbolaion*		*Nete Hyperbolaion*
T: *Paranete Hyperbolaion*	S′: *Parhypate Meson*	TT: *Paranete Hyperbolaion*	T: *Paranete Hyperbolaion*	*Nete Hyperbolaion*	TT: *Paranete Hyperbolaion*
T: *Trite Hyperbolaion*	S: *Hypate Meson*	Q: *Trite Hyperbolaion*	T: *Trite Hyperbolaion*	TS: *Paranete Hyperbolaion*	Q: *Trite Hyperbolaion*
S: *Hypate Hyperbolaion*	**End 3rd Tetrachord**	Q: *Hypate Hyperbolaion*	S: *Hypate Hyperbolaion*	S′: *Trite Hyperbolaion*	Q: *Hypate Hyperbolaion*
End 1st Tetrachord	**T:** *Paralichanos Hypaton* **Begin 4th Tetrachord**	**End 1st Tetrachord**	**End 1st Tetrachord**	S: *Hypate Hyperbolaion*	**End 1st Tetrachord**
T: *Paranete Diezeugmenon*	TS: *Lichanos Hypaton*	T: *Paranete Diezeugmenon*	T: *Paranete Diezeugmenon*	**End 1st Tetrachord**	T: *Paranete Diezeugmenon*
Begin 2nd Tetrachord	S′: *Parhypate Hypaton*	**Begin 2nd Tetrachord**	**Begin 2nd Tetrachord**	T: *Paranete Diezeugmenon*	**Begin 2nd Tetrachord**
T: *Nete Diezeugmenon*	S: *Hypate Hypaton/Nete Hyperbolaion*	TT: *Nete Diezeugmenon*	T: *Nete Diezeugmenon*	**Begin 2nd Tetrachord**	TT: *Nete Diezeugmenon*
T: *Parhypate Diezeugmenon*	**Begin 1st Tetrachord**	Q: *Parhypate Diezeugmenon*	T: *Parhypate Diezeugmenon*	TS: *Nete Diezeugmenon*	Q: *Parhypate Diezeugmenon*
S: *Trite-Hypate Diezeugmenon*	TS: *Paranete Hyperbolaion*	Q: *Trite-Hypate Diezeugmenon*	S: *Trite-Hypate Diezeugmenon*	S′: *Parhypate Diezeugmenon*	Q: *Trite-Hypate Diezeugmenon*
Begin 3rd Tetrachord	S′: *Parhypate Hyperbolaion*	**Begin 3rd Tetrachord**	**End 2nd Tetrachord**	S: *Trite-Hypate Diezeugmenon*	**End 2nd Tetrachord**
T: *Lichanos Meson*	S: *Trite-Hypate Hyperbolaion*	TT: *Lichanos Meson*	T: *Paranete Synemmenon*	**End 2nd Fourth**	T: *Paranete Synemmenon*
T: *Parhypate Meson*	**End 1st Tetrachord**	Q: *Parhypate Meson*	**Begin 3rd Tetrachord**	T: *Paranete Synemmenon*	**Begin 3rd Tetrachord**

TABLE 29 Nomenclature utilized for the *Timaeus* UPS (*cont.*)

Diatonic GPS	**Chromatic GPS**	**Enharmonic GPS**	**Diatonic LPS**	**Chromatic LPS**	**Enharmonic LPS**
S: *Hypate Meson*	T: *Paranete Diezeugmenon*	Q: *Hypate Meson*	T: *Nete Synemmenon*	**Begin 3rd Tetrachord**	TT: *Nete Synemmenon*
End 3rd Tetrachord	**Begin 2nd Tetrachord**	**End 3rd Tetrachord**	T: *Parhypate Synemmenon*	TS: *Nete Synemmenon*	Q: *Parhypate Synemmenon*
T: *Paralichanos Hypaton*	TS: *Nete Diezeugmenon*	T: *Paralichanos Hypaton*	S: *Trite-Hypate Synemmenon*	S′: *Parhypate Synemmenon*	Q: *Trite-Hypate Synemmenon*
Begin 4th Tetrachord	S: *Parhypate Diezeugmenon*	**Begin 4th Tetrachord**		S: *Trite-Hypate Synemmenon*	
T: *Lichanos Hypaton*	S′ *Trite-Hypate Diezeugmenon*	TT: *Lichanos Hypaton*			
T: *Parhypate Hypaton*	**Begin 3rd Tetrachord**	Q: *Parhypate Hypaton*			
S: *Hypate Hypaton*	TS: *Lichanos Meson*	Q: *Hypate Hypaton*			
End 4th Tetrachord	**End GPS system**	**End 4th Tetrachord**			

Following modern scholarly opinion that UPS was a descending two octave system starting [a^2] to [a], the *Timaeus* system contemplates a descending Dorian disdiapason with descending diatonic *Mixolydian* octave running from [e^2] to [e^1] at its center.[56] In Table 30, below, diatonic notes of *Timaeus* GPS are highlighted in yellow. Chromatic notes of *Timaeus* GPS are highlighted in blue. Enharmonic notes of *Timaeus* GPS are highlighted in green. When a note belongs to more than one scale, then it is highlighted in the applicable number of colors. Diatonic notes of *Timaeus* LPS are marked with a yellow asterisk *. Chromatic notes of *Timaeus* LPS are marked with a blue asterisk *.

56 See Appendix 3 and Chapter 6 of this study, particularly, to discern why it is a mistake to consider *tonoi* to be theoretically tied to specific pitch ranges. Compare standard UPS and the "neat" articulations of *Timaeus* UPS beginning from 41472 and 82944 on the same point, as both systems appear to cover much of the same tonal territory in the two octave spans beginning from those numbers, but simply arrange it differently.

Enharmonic notes of *Timaeus* LPS are marked with a green asterisk *. When a note belongs to more than one scale, it is marked with the applicable number of colored asterisks. Only elements so marked correspond to actual note values within *Timaeus* GPS and LPS. The numbers highlighted in red are notes implicit within the structure of "T," but they never independently manifest in *Timaeus* GPS or LPS, given the formulas for the diatonic, chromatic, and enharmonic Dorian scales in ancient Greek music. The "unused" elements point to theoretical possibilities for additional scales and UPS systems that may or may not have been realized by the ancient Greeks.

TABLE 30 *Timaeus UPS system I:* the four complete iterations of *Timaeus* UPS belonging to the octave chain of the first perfect disdiapason (384 to 1536)

[a^2] ***Nete Hyperbolaion*** **d.GPS, e.GPS, d.LPS, e.LPS; Chromatic *Lichanos Meson* c.GPS**	12288**	24576**	49152**	98304**
S′ = 2187/2048				
[a^{b2}] **Chromatic *Parhypate Meson* c.GPS**	13122	26244	52488	104976
S=256/243				
[g^2] ***Paranete Hyperbolaion*** **d.GPS, d.LPS; Chromatic *Hypate Meson* c.GPS; Chromatic *Nete Hyperbolaion* c.LPS**	13824**	27648**	55296 **	110592**
S′ = 2187/2048				
			59049	118098
S = 256/243				
[f^2] ***Trite Hyperbolaion*** **d.GPS, d.LPS Chromatic *Paralichanos Hypaton* c.GPS; Enharmonic *Paranete Hyperbolaion* e.GPS, e.LPS**	15552**	31104**	62208**	124416**
S = 256/243 [e^{*1}] **Enharmonic *Trite Hyperbolaion* e.GPS, e.LPS**	QT Division 15968 *	QT Division 31936 *	QT Division 63872 *	QT Division 127744 *
[e^1] ***Hypate Hyperbolaion*** **d. GPS, d.LPS; Chromatic *Paranete Hyperbolaion* c.LPS; Enharmonic *Hypate Hyperbolaion* e.GPS; e.LPS**	16384***	32768***	65536**	131072**
S′ = 2187/2048				

TABLE 30 *Timaeus UPS system I:* (384–1536) chain (*cont.*)

[e^{b1}] **Chromatic *Trite Hyperbolaion* c.LPS**	17496 *	34992 *	69984 *	139968 *
S = 256/243				
[d^{1}] **Tone of Disjunction/*Paranete Diezeugmenon* d.GPS, d.LPS** **Chromatic *Lichanos Hypaton* c. GPS** **Chromatic *Hypate Hyperbolaion* c.LPS; Enharmonic *Paranete Diezeugmenon* e.GPS, e.LPS**	18432***	36864***	73728***	147456***
S′ = 2187/2048				
[d^{b1}] **Chromatic *Parhypate Hypaton* c.GPS**	19683	39366	78732	157464
S = 256/243				
[c^{1}] ***Nete Diezeugmenon* d.GPS, d.LPS; Chromatic *Hypate Hypaton/Nete Hyperbolaion* c.GPS; Chromatic *Paranete Diezeugmenon* c.LPS.**	20736**	41472**	82944**	165888**
T=9/8				
[b^{b1}] ***Parhypate Diezeugmenon* d.GPS, d.LPS; Enharmonic *Nete Diezeugmenon* e.GPS, e.LPS**	23328**	46656**	93312**	186624**
S = 256/243 [*a#^{1}] **Enharmonic *Parhypate Diezeugmenon* e.GPS, e.LPS**	QT Division 23952 *	QT Division 47904 *	QT Division 95808 *	QT Division 191616 *
[a^{1}] ***Trite-Hypate Diezeugmenon* d.GPS, d.LPS; Chromatic *Paranete Hyperbolaion* c. GPS; Chromatic *Nete Diezeugmenon* c.LPS** **Enharmonic *Trite-Hypate Diezeugmenon* e.GPS, e.LPS**	24576***	49152***	98304***	196608***
S′ = 2187/2048				
[a^{b1}] **Chromatic *Parhypate Hyperbolaion* c.GPS; Chromatic *Parhypate Diezeugmenon* c.LPS**	26244 *	52488 *	104976 *	209952 *
S = 256/243				

TABLE 30 *Timaeus UPS system I:* (384–1536) chain (*cont.*)

[g^{1}] ***Lichanos Meson*** **d.GPS;** ***Paranete Synemmenon*** **d.LPS;** **Chromatic *Trite-Hypate Hyperbolaion* c.GPS** **Chromatic *Trite-Hypate Diezeugmenon* c.LPS** **Enharmonic *Paranete Synemmenon* e.LPS**	27648***	55296***	110592***	221184***
S′ = 2187/2048				
		59049	118098	236196
S = 256/243				
[f^{1}] ***Parhypate Meson*** **d.GPS;** ***Nete Synemmenon*** **d.LPS;** **Chromatic *Paranete Diezeugmenon* c.GPS; Chromatic *Paranete Synemmenon* c.LPS; Enharmonic *Lichanos Meson* e.GPS**	31104**	62208**	124416**	248832**
S = 256/243 **Enharmonic *Parhypate Meson* e.GPS**	QT Division 31936	QT Division 63872	QT Division 127744	QT Division 255488
[e^{1}] ***Hypate Meson*** **d.GPS** **Enharmonic *Hypate Meson* e.GPS**	32768	65536	131072	262144
S′ = 2187/2048				
[e^{b1}] ***Parhypate Synemmenon*** **d.LPS** **Enharmonic *Nete Synemmenon* e.LPS**	34992 **	69984 **	139968 **	279936 **
S = 256/243 [d#^{1}] **Enharmonic *Parhypate Synemmenon* e.LPS**	QT Division 35928 *	QT Division 71856 *	QT Division 143712 *	QT Division 287424 *
[d^{1}] **Tone of Disjunction/*Paralichanos Hypaton* d.GPS; *Trite-Hypate Synemmenon* d.LPS; Chromatic *Nete Diezeugmenon* c.GPS** **Chromatic *Nete Synemmenon* c.LPS** **Enharmonic *Paralichanos Hypaton* e.GPS; Enharmonic *Trite-Hypate Synemmenon* e.LPS**	36864***	73728***	147456***	294912***
S′ = 2187/2048				
[d^{b1}] **Chromatic *Parhypate Diezeugmenon* c.GPS; Chromatic *Parhypate Synemmenon* c.LPS**	39366 *	78732 *	157464 *	314928 *

TABLE 30 *Timaeus UPS system I:* (384–1536) chain (*cont.*)

S = 256/243				
[c^{1}] ***Lichanos Hypaton*** **d.GPS** **Chromatic *Trite-Hypate Diezeugmenon* c.GPS** **Chromatic *Trite-Hypate Synemmenon* c.LPS**	41472 *	82944 *	165888 *	331776 *
T = 9/8				
[b^{b1}] ***Parhypate Hypaton*** **d.GPS** **Enharmonic *Lichanos Hypaton* e.GPS**	46656	93312	186624	373248
S = 256/243 **Enharmonic *Parhypate Hypaton* e.GPS**	QT Division 47904	QT Division 95808	QT Division 191616	QT Division 383232
[a] ***Hypate Hypaton*** **d.GPS; Chromatic *Lichanos Meson* c.GPS; Enharmonic *Hypate Hypaton* e.GPS**	49152	98304	196608	393216

Note that only the first iteration of Table 30 includes in its articulation of *Timaeus* UPS all numbers in the *Timaeus* set occurring in Table 26 between the beginning and ending numbers of the sequence. Just as the diapason from 384 to 1536 was the first perfect disdiapason, so the disdiapason between 12288 and 49152 is the first *perfect* articulation of *Timaeus* UPS.

Observe, also, that all of the numbers in Table 30 are *Timaeus* numbers except for certain quarter tone values, necessary for articulating a QT enharmonic. Those quarter tone values are calculated in simple fashion from the *Timaeus* numbers just as they were calculated in Table 28. The values on either side of the QT are added and then divided by two to arrive at the QT value. Those who take the *Timaeus* number set as actually supporting only a semitone enharmonic can ignore these notes. They are included, just as they were in Table 28, because it is theoretically possible that a QT enharmonic existed in Plato's day. The same observation applies to Tables 31 through 34, below, as well.

Timaeus UPS iterations of the same kind as those in Table 30, with its clumsy chromatic GPS, are possible in descending Dorian two-octave sequences among the *Timaeus* numbers not belonging to the same octave chain as the perfect disdiapason stemming from 384. Table 31, entitled *Timaeus* UPS System I-A, presents one example of such a series of continuous iterations of *Timaeus* UPS in a single octave chain.

TABLE 31 *Timaeus UPS system I-A*: the complete iterations of *Timaeus* UPS belonging to an octave chain excluding the first perfect disdiapason (384 to 1536)

[a^{2}] ***Nete Hyperbolaion*** **d. GPS, e. GPS, d.LPS, e.LPS; Chromatic *Lichanos Meson* c.GPS**	73728**	147456**	294912**
S′ = 2187/2048			
[a^{b2}] **Chromatic *Parhypate Meson* c.GPS**	78732	157464	314928
S=256/243			
[g^{2}] ***Paranete Hyperbolaion*** **d.GPS, d.LPS; Chromatic *Hypate Meson* c.GPS; Chromatic *Nete Hyperbolaion* c.LPS**	82944**	165888**	331776**
S′ = 2187/2048			
S = 256/243			
[f^{2}] ***Trite Hyperbolaion*** **d.GPS, d.LPS Chromatic *Paralichanos Hypaton* c.GPS; Enharmonic *Paranete Hyperbolaion* e.GPS, e.LPS**	93312**	186624**	373248**
S = 256/243 [e^{*1}] **Enharmonic *Trite Hyperbolaion* e.GPS, e.LPS**	QT Division 95808 *	QT Division 191616 *	QT Division 383232 *
[e^{1}] ***Hypate Hyperbolaion*** **d. GPS, d.LPS; Chromatic *Paranete Hyperbolaion* c.LPS; Enharmonic *Hypate Hyperbolaion* e.GPS; e.LPS**	98304***	196608***	393216***
S′ = 2187/2048			
[e^{b1}] **Chromatic *Trite Hyperbolaion* c.LPS**	104976 *	209952 *	419904 *
S = 256/243			
[d^{1}] **Tone of Disjunction/*Paranete Diezeugmenon* d.GPS, d.LPS Chromatic *Lichanos Hypaton* c. GPS Chromatic *Hypate Hyperbolaion* c.LPS; Enharmonic *Paranete Diezeugmenon* e.GPS, e.LPS**	110592***	221184***	442368***
S′ = 2187/2048			
[d^{b1}] **Chromatic *Parhypate Hypaton* c.GPS**	118098	236196	472392

TABLE 31 *Timaeus UPS system I-A*: iterations beyond the (384–1536) chain (*cont.*)

S = 256/243			
[c^{1}] ***Nete Diezeugmenon*** **d.GPS, d.LPS; Chromatic *Hypate Hypaton/Nete Hyperbolaion* c.GPS; Chromatic *Paranete Diezeugmenon* c.LPS.**	124416**	248832**	497664**
T=9/8 In these two-octave sequences this tone is split, in contrast to the octave sequences of Table 30. [S = 256/243] = 131072, 262144 [S′ = 2187/2048]	See note at left re 131072.	See note at left re 262144.	
[b^{b1}] ***Parhypate Diezeugmenon*** **d.GPS, d.LPS; Enharmonic *Nete Diezeugmenon* e.GPS, e.LPS**	139968**	279936**	559872**
S = 256/243 [*a#^{1}] **Enharmonic *Parhypate Diezeugmenon* e.GPS, e.LPS**	QT Division 143712 *	QT Division 287424 *	QT Division 574848 *
[a^{1}] ***Trite-Hypate Diezeugmenon*** **d.GPS, d.LPS; Chromatic *Paranete Hyperbolaion* c. GPS; Chromatic *Nete Diezeugmenon* c.LPS; Enharmonic *Trite-Hypate Diezeugmenon* e.GPS, e.LPS**	147456***	294912***	589824***
S′ = 2187/2048			
[a^{b1}] **Chromatic *Parhypate Hyperbolaion* c.GPS; Chromatic *Parhypate Diezeugmenon* c.LPS**	157464 *	314928 *	629856 *
S = 256/243			
[g^{1}] ***Lichanos Meson*** **d.GPS;** ***Paranete Synemmenon*** **d.LPS;** **Chromatic *Trite-Hypate Hyperbolaion* c.GPS; Chromatic *Trite-Hypate Diezeugmenon* c.LPS** **Enharmonic *Paranete Synemmenon* e.LPS**	165888***	331776***	663552***
S′ = 2187/2048			
S = 256/243			
[f^{1}] ***Parhypate Meson*** **d.GPS; *Nete Synemmenon* d.LPS; Chromatic *Paranete Diezeugmenon* c.GPS; Chromatic *Paranete Synemmenon* c.LPS; Enharmonic *Lichanos Meson* e.GPS**	186624**	373248**	746496**

TABLE 31 *Timaeus UPS system I-A*: iterations beyond the (384–1536) chain (*cont.*)

S = 256/243 **Enharmonic *Parhypate Meson* e.GPS**	QT Division 191616	QT Division 383232	QT Division 766464
[e^{1}] ***Hypate Meson* d.GPS** **Enharmonic *Hypate Meson* e.GPS**	196608	393216	786432
S′ = 2187/2048			
[e^{b1}] ***Parhypate Synemmenon* d.LPS** **Enharmonic *Nete Synemmenon* e.LPS**	209952 **	419904 **	839808 **
S = 256/243 [d#^{1}] **Enharmonic *Parhypate Synemmenon* e.LPS**	QT Division 215568 *	QT Division 431136 *	QT Division 862272 *
[d^{1}] **Tone of Disjunction/*Paralichanos Hypaton* d.GPS; *Trite-Hypate Synemmenon* d.LPS; Chromatic *Nete Diezeugmenon* c.GPS** **Chromatic *Nete Synemmenon* c.LPS** **Enharmonic *Paralichanos Hypaton* e.GPS; Enharmonic *Trite-Hypate Synemmenon* e.LPS**	221184 ***	442368 ***	884736 ***
S′ = 2187/2048			
[d^{b1}] **Chromatic *Parhypate Diezeugmenon* c.GPS; Chromatic *Parhypate Synemmenon* c.LPS**	236196 *	472392 *	944784 *
S = 256/243			
[c^{1}] ***Lichanos Hypaton* d.GPS** **Chromatic *Trite-Hypate Diezeugmenon* c.GPS** **Chromatic *Trite-Hypate Synemmenon* c.LPS**	248832 *	497664 *	995328 *
T = 9/8 In these two-octave sequences this tone is split, in contrast to the octave sequences of Table 30. [S = 256/243] = 262144 [S′ = 2187/2048]	See note at left re 262144.		
[b^{b1}] ***Parhypate Hypaton* d.GPS** **Enharmonic *Lichanos Hypaton* e.GPS**	279936	559872	1119744

TABLE 31 *Timaeus UPS system I-A*: iterations beyond the (384–1536) chain (*cont.*)

S = 256/243 **Enharmonic *Parhypate Hypaton* e.GPS**	QT Division 287424	QT Division 574848	QT Division 1149696
[a] ***Hypate Hypaton* d.GPS; Chromatic *Lichanos Meson* c.GPS; Enharmonic *Hypate Hypaton* e.GPS**	294912	589824	1179648

Note, in Table 31, that just as the diapason from 384 to 1536 was the first perfect disdiapason, so the disdiapason between 294912 and 1179648 is a perfect articulation of *Timaeus* UPS, albeit not the first one possible in the *Timaeus* set. As mentioned above, descending Dorian two-octave sequences exist within the *Timaeus* numbers allowing a much neater presentation of the *Timaeus* UPS system than those presented in Tables 30 and 31. These sequences are not continuous with the perfect disdiapason descending from 384, but, nonetheless, permit the whole *Timaeus* UPS system, in all LPS and GPS genera, including the chromatic, to descend from the same value for *Nete Hyperbolaion*. One such continuous series of iterations is set forth in Table 32, entitled *Timaeus* UPS System II and another appears in Table 33, entitled *Timaeus* UPS System II-A. Note that the semitone sequence in these tables differs somewhat from that in Table 30. It follows the most completely articulated set of intervals, in Table 26, for the particular octave chains to which iterations of the new tables, respectively, pertain. The nomenclature used for the different tetrachords of the *Timaeus* UPS systems in Tables 32 and 33 is the same as that in Tables 30 and 31, but it is rearranged for the chromatic to show the chromatic Hyperbolaion tetrachord at the top of the system for GPS and chromatic scale origination from the same value as the diatonic and enharmonic genera for both LPS and GPS. As in Tables 30 and 31, numbers highlighted in red are part of the *Timaeus* set, but not part of *Timaeus* UPS.

TABLE 32 *Timaeus UPS system II*: complete "neat" iterations of *Timaeus* UPS in an octave chain excluding the first perfect disdiapason (384 to 1536)

[a^2] ***Nete Hyperbolaion* d.GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	41472***	82944**	165888***	331776***
T = 9/8				
[g^2] ***Paranete Hyperbolaion* d.GPS; d.LPS**	46656 *	93312 *	186624 *	373248 *

TABLE 32 *Timaeus UPS system II*: "neat" iterations outside the (384–1536) chain (*cont.*)

S = 256/243				
[f#^{2}] ***Paranete Hyperbolaion*** **c.GPS; c.LPS**	49152 *	98304 *	196608 *	393216 *
S′ = 2187/2048				
[f^2] ***Trite Hyperbolaion*** **d.GPS; d.LPS; c.LPS;** ***Parhypate Hyperbolaion*** **c.GPS;** ***Paranete Hyperbolaion*** **e.GPS; e.LPS**	52488***	104976***	209952***	419904***
S = 256/243 ***Trite Hyperbolaion*** **e.GPS; e.LPS**	QT Division 53892 *	QT Division 107784 *	QT Division 215568 *	QT Division 431136 *
[e^2] ***Hypate Hyperbolaion*** **d. GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	55296***	110592***	221184***	442368***
S′ = 2187/2048	59049	118098	236196	472392
S = 256/243				
[d^2] ***Paranete Diezeugmenon*** **d. GPS; c. GPS; e.GPS; d.LPS; c.LPS; e.LPS**	62208***	124416***	248832***	497664***
S = 256/243	65536	131072	262144	
S′ = 2187/2048				
[c^2] ***Nete Diezeugmenon*** **d.GPS; d.LPS**	69984 *	139968 *	279936 *	559872 *
S = 256/243				
[b^2] ***Nete Diezeugmenon*** **c.GPS; c.LPS**	73728 *	147456 *	294912 *	589824 *
S′ = 2187/2048				
[b^{b2}] ***Parhypate Diezeugmenon*** **d. GPS; d.LPS; c.GPS; c.LPS;** ***Nete Diezeugmenon*** **e.GPS; e.LPS**	78732***	157464***	314928***	629856***
S = 256/243 ***Parhypate Diezeugmenon*** **e.GPS; e.LPS**	QT Division 80838 *	QT Division 161676 *	QT Division 323352 *	QT Division 646704 *
[a^1] ***Trite-Hypate Diezeugmenon*** **d. GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	82944***	165888***	331776***	663552***

TABLE 32 *Timaeus UPS system II*: "neat" iterations outside the (384–1536) chain (*cont.*)

T = 9/8				
[g^{1}] ***Lichanos Meson*** **d. GPS; *Paranete Synemmenon* d.LPS; c.LPS; e.LPS**	93312***	186624***	373248***	746496***
S = 256/243				
[f♯^{1}] ***Lichanos Meson*** **c.GPS**	98304	196608	393216	786432
S′ = 2187/2048				
[f^{1}] ***Parhypate Meson*** **d.GPS; c.GPS; *Lichanos Meson* e.GPS: *Nete Synemmenon* d.LPS**	104976*	209952*	419904*	839808*
S = 256/243 ***Parhypate Meson*** **e.GPS**	QT Division 107784	QT Division 215568	QT Division 431136	QT Division 862272
[e^{1}] ***Hypate Meson*** **d.GPS; c.GPS; e.GPS; *Nete Synemmenon* c.LPS**	110592*	221184*	442368*	884736*
S′ = 2187/2048				
[e^{b1}]***Parhypate Synemmenon*** **d.LPS; c.LPS; *Nete Synemmenon* e.LPS**	118098***	236196***	472392***	944784***
S = 256/243 [e$^{b1–1}$] ***Parhypate Synemmenon*** **e.LPS**	QT Division 121257 *	QT Division 242514 *	QT Division 485028 *	QT Division 970056 *
[d^{1}] ***Paralichanos Hypaton*** **d.GPS; c.GPS; e.GPS; *Trite-Hypate Synemmenon* d.LPS; c.LPS; e.LPS**	124416***	248832***	497664***	995328***
S = 256/243	131072	262144		
S′ = 2187/2048				
[c^{1}] ***Lichanos Hypaton*** **d.GPS**	139968	279936	559872	1119744
S = 256/243				
[b^{1}] ***Lichanos Hypaton*** **c.GPS**	147456	294912	589824	1179648
S′ = 2187/2048				

TABLE 32 *Timaeus UPS system II*: "neat" iterations outside the (384–1536) chain (*cont.*)

[b^{b1}] ***Parhypate Hypaton*** **d.GPS; c.GPS; *Lichanos* Hypaton e.GPS**	157464	314928	629856	1259712
S = 256/243 ***Parhypate Hypaton*** **e.GPS**	QT Division 161676	QT Division 323352	QT Division 646703	QT Division 1293408
[a] ***Hypate Hypaton*** **d.GPS; c.GPS; e.GPS**	165888	331776	663552	1327104

Note that the first number in Table 32, 41472, is the fourth multiple of 10368 (27 × 384), as well as the number that, squared, yields the last element of the *Timaeus* set. One can almost calculate an additional complete series in the same octave chain from a starting value of 20736, but not quite. Enharmonic LPS is incomplete in the disdiapason from 20736 to 41472. Observe, also, that although the series in Table 32 allows a much neater presentation of *Timaeus* UPS than the series of Tables 30 and 31 allow, no sequence in Table 32 allows a "perfect UPS," as defined above. Although all elements of UPS are present, distorting elements (the numbers highlighted in red) appear in each sequence.

Table 33 exhibits an entirely separate "neat" set of complete iterations of *Timaeus* UPS in yet another octave chain excluding the first perfect disdiapason. It begins from 13824.

TABLE 33 *Timaeus UPS system II-A*: complete "neat" iterations of *Timaeus* UPS in another octave chain excluding the first perfect disdiapason (384 to 1536)

[a^{2}] ***Nete Hyperbolaion*** **d.GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	13824***	27648***	55296***	110592***
T = 9/8 Divided for third and fourth columns into the following sequence: S′ = 2187/2048; S = 256/243			59049 for S′ = 2187/2048	118098 for S′ = 2187/2048
[g^{2}] ***Paranete Hyperbolaion*** **d.GPS; d.LPS**	15552 *	31104 *	62208 *	124416 *
S = 256/243				
[f#^{2}] ***Paranete Hyperbolaion*** **c.GPS; c.LPS**	16384 *	32768 *	65536 *	131072 *
S′ = 2187/2048				

TABLE 33 *Timaeus UPS system II-A*: other "neat" iterations of *Timaeus* UPS (*cont.*)

[f^2] ***Trite Hyperbolaion*** **d.GPS; d.LPS; c.LPS; *Parhypate Hyperbolaion* c.GPS; *Paranete Hyperbolaion* e.GPS; e.LPS**	17496***	34992***	69984***	139968***
S = 256/243 ***Trite Hyperbolaion*** **e.GPS; e.LPS**	QT Division 17964 *	QT Division 35928 *	QT Division 71856 *	QT Division 143712 *
[e^2] ***Hypate Hyperbolaion*** **d. GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	18432***	36864***	73728***	147456***
S′ = 2187/2048	19683	39366	78732	157464
S = 256/243				
[d^2] ***Paranete Diezeugmenon*** **d. GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	20736***	41472***	82944***	165888***
T=9/8 In Table 33 this is split by the semitone sequence: S = 256/243; S′ = 2187/2048				
[c^2] ***Nete Diezeugmenon*** **d.GPS; d.LPS**	23328 *	46656 *	93312 *	186624 *
S = 256/243				
[b^2] ***Nete Diezeugmenon*** **c.GPS; c.LPS**	24576 *	49152 *	98304 *	196608 *
S′ = 2187/2048				
[b^{b2}] ***Parhypate Diezeugmenon*** **d. GPS; d.LPS; c.GPS; c.LPS; *Nete Diezeugmenon* e.GPS; e.LPS**	26244***	52488***	104976***	209952***
S = 256/243 ***Parhypate Diezeugmenon*** **e.GPS; e.LPS**	QT Division 26946 *	QT Division 53892 *	QT Division 107784 *	QT Division 215568 *
[a^1] ***Trite-Hypate Diezeugmenon*** **d. GPS; c.GPS; e.GPS; d.LPS; c.LPS; e.LPS**	27648***	55296***	110592***	221184***
T = 9/8 Divided for all but the first column into the following sequence: S′ = 2187/2048; S = 256/243		59049 for S′ = 2187/2048	118098 for S′ = 2187/2048	236196 for S′ = 2187/2048

TABLE 33 *Timaeus UPS system II-A*: other "neat" iterations of *Timaeus* UPS (*cont.*)

[g^{1}] ***Lichanos Meson* d. GPS; *Paranete Synemmenon* d.LPS; c.LPS; e.LPS**	31104***	62208***	124416***	248832***
S = 256/243				
[f#^{1}] ***Lichanos Meson* c.GPS**	32768	65536	131072	262144
S′ = 2187/2048				
[f^{1}] ***Parhypate Meson* d.GPS; c.GPS; *Lichanos Meson* e.GPS: *Nete Synemmenon* d.LPS**	34992*	69984*	139968*	279936*
S = 256/243 ***Parhypate Meson* e.GPS**	QT Division 35928	QT Division 71856	QT Division 143712	QT Division 287424
[e^{1}] ***Hypate Meson* d.GPS; c.GPS; e.GPS; *Nete Synemmenon* c.LPS**	36864*	73728*	147456*	294912*
S′ = 2187/2048				
[e^{b1}]***Parhypate Synemmenon* d.LPS; c.LPS; *Nete Synemmenon* e.LPS**	39366***	78732***	157464***	314928***
S = 256/243 [e^{b1-1}] ***Parhypate Synemmenon* e.LPS**	QT Division 40419 *	QT Division 80838 *	QT Division 161676 *	QT Division 323352 *
[d^{1}] ***Paralichanos Hypaton* d.GPS; c.GPS; e.GPS; *Trite-Hypate Synemmenon* d.LPS; c.LPS; e.LPS**	41472***	82944***	165888***	331776***
T=9/8 In Table 33 this is split by the semitone sequence: S = 256/243; S′ = 2187/2048				
[c^{1}] ***Lichanos Hypaton* d.GPS**	46656	93312	186624	373248
S = 256/243				
[b^{1}] ***Lichanos Hypaton* c.GPS**	49152	98304	196608	393216
S′ = 2187/2048				
[b^{b1}] ***Parhypate Hypaton* d.GPS; c.GPS; *Lichanos Hypaton* e.GPS**	52488	104976	209952	419904

TABLE 33 *Timaeus UPS system II-A*: other "neat" iterations of *Timaeus* UPS (*cont.*)

S = 256/243 ***Parhypate Hypaton*** **e.GPS**	QT Division 53892	QT Division 107784	QT Division 215568	QT Division 431136
[a] ***Hypate Hypaton*** **d.GPS; c.GPS; e.GPS**	55296	110592	221184	442368

Table 33 clearly also allows for four complete iterations of *Timaeus* UPS, although no "perfect" one undistorted by foreign elements. There is no earlier octave in the same octave chain that could be a starting value for a *Timaeus* UPS system. Observe that the presentation of *Timaeus* UPS in this chapter departs somewhat from the approach to alternative UPS systems set forth in Appendix 4. It does not attempt to assign a different note span to *Timaeus* UPS than the A-a-A system assumed for the standard system. Indeed, the standard UPS system presented first, above in this chapter, covers much of the same tonal range as two of the "neat" articulations *Timaeus* UPS beginning from 41472 and 82944; it is simply organized differently.

The above presentation of *Timaeus* UPS also does not assume that LPS has to be related to GPS in a manner exactly analogous to the way in which LPS is related to GPS in the standard system. Indeed, as explained above, *Timaeus* UPS could not be accounted for as a descending system in that case, but would have to be an ascending system. There is no compelling reason to assume all of the limitations of standard UPS for *Timaeus* UPS, absent any evidence of the use of *Timaeus* UPS in ancient Greek music; and it seems most important to preserve the scholarly preference for construing perfect systems as descending. Because there is no existing evidence limiting the interpretation of *Timaeus* UPS, all of the theoretical possibilities for it are open. The presentation of *Timaeus* UPS, in this chapter, together with the different approach to the presentation of alternative perfect systems made in Appendix 4, should feed the reader's imagination about the range of possibilities. One can only speculate about the actual issues the ancients considered given our fragmentary evidence.

9 Possibilities for Modulation among Different Perfect Systems Arising within the *Timaeus* Numbers

The easiest demonstration of possibilities for modulation among different perfect systems articulable on the basis of the *Timaeus* numbers is made by

comparing standard UPS with the various articulations of *Timaeus* UPS to determine where they cover the same tonal range. In the areas of overlap, it is not difficult to discern how a fitting instrument appropriately calibrated in semitone intervals might be constructed to permit easy passage from one UPS system to another. The interested reader should also see the discussion in Appendix 3 on this point. As far as modern scholars know, ancient Greeks used only one version of the perfect system and were not interested in modulations between different alternatives. Because our evidence is so fragmentary, one cannot be sure that creative musicians did not attempt such feats, but it is speculative to assume that they did just because it is possible in theory.

10 The Primary *Timaeus* Scale

The STN, 384, is highlighted in light blue in Table 26. All subsequent light blue numbers in the table are starting tones for a repeating chain of ten complete octaves, having the same pattern. These complete octaves are interrupted by chromatic numbers, i.e., numbers not belonging to the original diatonic pattern, after [1536], the end of the first and only perfect disdiapason. Chromatic factors ("CF") of 1719926784, the last number of the set, are in gray. Chromatic nonfactors ("CNF") of 1719926784 are in olive green. The chain of ten complete a.Lydian/d.Dorian diatonic octaves concludes with 393,216; so the entire primary *Timaeus* scale includes all numbers between 384 and 393216 conforming to the specified diatonic pattern, as well as all numbers between 384 and 393216 belonging to *Timaeus* UPS.

The easiest way to present the primary *Timaeus* scale is to use the descending form of *Timaeus* UPS, as given in Table 30 above, this time, however, (a) making a continuous presentation from 384 through 393216, rather than treating each two-octave system within the chain as a separate UPS system, if it is capable of articulating UPS, and (b) also taking into account the two-octave sequences incomplete for UPS but complete for diatonic GPS. This kind of presentation of the primary scale will make clear how and when diatonic LPS, chromatic GPS, and enharmonic sequences of both GPS and LPS all arise in the chain of ten complete diatonic octaves from 384 through 393216, as a function of distortions attributable to the chromatic elements creeping into the system as the diatonic octaves repeat.

In Table 34 below, each new column is in direct continuity with the column preceding, but each occurs two octaves in pitch below the beginning of the column preceding it. The reader should interpret [a^2], [a^1], and [a] in the table

accordingly. The same nomenclature and markings are relevant to Table 34 as were relevant for Tables 31 through 34. Numbers belonging to diatonic *Timaeus* GPS are highlighted in yellow; those belonging to chromatic *Timaeus* GPS are highlighted in blue; and those belonging to enharmonic *Timaeus* GPS are highlighted in green. Numbers belonging to diatonic *Timaeus* LPS are marked with asterisks highlighted in yellow; those belonging to chromatic *Timaeus* LPS are marked with asterisks highlighted in blue; and those belonging to enharmonic *Timaeus* LPS are marked with asterisks highlighted in green. Numbers in highlighted in red occur in the set of *Timaeus* numbers but are not part of the primary scale. As Chapter 6 will show, these numbers belong to other scales articulable on the basis of the *Timaeus* numbers.

Bear in mind that the only complete iterations of *Timaeus* UPS in the continuous primary scale presented in Table 34 proceed from 24576 and 98304. GPS Chromatic is missing notes in the series proceeding from 384, 1536, and 6144, GPS Enharmonic has a missing note in the series proceeding from 384; LPS diatonic is missing a note in the series proceeding from 384, LPS Chromatic is missing notes in the series pertaining to 384 and 1536; and LPS Enharmonic has missing notes in the series proceeding from 384. All numbers consistent with chromatic or enharmonic patterns in the series proceeding from 384, 1536, and 6144 are marked as belonging to them.

Observe, also, that Table 34 does not present the numbers of the *Timaeus* set prior to 384. It presents only the ten completely articulated descending Dorian (or ascending Lydian) diatonic octaves between 384 and 393216 together with their related enharmonic and chromatic variants as they arise. The numbers prior to 384 account for the rise to the primary scale, while the numbers following 393216 mark its degeneration.

TABLE 34 Primary *Timaeus* scale descending from 384

[a^2] ***Nete Hyperbolaion*** **d. GPS, e. GPS, d.LPS, e.LPS; Chromatic *Lichanos Meson* c.GPS**	384**	1536**	6144**	24576**	98304**
S′ = 2187/2048					
[a^{b2}] **Chromatic *Parhypate Meson* c.GPS**			6561	26244	104976
S = 256/243					
[g^2] ***Paranete Hyperbolaion*** **d.GPS, d.LPS; Chromatic *Hypate Meson* c.GPS; Chromatic *Nete Hyperbolaion* c.LPS**	432**	1728**	6912**	27648**	110592**

TABLE 34 Primary *Timaeus* scale descending from 384 (*cont.*)

S′ = 2187/2048					
					118098
S = 256/243					
[f²] ***Trite Hyperbolaion*** d.GPS, d.LPS; Chromatic ***Paralichanos Hypaton*** c.GPS; Enharmonic ***Paranete Hyperbolaion*** e.GPS, e.LPS	486**	1944**	7776**	31104**	124416**
S = 256/243 [e*¹] **Enharmonic *Trite Hyperbolaion* e.GPS, e.LPS**	QT Division 499 *	QT Division 1996 *	QT Division 7984 *	QT Division 31936 *	QT Division 127744 *
[e¹] ***Hypate Hyperbolaion*** d. GPS, d.LPS; Chromatic ***Paranete Hyperbolaion*** c.LPS; Enharmonic ***Hypate Hyperbolaion*** e.GPS; e.LPS	512***	2048***	8192***	32768***	131072***
S′ = 2187/2048					
[e^{b1}] **Chromatic *Trite Hyperbolaion* c.LPS**		2187 *	8748 *	34992 *	139968 *
S = 256/243					
[d¹] **Tone of Disjunction/*Paranete Diezeugmenon*** d.GPS, d.LPS **Chromatic *Lichanos Hypaton*** c. GPS; **Chromatic *Hypate Hyperbolaion*** c.LPS; **Enharmonic *Paranete Diezeugmenon*** e.GPS, e.LPS	576***	2304***	9216***	36864***	147456***
S′ = 2187/2048					
[d^{b1}] **Chromatic *Parhypate Hypaton* c.GPS**				39366	157464
S = 256/243					
[c¹] ***Nete Diezeugmenon*** d.GPS, d.LPS; **Chromatic *Hypate Hypaton/Nete Hyperbolaion*** c.GPS; **Chromatic *Paranete Diezeugmenon*** c.LPS.	648**	2592**	10368**	41472**	165888**
T=9/8					
[b^{b1}] ***Parhypate Diezeugmenon*** d.GPS, d.LPS; **Enharmonic *Nete Diezeugmenon*** e.GPS, e.LPS	729**	2916**	11664**	46656**	186624**

TABLE 34 Primary *Timaeus* scale descending from 384 (*cont.*)

S = 256/243 [*a#^{1}] **Enharmonic *Parhypate Diezeugmenon* e.GPS, e.LPS**		QT Division 2994 *	QT Division 11976 *	QT Division 47904 *	QT Division 191616 *
[a^1] ***Trite-Hypate Diezeugmenon* d.GPS, d.LPS; Chromatic *Paranete Hyperbolaion* c. GPS; Chromatic *Nete Diezeugmenon* c.LPS; Enharmonic *Trite-Hypate Diezeugmenon* e.GPS, e.LPS**	768***	3072***	12288***	49152***	196608***
S′ = 2187/2048					
[a^{b1}] **Chromatic *Parhypate Hyperbolaion* c.GPS Chromatic *Parhypate Diezeugmenon* c.LPS**			13122 *	52488 *	209952 *
S = 256/243					
[g^1] ***Lichanos Meson* d.GPS; *Paranete Synemmenon* d.LPS; Chromatic *Trite-Hypate Hyperbolaion* c.GPS Chromatic *Trite-Hypate Diezeugmenon* c.LPS Enharmonic *Paranete Synemmenon* e.LPS**	864***	3456***	13824***	55296***	221184***
S′ = 2187/2048					
				59049	236196
S = 256/243					
[f^1] ***Parhypate Meson* d.GPS; *Nete Synemmenon* d.LPS; Chromatic *Paranete Diezeugmenon* c.GPS; Chromatic *Paranete Synemmenon* c.LPS; Enharmonic *Lichanos Meson* e.GPS**	972**	3888**	15552**	62208**	248832**
S = 256/243 **Enharmonic *Parhypate Meson* e.GPS**	QT Division 998	QT Division 3992	QT Division 15968	QT Division 63872	QT Division 255488
[e^1] ***Hypate Meson* d.GPS Enharmonic *Hypate Meson* e.GPS**	1024	4096	16384	65536	262144
S′ = 2187/2048					
[e^{b1}] ***Parhypate Synemmenon* d.LPS; Enharmonic *Nete Synemmenon* e.LPS**		4374 **	17496 **	69984 **	279936 **

TABLE 34 Primary *Timaeus* scale descending from 384 (*cont.*)

S = 256/243 [d#^{1}] **Enharmonic *Parhypate Synemmenon* e.LPS**		QT Division 4491 *	QT Division 17964 *	QT Division 71856 *	QT Division 287424 *
[d^{1}] **Tone of Disjunction/*Paralichanos Hypaton* d.GPS; *Trite-Hypate Synemmenon* d.LPS; Chromatic *Nete Diezeugmenon* c.GPS Chromatic *Nete Synemmenon* c.LPS; Enharmonic *Paralichanos Hypaton* e.GPS; Enharmonic *Trite-Hypate Synemmenon* e.LPS**	1152***	4608***	18432***	73728***	294912***
S′ = 2187/2048					
[d^{b1}] **Chromatic *Parhypate Diezeugmenon* c.GPS; Chromatic *Parhypate Synemmenon* c.LPS**			19683 *	78732 *	314928 *
S = 256/243					
[c^{1}] ***Lichanos Hypaton* d.GPS Chromatic *Trite-Hypate Diezeugmenon* c.GPS Chromatic *Trite-Hypate Synemmenon* c.LPS**	1296*	5184*	20736 *	82944 *	331776 *
T = 9/8					
[b^{b1}] ***Parhypate Hypaton* d.GPS; Enharmonic *Lichanos Hypaton* e.GPS**	1458	5832	23328	93312	373248
S = 256/243 **Enharmonic *Parhypate Hypaton* e.GPS**	QT Division 1497	QT Division 5988	QT Division 23952	QT Division 95808	QT Division 383232
[a] ***Hypate Hypaton* d.GPS; Chromatic *Lichanos Meson* c.GPS; Enharmonic *Hypate Hypaton* e.GPS**	1536	6144	24576	98304	393216

Clearly, the primary *Timaeus* scale presented by this study in Table 34 is much more extensive than the scale as it has been interpreted, to this point, by other scholars. It comprises a ten-octave tonal space, rather than the standard four octaves, a fifth, and a whole tone; and it encompasses enharmonic and chromatic elements, as well as diatonic elements. It is worth comparing with selected modern interpretations.

11 Some Other Modern Interpretations of the *Timaeus* Numbers and *Timaeus* Scale

The *Timaeus* number set offered by this study agrees number per number with Ernest McClain's from 1 through 20,736. That is where McClain stopped in accord with the usual interpretation limiting the number extension to four fully articulated diatonic octaves, a fifth, and a tone. He started counting octaves, not at the first fully articulated octave beginning with 384, but with the next one, beginning with 768, following the alternative traditional interpretation (768 × 27 = 20736). He actually recognized all the numbers required for five octaves, a fifth, and a tone, plus all intervening numbers representing chromatic elements entering the scale originating from the octave repetitions starting with 1536.[57] Luc Brisson stopped at 10,368, since he began counting the four octaves, fifth, and a tone with the octave beginning from 384; he allowed into the set only the numbers necessary to articulate the diatonic octaves. In contrast to McClain, he made no reference to chromatic numbers arising after the octave ending with 1536.[58]

The results of this study, as mentioned in preceding chapters, differ from the standard options for limiting the octave extension to four octaves, a fifth, and a tone, in rejection of the idea that "27" has the significance usually attributed to it. The reader may recall that both Brisson and Cornford rightly complained that such a limitation makes no musical sense. Further, if one accepts the standard limitation, then one finds oneself hopelessly stuck between 10368 or 20736 as the number of the ending tone. There are no decision criteria that could definitively legitimate one ending over the other. It depends upon the starting point of the scale.

The matter of the starting point both for interpreters such as McClain, Brisson, and Cornford, and for others, not rejecting the standard limitations, should depend, probably, upon whether one takes the first and only undistorted diatonic disdiapason between 384 and 1536 or the first possible articulation of diatonic *Timaeus* UPS between 768 and 3072 as the primary phenomenon. Since the perfection of the first possible disdiapason is all together missed if one begins from 768 and since diatonic *Timaeus* UPS is a secondary phenomenon in relation to it, this study opts for 384 as the beginning of the primary scale.

57 Ibid., 61 and 63.

58 Brisson, *Le même et l'autre*, 320–322.

Regarding the standard limitation, it is worth considering that Plato's genius precludes the conclusion that he furnished his reader with something nonsensical; rather, he gave the reader a labyrinthine riddle to solve. As in any labyrinth, one might take various seemingly plausible paths, attempting to find the one leading to its center. The paths that come to a dead end are wrong just because they do so. In the context of the *Timaeus*, the fact that a particular interpretation leads to an impasse means that it could not be the correct solution. Because the interpretations limiting the octave extension to four octaves, a fifth, and a tone do so, they must be wrong. Hermeneutical humility requires, in such a case, that the analyst simply admit that he is not yet finished with the text—or, perhaps more accurately, that the text is not yet done with him.

Plato would have required an analyst to keep searching until he found a path that allowed him to keep going in the face of an obvious impasse. The ability of a method to allow one to continue progressing with the literal interpretation of the text dictates whether or not it is correct. It is at least, aesthetically pleasing, given Plato's Pythagorean preoccupation with triads, the *tetractys*, and the Decad, that the current study offers, as the primary *Timaeus* scale, a decad of octaves, comprehending, in the end, a triad of octave genera. After some continued necessary exploration of the further musical significance of the *Timaeus* numbers in the remainder of Chapter 5 and in Chapter 6, the study picks up in Chapter 7 with the Plato's cutting of the fabric at the end of 36 B. It continues to arrive at results that assist the further literal interpretation of the text in compelling ways.

12 The Feature of Ascending/Descending Ambiguity in Plato's Scale

Although Table 34 presents the primary *Timaeus* scale as a descending Dorian scale honoring the prevailing scholarly prejudice, it could, as previously indicated, represent an ascending Lydian scale, instead; the structure of each of these scales is the same. In the ascending case, the note names for the scale would all be reversed in direction and undergo other slight adjustments to reflect the scale's ascending character. If one chose an ascending Lydian interpretation, one might wish to go back in the analysis offered here and create a *Timaeus* GPS that would more closely parallel the structure of standard GPS, as has been suggested is possible above. Having been shown the principles of such construction, the reader is free to pursue that project.

It is actually impossible to make a definitive either/or determination in favor of a descending or ascending possibility. Pseudo-Plutarch stated, upon his anal-

ysis of the *Myth of Er* in the *Republic*, that Plato held the Dorian mode in special esteem but did not at all ignore the Lydian, as he knew it was used by the tragic poets.[59]

Plato might have intended the scale to be construed in a "both/and," rather than "either/or" fashion. Handschin opined that it would have been characteristic of Plato's style to have left the question open.[60] He further observed that Pseudo-Timaeus, Proclus, and Psellus all left the question open, rather than deciding it.[61]

Ernest McClain's comments are certainly consistent with a both/and interpretation. He noted Plato's fondness for reciprocal relations, identifying "the rigorous application of reciprocity, or the study of opposites" as one of Plato's chief musical theses. Citing the *Republic* to that effect, he stated as follows:

> Some things are apt to summon thought, while others are not .... Apt to summon it [are] those that strike the sense at the time as their opposites.
>
> *Republic* 524 d
>
> Platonists have always known that in Pythagorean ratio theory numbers function reciprocally as both *multiples* and *submultiples* of some basic unit (n and 1/n). The secret to Platonic mathematical riddles is that we must study *reciprocals*, compressed to one model octave. The game with reciprocals illustrates a theory of perception: *qualities* depend on sensation, which depend in turn upon a theory of "flesh, or the mortal part of the soul," a dilemma which requires that we "assume the existence of sensation ... and afterward turn back to examine what we have assumed" (*Timaeus* 61c, d). It is this turning back to criticize one's initial assumptions which separates Plato from all philosophy developed from "first principles," as Western philosophy tried repeatedly from Aristotle onwards. No assumption we can make about Plato's tone-numbers makes any sense until we have "turned back" to study them also from the opposite point of view.[62]

The Dorian and Lydian diatonic scales, as noted, are exactly reciprocal. Their simultaneous representation would nicely illustrate the insight central to GPS that all *tonoi* can be expressed as transpositions of the Dorian scale.

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59 Pseudo-Plutarque *Musica* 17.

60 Handschin, "Timaeus Scale," 15.

61 Ibid., 21.

62 McClain, *Pythagorean Plato*, 7–8.

13 Significance of the Chromatic Invasion for the Primary *Timaeus* Scale

13.1 *Emergence of the Entire Unmodulating System in All Three Genera*

The chromatic invasion of the primary diatonic pattern between 384 and 393216 greatly enriches the significance of the *Timaeus* set for the primary descending Dorian or ascending Lydian scale. It ultimately makes possible multiple complete iterations of *Timaeus* UPS, as exemplified in Tables 30 through 33, above, not to mention multiple complete iterations of standard UPS, as exemplified in Table 28 above.

LPS Diatonic and Enharmonic, in addition to GPS and LPS chromatic, depend upon the chromatic invasion before their full articulation becomes possible. As noted previously, the first instance of diatonic *Timaeus* LPS cannot even arise in the number set until after the chromatic element 2187 enters the number sequence: 768 T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536 T 1728 T 1944 T 2187 S 2304. This is also true of standard LPS, built on a different pattern, the first instance of which is 1728 T 1944 T 2187 S 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608.

The first possible *Timaeus* LPS sequence actually overlaps the two columns headed "384" and "1536" in Table 34 but is not represented (1024, 1296, 1458 1536 are not marked as LPS diatonic) because all calculations for the primary scale depend upon the *ordo* of genera as they emerge when one considers 384 as its origin. A different *ordo* of emergence obtains when one takes 768 as the beginning of the primary scale, as some interpreters were wont to do. Perhaps one reason they preferred it was precisely because diatonic LPS and GPS can both be articulated immediately from that alternative origin. In Table 34, LPS diatonic emerges first in the column headed "1536" but still clearly requires the chromatic element 4374 for its complete articulation: 1536 T 1728 T 1944 S 2048 T 2304 T 2592 T 2916 S 3072 T 3456 T 3888 T 4374 S 4608.

Enharmonic *Timaeus* LPS in Table 34, also originating in the column headed "1536," depends upon the same chromatic element as follows: 1536 TT 1944 Q [1996] Q 2048 T 2304 TT 2916 Q [2994] Q 3072 T 3456 TT 4374 Q [4491] Q 4608, where numbers in brackets are those that must be calculated for the split of a 256/243 semitone between the numbers on either side of it. This sequence also represents the very first emergence of Enharmonic *Timaeus* LPS in the number set, as Enharmonic *Timaeus* LPS cannot be articulated descending from 768. Both Enharmonic LPS and GPS in the standard model depend upon the emergence of chromatic elements, as Table 28 demonstrates in its entries for [f^1], enharmonic *lichanos* meson LPS and GPS, and [c^1], enharmonic *lichanos* hypaton LPS and GPS.

13.2 *Other Diatonic Possibilities Coincident with the Primary* Timaeus *Scale*

In addition to allowing the complete articulations of alternative UPS systems, not all of which have been exemplified in the consideration, above, of standard UPS and *Timaeus* UPS, the chromatic invasion besets the primary scale with ambiguity. After an initial perfect disdiapason solely reflecting a d.Dorian/a.Lydian diatonic scale, the primary scale undergoes distinctive distortions with the introduction of chromatic elements. With the introduction of the first chromatic element in the third octave, a d.Mixolydian/a.Hypolydian diatonic scale emerges as a distinct interpretational possibility for the affected octave, in addition to the original possibility. The second chromatic element produces no further distortions. With the introduction of the third chromatic element in the sixth octave, an arrangement of the octave in an unaccustomed diatonic pattern, TTTTSTS, not representing any standard scale arrangement, is also possible, along with the other two. The introduction of a fourth chromatic element in the eighth octave does not further distort the original pattern. The three octave arrangements mentioned remain intact through the tenth octave, ending with 393216.

The original descending Dorian or ascending Lydian diatonic pattern of the primary scale disappears with its final articulation in the tenth octave but leaves the secondary d.Mixolydian/a.Hypolydian diatonic scale, intact, along with the unaccustomed sequence TTTTSTS, for an additional octave in the first incomplete sequence of Table 26 vis-à-vis the original pattern. Both of these patterns terminate with the second incomplete sequence of Table 26, as it is not a complete octave. This sequence fails as an octave because it loses the eighth element of the original d.Dorian/a.Lydian diatonic pattern.

From the third through sixth incomplete sequences in Table 26, new fully articulated octave patterns are calculable from the remaining seventh element of the original d.Dorian/a.Lydian diatonic pattern. The third incomplete sequence (from the standpoint of the original d.Dorian/a.Lydian diatonic pattern) is an octave interval allowing the full articulation of a d.Hypodorian/a.Hypophrygian diatonic sequence; a d.Phrygian/a.Phrygian diatonic sequence; a d.Hypophrygian/a.Hypdorian diatonic sequence, and an unaccustomed diatonic sequence in the pattern. TTSTSTT. The fourth and fifth incomplete sequences (from the standpoint of the original d.Dorian/a.Lydian diatonic pattern) are likewise octave intervals. In these intervals, the remaining articulable patterns are the d.Hypodorian/a.Hypophrygian diatonic and d.Phrygian/a.Phrygian diatonic sequences. The sixth incomplete octave sequence (from the standpoint of the original d.Dorian/a.Lydian diatonic pattern) is still an octave interval exhibiting a full d.Hypodorian/a.Hypophrygian diatonic octave.

The seventh and eighth incomplete sequences (from the standpoint of the original d.Dorian/a.Lydian diatonic pattern) are octaves, but they are not fully articulated because of the undivided TS interval at the end of each. Were the TS interval divided in these octaves, each of them would articulate a new d.Dorian/a.Lydian pattern.

The ninth incomplete sequence (from the standpoint of the original d.Dorian/a.Lydian diatonic pattern) is an octave, but it is not fully articulated. It consists in a fifth comprised of a T T (TS) sequence, together with an undivided fourth. It is too ambiguous to assign a specific diatonic octave structure.

The tenth incomplete sequence (from the standpoint of the original d.Dorian/a.Lydian diatonic pattern) is not an octave interval but consists in the following sequence amounting to a tone and three octaves: a tone, an undivided fourth, an undivided fifth, another undivided fourth, another undivided fifth, and an undivided octave.

Since multiple interpretational possibilities attend the primary scale both as it progresses through ten complete octaves, and then as it degenerates, in the first and, then, in the third through eighth incomplete sequences from the standpoint of the original pattern, some might seek to represent, in one table, all elements, appropriately and distinctly labeled, belonging to all possible patterns. Such a table would not, however, present a distinct primary scale, but, rather, multiple scales within one table, with various scales running concurrently at different points of the presentation. Further it would ignore the true hiatus that occurs with the second incomplete sequence and would give equal status to phenomena unequal in importance.

Because the descending Dorian (ascending Lydian) pattern emerges first among the *Timaeus* numbers, that pattern has primacy and should receive a distinct presentation, together with its associated chromatic and enharmonic phenomena, as it has in Table 34. In addition to the *Timaeus* UPS system represented in Tables 30 through 34, based upon the descending Dorian disdiapason, it is clear that one also might attempt to construct a prominent secondary *Timaeus* UPS system. This would be based upon the descending Mixolydian pattern (with its descending Dorian heart starting below the upper fifth and ending above the final fourth) that runs concurrently with the primary scale from octaves three through ten and then in the first incomplete sequence.

Other, stranger possibilities for a tertiary system too remote to speculate upon attend the unaccustomed pattern TTTTSTS that is also possible as a concurrent pattern with the original pattern from octaves six through ten and with the Mixolydian octave in the first incomplete sequence. These possibilities would have no precedent in ancient Greek music.

One could even attempt to construct fourth order *Timaeus* UPS systems based upon the patterns that emerge as the original d.Dorian/a.Lydian pattern degenerates; but these would not have the same status as those occurring before the marked hiatus comprising the second incomplete sequence of Table 26.

14 The Orderliness of the Chromatic Invasion within the Primary Scale

The chromatic invasion accounting for the remarkable results discussed above, in this chapter, is anything but chaotic. Table 26 shows that the eight complete octaves containing chromatic elements exhibit a slow but orderly creep of those distortions into the primary scale as it progresses. For example, the third complete octave beginning with 1536 and ending with 3072 and the fourth complete octave beginning with 3072 and ending with 6144 each exhibit one chromatic element apiece: 2187 and 4374, respectively. Then the fifth complete octave beginning with 6144 and ending with 12288 exhibits two chromatic elements: 6561 and 8748. The sixth complete octave beginning with 12288 and ending with 24576, exhibits three chromatic factors: 13,122; 17,496; and 19,683. Two of these are like those in the previous octave in being factors of 1719926784 and one is unlike, being a nonfactor of 1719926784. The seventh complete octave, beginning with 24576 and ending with 49152, is like the sixth in that it contains three chromatic elements, only two of which are factors of 1719926784: 26,244; 34,992; and 39,366. The eighth, ninth, and tenth complete octaves each display four chromatic elements each, only two of which are factors of 1719926784. The chromatic elements of the eighth complete octave, beginning with 49,152 and ending with 98,304, are 52,488; 59,049; 69,984; and 78,732. Those of the ninth complete octave, beginning with 98304 and ending with 196,608, are 104,976; 118,098; 139,968; and 157,464. Those of the tenth complete octave, beginning with 196,608 and ending with 393,216 are 209,952; 236,136; 279,936; and 314,928.

Taking each completely articulated disdiapason within the ten completely articulated octaves of the primary scale as a basic duadic unit, there are clearly four chromatic disdiapasonic units in the model diatonic octave string before it begins to degenerate. The ratios of the chromatic elements within the four *disdiapasons* may be represented as follows:

(A) 1:1
(B) 2:3

(C) 3:4
(D) 4:4

Each of the ratios in (A)–(D) is important on its face, representing, respectively, unity, the sesquialter ratio, the sesquitertian ratio, and unity again. They are also important in relationship to each other. The ratio 1:4 between the parallel and cross elements of (A) and (D) is the double octave ratio of the *disdiapason*, while the internal structure of each unit reflects the identity proportion and, therefore, unity.

The internal structures of (B) and (C) represent the sesquialter ratio of the fifth and the sesquitertian proportion of the fourth, respectively, as do the relations of their parallel elements vis-à-vis each other. The cross proportions represent identity and the octave ratio; but considered in further relation, ($\{3 \times 3\}/\{2 \times 4\}$), reproduce the 9/8 interval of the whole tone, as well as represent a perfect square (three squared is nine) and a perfect cube (two cubed is eight), those special numbers so prominent within the *Timaeus* fabric.

Both the parallel and cross-proportions between (A) and (B) reproduce the double and triple intervals. The parallel proportions between (C) and (D) reproduce both the sesquitertian ratio of the fourth and identity, as do the cross proportions; and the cross proportions in further relation ($\{4 \times 4\}/\{3 \times 4\}$) also reproduce the sesquitertian proportion.

The parallel proportions between (B) and (D) reproduce the octave and the sesquitertian ratio, as do their cross proportions, while the cross proportions in further relation ($\{3 \times 4\}/\{2 \times 4\}$) reproduce the sesquialter proportion. Because the introduction of disorderly elements into the octaves respects the musical proportions, the very process of octave disruption occurs in an orderly way. It refers, in its own way, to the cosmic limits important to Plato.

The repetition of the quadruple ratio of the *disdiapason* between the two most extreme pairs of chromatic *disdiapasons* suggests a disdiapasonic macrostructure for the fully differentiated universe—exactly in line with Favonius Eulogius' representations of the cosmic structure in his vastly later commentary on Cicero's *In somnium Scipionis*, a text in dialogue with Plato's *Timaeus*.[63] The chromatic progression articulates a *disdiapason* of *disdiapasons*, a fourfold progression to a *disdiapason* from elements themselves comprising unit *disdiapasons*. As noted above, the Decad is the appropriate structure describing such a fourfold expansion.

63 Eulogius *somnio Scipionis* 25.

Since a *disdiapason* is a duadic unit, the expansion has a very interesting mathematical significance. The repetition of the quadruple proportion between the two most extreme of the duadic units constituting chromatic *disdiapasons* reflects the progression of the duad to a decad. Plato's earlier numerical divisions allowed his readers to see only the fourfold progression of the monad to a *disdiapason* within the decadic structure—in other words, the expansion of the monad to a decad. The chromatic phenomena mean that Plato's division from 35 A–36 D allows a complete progression from monad to duad; further, it assigns a kind of equivalency between monad and duad as principles of the Decad, in that the text expresses first the monad progressing to a decad and then the duad progressing to a decad.

The inclusion of even the duad within decadic order indicates a relation of order to disorder that Plato may especially have sought to underscore. Disorder, it seems, is always contained by order and never has the upper hand. The *Philebus* echoes the point in stressing that even the duad of more and less is really a unity, though it displays multiplicity (Plato, *Philebus*, 26 D). Further, the *harmonia* of the universe extends to a disdiapason of duadic units; for only the latter limitation accomplishes a complete cycle of progression of the monad to the duad.

15 Orderly Rise and Fall of Fifth Periodicity with the Decay of the Primary Scale

After the tenth completely articulated diatonic octave of the primary *Timaeus* scale, the *Timaeus* numbers exhibit a decad of increasingly incomplete diatonic octaves vis-à-vis the model octave chain descending from 384. A study of this degeneration indicates that the ultimate meaning of the chromatic invasion for the *Timaeus* scale is the eventual replacement of octave periodicity by fifth periodicity.[64]

"Fifth periodicity" is the periodicity appropriate to recurring fifths—diapentic periodicity, to use the ancient vocabulary. The eminent musicologist, Jacques Handschin noted the phenomenon, while considering several different options for understanding the *Timaeus* scale (see his Form I). Comparing those options Handschin wrote:

64 See the notes to the text above and below, as well as the glossary, clarifying the musical points.

> … In looking again at our diagram we see that in I and II all the fourths contained in the Platonic division are made use of and all the Platonic points of support stand again as such (of course with the exception of the fourth 6:4 1/2 [6:9/2] and the tone 4 1/2 [9/2]), which is not the case in III—the difference between I and II being only that the primary tone 27 is in I, employed as delimiting tone, and not in II. The form I, which so largely replaces octave periodicity with **fifth periodicity** is much like the scale of *Musica Enchiriadis*.
>
> Summarizing our argument, we can say that Plato not having contented himself with dividing the octave but having divided also the twelfth, has offended against octave periodicity, and the consequence is that if we followed him strictly, we should transgress the limits of diatonicism. It can be doubted whether Plato was aware of all the consequences of what he had said.[65] [Emphasis in bold added.]

It is possible that Handschin mistakenly deemed Plato ignorant that his divisions would ultimately offend against octave periodicity. He may, instead, have aimed to limit octave periodicity in ways that would fit his cosmological objectives.

When one analyzes the numbers belonging to the ten incomplete sequences identified in Table 26, in relation to the original, primary diatonic model octave pattern, one finds that the last interval to disappear in the degenerating model scale is "the fifth."[66] To discern this fact from Tables 35 and 36, below, one must specify the steps of the original diatonic scale that fall out, in these sequences, as they progress. Table 35 makes this recitation for the original scale interpreted as an ascending Lydian scale; and Table 36 does so for the original scale interpreted as a descending Dorian scale.

Be aware, in relation to Table 35, that the steps of the complete TTSTTTS pattern of the model octave string (Δ-string in later chapters), understood as an ascending Lydian scale, have the following Greek names: *hypate-T-parhypate-T-lichanos-S-mese-T-paramese*--T-*trite-T-paranete*--*S-nete*. In relation to Table 36, assuming a descending Dorian order of the scale, the Greek names of the steps, in order, are *nete-T-paranete-T-trite-S-paramese-T-mese-T-lichanos*-T-*parhypate-*

65 Handschin, "Timaeus Scale," 23–24.

66 The term "fifth" is used for ease of expression and not to import modern connotations. Handschin and others use the term "fifth" because it is the modern term most closely analogous to the "diapente" of ancient vocabulary.

TABLE 35 The rise and fall of "fifth" periodicity in the incomplete sequence for the ascending Lydian

(11)	*hypate parhypate lichanos* [ ] *paramese trite paranete nete*[67]
(12)	*hypate parhypate lichanos* [ ] *paramese trite paranete* [ ]
(13)	[ ] *parhypate lichanos* [ ] *paramese trite paranete* [ ]
(14)	[ ] *parhypate lichanos* [ ] [ ] *trite paranete* [ ]
(15)	[ ] *parhypate lichanos* [ ] [ ] *trite paranete* [ ]
(16)	[ ] [ ] *lichanos* [ ] [ ] *trite paranete* [ ]
(17)	[ ] [ ] *lichanos* [ ] [ ] [ ] *paranete* [ ]
(18)	[ ] [ ] *lichanos* [ ] [ ] [ ] *paranete* [ ]
(19)	*paranete*
(20)	[ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

TABLE 36 The rise and fall of "fifth" periodicity in the incomplete sequences for the descending Dorian

(11)	*nete paranete trite* [ ] *mese lichanos parhypate hypate*[68]
(12)	*nete paranete trite* [ ] *mese lichanos parhypate* [ ]
(13)	[ ] *paranete trite* [ ] *mese lichanos parhypate* [ ]
(14)	[ ] *paranete trite* [ ] [ ] *lichanos parhypate* [ ]
(15)	[ ] *paranete trite* [ ] [ ] *lichanos parhypate* [ ]
(16)	[ ] [ ] *trite* [ ] [ ] *lichanos parhypate* [ ]
(17)	[ ] [ ] *trite* [ ] [ ] [ ] *parhypate* [ ]
(18)	[ ] [ ] *trite* [ ] [ ] [ ] *parhypate* [ ]
(19)	*parhypate*
(20)	[ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

S-hypate. The magnitudes of intervals between steps having the same names, in the two scales, are different just because the names apply to different parts of the TTSTTTS pattern, depending upon the ascending or descending perspective. For example, in the ascending Lydian scale, the interval between *paranete* and *nete* is a semitone. In the descending Dorian scale, it is a tone. The names themselves, then, do not imply intervals of absolute magnitude.

67 The missing note is *mese*.
68 Ibid.

Table 35 shows that three possible *ordos* of "fifth" periodicity, marked by "hypate–paramese," "parhypate–trite," and "lichanos–paranete," appear to be surfacing in the first incomplete sequence, the only one still spanning an octave. This sequence is truly ambiguous as between octave and fifth periodicity. The two types of periods overlap; but octave periodicity in the original *ordo* is clearly giving way to fifth periodicity in that *ordo*, since by the second sequence, the octave has disappeared and the fifth is clearly ubiquitous in a positive triad of fifths. This triad of fifths degenerates to a duad of fifths and then a monad.

Again, as Table 36 shows, the new *ordo* of "fifth" periodicity, marked by "nete–mese," "paranete–lichanos," and "trite–parhypate," appears to surface in the first incomplete sequence, the only one still spanning an octave. This sequence is truly ambiguous as between octave and fifth periodicity. The two types of periods overlap; but octave periodicity in the original *ordo* is clearly giving way to fifth periodicity in that *ordo*, since by the second sequence, the octave has disappeared and a clear triad of fifths of the kind identified is clearly ubiquitous. This triad of fifths disintegrates to a duad of fifths and then a monad before finally disappearing.

16 Grammar of Chromaticity in the Rise and Fall of Fifth Periodicity

The study above has demonstrated that all of the chromatic elements in the *Timaeus* set through the end of the tenth complete octave of the primary scale were needed to demarcate an orderly progression of disorderly elements into the primary diatonic scale as it continued past the first disdiapason. Chromaticity has a different but equally important function in relationship to the rise and fall of fifth periodicity.

The number of chromatic elements in each incomplete sequence remains constant, through the eighth incomplete sequence, at 4. Thus, when incomplete sequences are taken in pairs, the ratio of chromatic elements in each pair's two sequences is 4:4, for the first four pairs. The ratio for the sequences of the fifth pair is 3:6. In each of the sequences of the fifth pair, fifth periodicity vis-à-vis the original diatonic *Timaeus* pattern is nonexistent. Only one element continuous with the original diatonic scale remains, in the *Timaeus* number set, in the ninth sequence and none remains in the tenth. All traces of the original scale have vanished with the tenth sequence.

The uniform 4:4 ratio of chromatic elements, in the sequences of the first four pairs is the limiting balance under which fifth periodicity can be preserved in them. When the monadic unity, in the first four pairs, proceeds to duality,

represented by the fifth pair (4:4, i.e., 1:1⇒3:6, i.e., 1:2), fifth periodicity disappears and, ultimately, all elements continuous with the original scale. The move to the duad, as to chromaticity, serves as fitting punctuation marking the end of the number set.

The move from one to four pairs of sequences representing unity, as to their chromatic elements, is quite in line with the normal grammar of generation in the *Timaeus* text permitting a new phenomenon to arise as a duad only after a progression to four. The new phenomenon in this case is the end of the set. All of the chromatic numbers, therefore, appear to be necessary in Plato's finite universe; one may, therefore, conclude that all properly belong to the *Timaeus* number set. Chapter 6 buttresses this conclusion, in its consideration of the further musical significance of the chromatic numbers.

17 Another Look at the Crantor Matrix

One can relate the observations, made in this chapter about the primary *Timaeus* scale and its degeneration, to the original matrix pattern made by the numbers, as indicated in Figure 13, below. Rather than aligning the pattern to the center, however, as in Figure 12, Figure 13 aligns the pattern to the left.

18 The Decad in the Rise, Wax, and Wane of the Primary *Timaeus* Scale

Regardless of whether one views the primary *Timaeus* scale as an ascending Lydian or descending Dorian scale, the *disdiapason* and Decad remain the predominant patterns of the *Timaeus*. The decad of octaves and decad of incomplete sequences evidencing the emergence and disappearance of fifth periodicity is the highest level pairing articulated by the *Timaeus* numbers from the standpoint of the primary scale. They stand in the 2:1 relation of the octave and jointly articulate a duad of *disdiapasons* just because they both are decadic units. As noted, above, however, a duad of *disdiapasons* is a tetrad of *diapasons* (themselves *duads*); and this sequence implies the fourfold progression of the *duad* to an overarching decad. The cosmic limit, then, is marked by the duad's self-return.

This well-structured decadic grammar, in Plato's text, of the relationship between octave and fifth periodicity vis-à-vis the primary *Timaeus* scale sets the harmonic bounds of the finite model universe that he was constructing as a paradigm for the actual cosmos. It fit the cosmos, according to the best thought

Key: All dots, *f*'s and ∫'s belong to the number set.
The *f*'s indicate chromatic numbers that are factors of 1719926784.
The ∫'s indicate chromatic numbers that are not factors of 1719926784.
Characters highlighted in purple represent numbers that are both perfect squares and perfect cubes.
Characters highlighted in green only are perfect squares.
Characters highlighted in yellow only are perfect cubes.

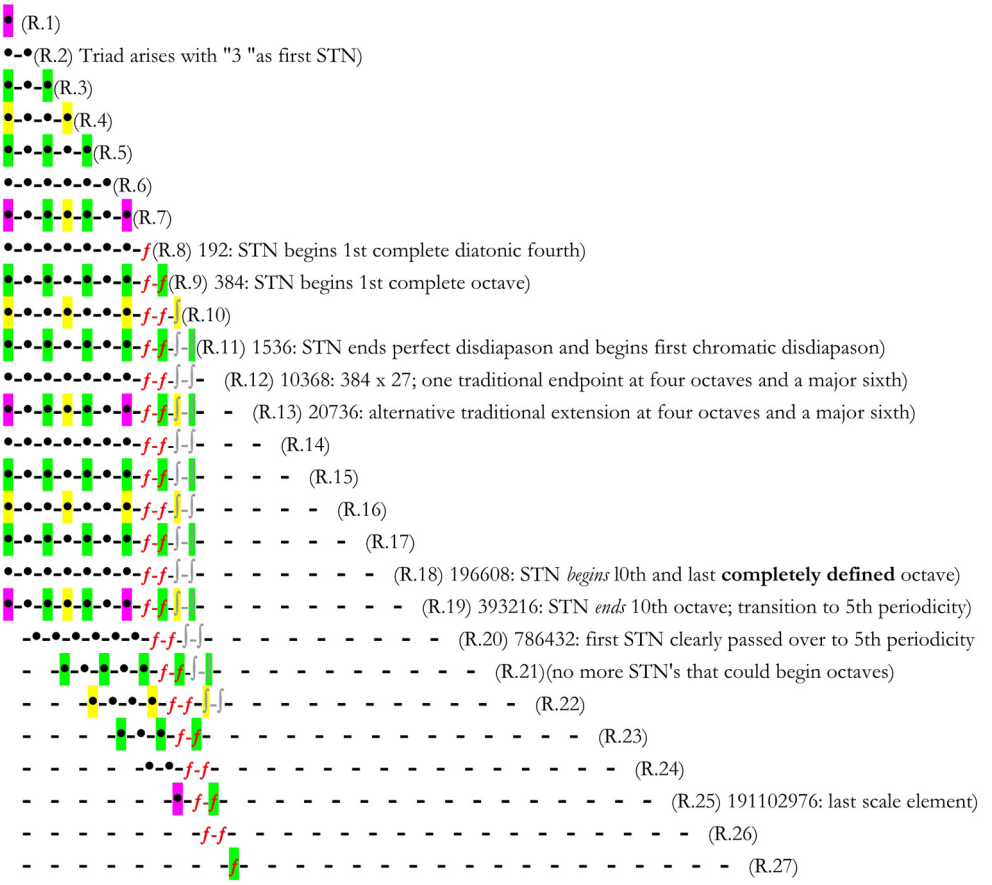

FIGURE 13 Dot chart relating analysis of *Timaeus* table numbers to Crantor pattern emerging from *Timaeus*

of his day, in uncanny ways that cannot be appreciated without an examination of the further musical significance of the *Timaeus* numbers. If Plato had correctly defined the bounds of the universe, then, among other things, all *harmonia* possible within it had to be tied to the phenomena associated with the primary *harmonia*. Chapter 6 shows that all harmonic phenomena possible in the text, including many secondary *Timaeus* scales, are born from permutations of the primary d.Dorian/a.Lydian diatonic octave structure assimilating the invasion by chromatic elements, as that structure repeats.

CHAPTER 6

The Further Musical Significance of Plato's Number Matrix: the Many Secondary *Timaeus* Scales and Associated Musical Phenomena

This study has shown that the primary *Timaeus* scale begins as a descending Dorian or ascending Lydian diatonic octave scale, incorporating, as it progresses, chromatic and enharmonic variants sufficient for several iterations of a fully articulated *Timaeus* UPS in three genera. The study has followed both the development and degeneration of that scale. The scale starts with the model diatonic octave, i.e., the first complete octave that emerges in the *Timaeus* set, beginning at 384, and ends at 393,216, the final element of the tenth complete octave preserving the original diatonic pattern. Since the primary scale arises from the very first complete octave, it has precedence within the *Timaeus* set. It is not, however, the only possible *Timaeus* scale.

As the model octave scale repeats, all numbers in the sequence belonging to the original pattern become independent bases for continuing octave trains of their own, following not the descending Dorian/ascending Lydian pattern, but, instead, exemplifying other possibilities for the diatonic scale. As these trains continue, they, too, develop chromatic and enharmonic variants, fitted to alternative UPS systems. It would take far too much time and space to demonstrate the alternative UPS systems fitted to the variants for each of the secondary diatonic trains. The reader can work these out for himself imitating the method for constructing a UPS system demonstrated in Chapter 5 for *Timaeus* UPS. For the sake of simplicity, this chapter separately addresses the secondary diatonic, chromatic, and enharmonic *Timaeus* scales hidden within the *Timaeus* number set.

1 The Many Secondary Diatonic *Timaeus* Scales Hidden in the Fabric

To find the secondary diatonic octave chains in the *Timaeus* number set, one locates the model octave beginning at 384 but starts constructing octaves from an STN of 432, instead of 384. One observes the octave pattern that emerges in the sequence from 432 to 864 and moves through the entire *Timaeus* set, tracking that pattern and any others that emerge as far as possible. Upon the completion of this process, one returns to the model octave beginning at 384,

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TABLE 37 Descending diatonic octave patterns

Octave species	**Diatonic**
Mixolydian	TTTSTTS
Lydian	STTTSTT
Phrygian	TSTTTST
Dorian	TTSTTTS
Hypolydian	STTSTTT
Hypophrygian	TSTTSTT
Hypodorian	TTSTTST

but now constructs octaves from an STN of 486 and so on and so forth, following the new patterns as far as it is possible to continue them on the basis of numbers in the *Timaeus* set. When one follows such a procedure with each of the numbers representing a step in the original descending Dorian/ascending Lydian scale, one derives diatonic octave chains different both from each other and from the primary one grounded upon the model octave. Indeed, distinct diatonic octave chains of all seven *tonoi* of ancient Greek music emerge, in the exercise, in both ascending and descending form. Appendix 7 catalogues, in detail, all of the octave chains originating from steps in the model scale.

It eventually occurs to an analyst that, in addition to calculating diatonic octave strings starting from 384 and other steps of the model octave, moving sequentially through the set to follow the octave chains to completion, one can also find octave chains based upon the chromatic numbers. Appendices 5 and 6 catalogue those octave chains.

The reader can verify the catalogues comprised by Appendices 5, 6, and 7 upon his own observation of the diatonic patterns of each of the *tonoi* and independent work with the *Timaeus* numbers. For the reader's convenience, Table 37 appears above, listing the descending diatonic octave patterns dominant in Plato's day.[1]

Upon his own verification, the reader should agree that Appendices 5, 6, and 7 together show that each of the ten diatonic octave chains discoverable from the *Timaeus* numbers (the Decad again), beyond the model octave chain, begins with a characteristic principal sequence that degenerates and metamor-

1 Reese, *Music*, 30 and 41.

phoses with repetition. A summary of what happens is set forth in Table 38 immediately below.

In Table 38, the model octave chain is "Δ-string." The chain beginning from the model octave's second element (432) is "α-string." The one beginning from its third element (486) is "β-string." The one beginning from its fourth element (512) is "γ-string." The one beginning from its fifth element (576) is "δ-string." The one beginning from its sixth element (648) is "ε-string." The one beginning from its seventh element (729) is "ζ-string." The four chromatic chains independent of the model octave are designated as "CF1," "CF2," "CNF1," and "CNF2," where "CF" indicates chromatic factor and "CNF" indicates chromatic nonfactor of 1719926784. These chains start from 2187, 6561, 19683, and 59049, respectively. The material presented concerning "Δ-string" is already familiar to the reader from Chapter 5.

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus*

Δ-string: This is the primary *Timaeus* scale of the text. After an initial perfect disdiapason beginning from 384 that solely reflects a d.Dorian/a.Lydian diatonic scale (TTSTTTS), the Δ-string undergoes distinctive distortions with the introduction of various chromatic elements. With the introduction of the first chromatic element in the third octave, a d.Mixolydian/a.Hypolydian diatonic scale (TTTSTTS) (characteristic of the γ-string) emerges as a distinct interpretational possibility for the affected octave, in addition to the original possibility. The second chromatic element in the fifth octave produces no further distortions. With the introduction of the third chromatic element in the sixth octave, an arrangement of the octave in an unaccustomed diatonic pattern, TTTTSTS, not representing any standard scale arrangement, is also possible, along with the other two. The introduction of a fourth chromatic element in the eighth octave does not further distort the original pattern of the Δ-string. The three octave arrangements mentioned remain intact through the tenth octave, ending with 393216.
The original d.Dorian/a.Lydian diatonic pattern of the Δ-string disappears with its final articulation in the tenth octave but leaves the secondary d.Mixolydian/a.Hypolydian diatonic scale (imitating γ-string), intact, along with the unaccustomed sequence TTTTSTS, for another octave. These sequences terminate upon their articulation in the eleventh octave, so that the eleventh is the last octave in the chain. There are, all together, in the Δ-string, ten (10) d.Dorian/a.Lydian diatonic octaves; nine (9) d.Mixolydian/a.Hypolydian diatonic octaves, and six (6) unaccustomed patterns of the type TTTTSTS.

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

α-string: After an initial perfect disdiapason beginning from 432 that solely reflects a d.Phrygian/a.Phrygian scale (TSTTTST), the α-string undergoes distinctive distortions with the introduction of various chromatic elements. With the introduction of the first chromatic element in the third octave a d.Hypodorian/a.Hypophrygian scale (TTSTTST) (characteristic of the δ-string) emerges in addition to the original possibility. The second chromatic element in the fourth octave allows two further possibilities: (a) a d.Dorian/a.Lydian scale imitating the Δ-string and (b) an unaccustomed arrangement of the octave in the pattern TSTTTTS. Two more possibilities for scale interpretation arise with the entrance of the third chromatic element into the scale in the sixth octave: (a) d.Mixolydian/a.Hypolydian scale (TTTSTTS) imitating the γ-string and (b) a second unaccustomed arrangement of the octave in the pattern TTTSTST. The introduction of the fourth chromatic element in the eighth octave does not produce additional possibilities for tone sequences in the α-string.

The α-string begins to degenerate after the emergence of the fourth chromatic element, in the eleventh octave, with the disappearance of the d.Phrygian/a.Phrygian diatonic pattern and the unaccustomed pattern TSTTTTS. The d.Hypodorian/a.Hypophrygian pattern and the unaccustomed pattern TTTSTST are the second to disappear, in the twelfth octave. The third patterns to disappear are the d.Dorian/a.Lydian, in the fourteenth octave of the chain, so that the last complete octave of the string reflects the d.Mixolydian/a.Hypolydian pattern characteristic of the γ-string. The string disappears as a string of complete octave sequences after fourteen octaves. There are all together in the α-string ten (10) d.Phrygian/a.Phrygian diatonic octaves; nine (9) d.Hypodorian/a.Hypophrygian diatonic octaves; ten (10) d.Dorian/a.Lydian diatonic octaves; nine (9) d.Mixolydian/a.Hypolydian diatonic octaves; seven (7) unaccustomed patterns of the type TSTTTTS; and six (6) unaccustomed patterns of the type TTTSTST.

β-string: This string begins by running in the exact opposite direction of Δ-string. After an initial perfect disdiapason beginning from 486 that reflects solely a d.Lydian/a.Dorian scale (STTTSTT), the β-string undergoes distinctive distortions with the introduction of each chromatic element. With the introduction of the first chromatic element in the third octave, a d.Hypophrygian/a.Hypodorian pattern (TSTTSTT) imitating the ε-string becomes possible in addition to the original sequence. The introduction of a second chromatic element in the fourth octave permits an additional d.Phrygian/a.Phrygian pattern (TSTTTST) imitating the α-string, as well as an unaccustomed arrangement of the octave represented by the pattern STTTTST. The third chromatic element in the sixth octave introduces a d.Hypodorian/a.Hypophrygian scale in imita-

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

tion of the δ-string, as well as another unaccustomed arrangement of the octave, represented by the pattern TTSTSTT. The fourth chromatic element in the seventh octave allows a d.Dorian/a.Lydian scale (TTSTTTS) imitating the Δ-string and two additional unaccustomed arrangements of the octave, represented, respectively, by the patterns STTTTTS and TSTTTTS. By the time of the introduction of the fourth chromatic element, then, nine patterns compete for recognition in the β-string. Note that, of these patterns, the Phrygian is its own opposite; and the following pairs run in opposite directions: (1) d.Lydian/a.Dorian and d.Dorian/a.Lydian; (2) d.Hypophrygian/a.Hypodorian and d.Hypodorian/a.Hypophrygian; and (3) the two unaccustomed patterns STTTTST and TSTTTTS.

As the octaves continue to repeat, the β-string degenerates, losing, first, in the eleventh octave, its characteristic d.Lydian/a.Dorian pattern and the two unaccustomed patterns STTTTST and STTTTTS. In the twelfth octave the d.Hypophrygian/a.Hypodorian and the unaccustomed pattern TTSTSTT disappear. In the fourteenth octave the d.Phrygian/a.Phrygian pattern and the unaccustomed pattern TSTTTTS are lost. In the fifteenth octave the d.Hypodorian/a.Hypophrygian patterns disappear. The final complete octave of the β-string exhibits the d.Dorian/a.Lydian pattern characteristic of the Δ-string. The string disappears as a string of complete octaval sequences after sixteen octaves. There are, all together, in β-string, ten (10) d.Lydian/a.Dorian diatonic octaves; nine (9) d.Hypophrygian/a.Hypodorian diatonic octaves; ten (10) d.Phrygian/a.Phrygian diatonic octaves; nine (9) d.Hypodorian/a.Hypophrygian diatonic octaves; ten (10) d.Dorian/a.Lydian diatonic octaves; seven (7) unaccustomed patterns of the type STTTTST; six (6) unaccustomed patterns of the type TTSTSTT; four (4) unaccustomed patterns of the type STTTTTS; and seven (7) unaccustomed patterns of the type TSTTTTS.

γ-string: The γ-string begins from 512 with a perfect disdiapason reflecting a d.Mixolydian/a.Hypolydian scale (TTTSTTS). The introduction of the first chromatic element into the scale, in the third octave, does not affect the possibilities for the tone sequence of the octave. The second chromatic element in the fourth octave allows the introduction of the unaccustomed pattern TTTTSTS. The third chromatic element in the sixth octave does not further alter the possibilities for octave arrangement; but the fourth in the seventh octave allows the introduction of another unaccustomed pattern: TTTTTSS. Interestingly, γ-string is the string with the stablest pattern. The Δ-string is next in stability, showing a tendency to shift over time only to the γ-string pattern. The γ-string disappears abruptly, in all of its patterns, as a string of complete octaval sequences after nine octaves. There are, all together, in the γ-string, nine (9)

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

d.Mixolydian/a.Hypolydian diatonic octaves; six (6) unaccustomed patterns of the type TTTTSTS; and three (3) unaccustomed patterns of the type TTTTTSS.

δ-string: The δ-string, beginning from 576, is the first string that does not form a perfect disdiapason in its characteristic sequence, namely, the d.Hypodorian/a.Hypophrygian scale (TTSTTST). The first chromatic element in the second octave allows, in addition to the original sequence, the d.Dorian/a.Lydian tone sequence (TTSTTTS), imitating the Δ-string. The introduction of the second chromatic element in the fourth octave allows two additional sequences, TTTSTST, an unaccustomed pattern, and a d./Mixolydian/a.Hypolydian pattern (TTTSTTS) imitating the γ-string. The third chromatic element in the sixth octave introduces no new variations; but the fourth in the seventh permits two additional unaccustomed sequences, namely: TTTTSST and TTTTSTS.

As the δ-octaves continue to repeat after the introduction of the fourth chromatic element, the δ-string degenerates, losing in the tenth octave, its own characteristic d.Hypodorian/a.Hypophrygian sequence, along with TTTSTST and TTTTSST, the unaccustomed patterns. In the twelfth octave, it loses the d.Dorian/a.Lydian sequence imitating the Δ-string, leaving only the d.Mixolydian/a.Hypolydian pattern. What began, then, as a d.Hypodorian/a.Hypophrygian scale ends as a d.Mixolydian/a.Hypolydian scale imitating the γ-string. The string disappears as a string of complete octaval sequences after twelve octaves. There are, all together, in the δ-string nine (9) d.Hypodorian/a.Hypophrygian diatonic octaves; ten (10) d.Dorian/a.Lydian diatonic octaves; nine (9) d.Mixolydian/a.Hypolydian diatonic octaves; six (6) unaccustomed sequences of the type TTTSTST; six (6) unaccustomed sequences of the type TTTTSTS; and three (3) unaccustomed sequences of the type TTTTSST.

ε-string: This string starts from 648 by running in the opposite direction of δ-string. After an initial octave reflecting a d.Hypophrygian/a.Hypodorian scale (TSTTSTT), chromatic elements enter the ε-string, causing it to exhibit variations additional to the basic pattern laid by the first octave. With the first chromatic element in the second octave, the d.Phrygian/a.Phrygian pattern (TSTTTST) characteristic of the α-string becomes possible. The second chromatic element in the fourth octave allows the d.Hypodorian/a.Hypophrygian scale (TTSTTST) characteristic of the δ-string and the unaccustomed pattern TTSTSTT. The third chromatic element in the fifth octave gives rise to the unaccustomed pattern TSTTTTS and d.Dorian/a.Lydian pattern (TTSTTTS) characteristic of the Δ-string. The fourth chromatic element allows two additional unaccustomed patterns: TTTSSTT, TTTSTST, and the d.Mixolydian/a.Hypolydian pattern (TTTSTTS), characteristic of the γ-string. Note that, among the competing octave

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

patterns in the string, the Phrygian is its own opposite and the d.Hypophrygian/a.Hypodorian and d.Hypodorian/a.Hypophrygian run in opposite directions.

Repetitions of the octave beyond the occurrence of the fourth chromatic element evidence the gradual degeneration of the ε-string patterns. In the tenth octave, the d.Hypophrygian/a.Hypodorian pattern disappears, along with the two unaccustomed sequences TTSTSTT and TTTSSTT. In the twelfth octave, the d.Phrygian/a.Phrygian pattern and unaccustomed pattern TSTTTTS disappear. In the fourteenth octave, the d.Hypodorian/a.Hypophrygian pattern and the unaccustomed pattern TTTSTST disappear. In the fifteenth octave the d.Dorian/a.Lydian pattern disappears. The last intact pattern is the d.Mixolydian/a.Hypolydian scale characteristic of the γ-string. What began as a d.Hypophrygian/a.Hypodorian scale comes to rest, at last, in the pattern of the γ-string. For a discussion of the relationship of the γ-string to the monad and of Δ-string to the triad, see the text below. The string disappears as a string of complete octaval sequences after fifteen octaves. There are, all together, in the ε-string, nine (9) d.Hypophrygian/a.Hypodorian diatonic octaves; ten (10) d.Phrygian/a.Phrygian diatonic octaves; nine (9) d.Hypodorian/a.Hypophrygian diatonic octaves; ten (10) d.Dorian/a.Lydian diatonic octaves; nine (9) d.Mixolydian/a.Hypolydian diatonic octaves; six (6) unaccustomed patterns of the type TTSTSTT; seven (7) unaccustomed patterns of the type TSTTTTS; six (6) unaccustomed patterns of the type TTTSTST; and three (3) unaccustomed patterns of the type TTTSSTT.

ζ-string: This string begins from 729 as a d.Hypolydian/a.Mixolydian (STTSTTT) scale and so runs in the opposite direction of the γ-string. In this character, ζ-string has a special nature as the limit to the extension of strings founded upon the model octave. The γ-string is the one string with "proto-elements" bearing a primary monadic relation to the Δ-string (see the text below). No other string besides the ζ-string, among the strings based upon the model octave, runs directly opposite the γ-string; and all of the chromatic strings imitate the ζ-string in that respect. The ζ-string's reversal of the fundamental scale direction set by the only string bearing a monadic relation to the Δ-string, in fact, indicates the natural outer bound or limit of relational cohesion within the set of octaves derived by taking each successive step of the model octave (first complete octave of Δ-string) as the stem for an octave series. In its function as limiting string, as this study shows, it also constitutes the bound between two parts of a fabric of numbers defined by the proposed *Timaeus* set.

After an initial octave comprising a d.Hypolydian/a.Mixolydian (STTSTTT) pattern, the ζ-string admits variation with the introduction of chromatic elements. The first

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

chromatic element in the second octave permits the d.Lydian/a.Dorian pattern (STTTSTT) characteristic of the β-string. The second chromatic element in the fourth octave allows the unaccustomed pattern TSTSTTT and the d.Hypophrygian/a.Hypodorian pattern (TSTTSTT) characteristic of the ε-string. The third chromatic element in the fifth octave permits the unaccustomed sequence STTTTST and the d.Phrygian/a.Phrygian pattern (TSTTTST) characteristic of the α-string. The fourth chromatic element in the seventh octave permits two additional unaccustomed arrangements, namely, TTSSTTT and TTSTSTT, as well as the d.Hypodorian/a.Hypophrygian pattern (TTSTTST) characteristic of the δ-string. Note that, among the competing patterns in the string, the Phrygian is its own opposite; and the d.Hypophrygian/a.Hypodorian and d.Hypodorian/a.Hypophrygian patterns run in opposite directions.

Repetitions of ζ-string octaves beyond the introduction of the fourth chromatic element witness the gradual degeneration of possible ζ-string tone sequences. In the tenth octave, the d.Hypolydian/a.Mixolydian pattern and the two unaccustomed sequences, TSTSTTT and TTSSTTT disappear. In the twelfth octave, the d.Lydian/a.Dorian pattern and the unaccustomed pattern STTTTST disappear. In the thirteenth octave, the d.Hypophrygian/a.Hypodorian pattern and unaccustomed pattern TTSTSTT disappear. In the fifteenth octave, the d.Phrygian/a.Phrygian pattern is lost. The last intact pattern is the d.Hypodorian/a.Hypophrygian pattern characteristic of the δ-string. Unlike any of the other strings, then, the ζ-string comes to rest neither in the γ-string, the string bearing a special relation to the monad, nor in the Δ-string, the string with a primary triadic character (see the text below). Rather, it comes to rest in a string that has a secondary triadic character; for δ-string stands in the same relation to ζ-string, as Δ-string stands to γ-string (see the text below). The string disappears as a string of complete octaval sequences after fifteen octaves. There are, all together, in ζ-string, nine (9) d.Hypolydian/a.Mixolydian diatonic octaves; ten (10) d.Lydian/a.Dorian diatonic octaves; nine (9) d.Hypophrygian/a.Hypodorian diatonic octaves; nine (9) d.Hypodorian/a.Hypophrygian diatonic octaves; ten (10) d.Phrygian/a.Phrygian diatonic octaves; six (6) unaccustomed patterns of the type TSTSTTT; seven (7) unaccustomed patterns of the type STTTTST; six (6) unaccustomed patterns of the type TTSTSTT; and three (3) unaccustomed patterns of the type TTSSTTT.

CF1: CF1 begins from 2187 with the same primary d.Hypolydian/a.Mixolydian pattern (STTSTTT) as ζ-string. As CF1 octaves repeat, however, additional patterns become possibilities in the following order: (a) the d.Lydian/a.Dorian pattern characteristic of the β-string emerges in the second octave; (b) the d.Hypophrygian/a.Hypodorian pattern characteristic of ε-string emerges in the fourth octave, along with the unaccus-

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

tomed sequence TSTSTTT; and (c) the d.Phrygian/a.Phrygian pattern characteristic of α-string arises in the fifth octave, along with the unaccustomed sequence STTTTST.

The scale begins to degenerate in the tenth octave with the loss of the d.Hypolydian/a.Mixolydian and unaccustomed TSTSTTT patterns. In the twelfth octave the d.Lydian/a.Dorian pattern disappears, along with the unaccustomed pattern STTTTST. In the thirteenth octave, the d.Hypophrygian/a.Hypodorian pattern disappears, leaving only the d.Phrygian/a.Phrygian pattern. The string disappears as a string of complete octaval sequences after fourteen octaves. There are, all together in CF1, nine (9) d.Hypolydian/a.Mixolydian diatonic octaves; ten (10) d.Lydian/a.Dorian diatonic octaves; nine (9) d.Hypophrygian/a.Hypodorian diatonic octaves; ten (10) d.Phrygian/a.Phrygian diatonic octaves; six (6) unaccustomed sequences of the type TSTSTTT and seven (7) unaccustomed sequences of the type STTTTST.

CF2: CF2 begins from 6561 with the d.Hypolydian/a.Mixolydian pattern (STTSTTT) characteristic of ζ-string and the unaccustomed pattern STSTTTT. It is the first string to begin in an ambiguous fashion. In the second octave, the d.Lydian/a.Dorian pattern (STTTSTT) characteristic of the β-string becomes possible, in addition to the others. In the fourth octave, three more patterns emerge as possibilities: the d.Hypophrygian/a.Hypodorian pattern (TSTTSTT) characteristic of ε-string and two more unaccustomed patterns, TSSTTTT and TSTSTTT. These patterns remain stable for two more octaves.

In the seventh octave, CF2 begins to break down with the loss of the unaccustomed patterns, STSTTTT and TSSTTTT. In the tenth octave, the d.Hypolydian/a.Mixolydian pattern disappears, along with the unaccustomed pattern TSTSTTT. In the twelfth octave the d.Lydian/a.Dorian pattern drops out, leaving only the d.Hypophrygian/a.Hypodorian pattern. The octave string disappears after twelve octaves. There are, all together, in CF2 nine (9) d.Hypolydian/a.Mixolydian diatonic octaves; ten (10) d.Lydian/a.Dorian diatonic octaves; nine (9) d.Hypophrygian/a.Hypodorian diatonic octaves; six (6) unaccustomed sequences of the type STSTTTT; six (6) unaccustomed patterns of the type TSTTSTT; and three (3) unaccustomed patterns of the type TSSTTTT.

CNF1: This string, beginning from 19683, exhibits, in its first octave, both the d.Hypolydian/a.Mixolydian pattern (STTSTTT) characteristic of ζ-string and the unaccustomed pattern STSTTTT; so, like CF2, it begins ambiguously. It gives rise in the second octave to the d.Lydian/a.Dorian pattern (STTTSTT) characteristic of the β-string. CNF1 begins to degenerate in the seventh octave with the disappearance of the unaccustomed

TABLE 38 Behavior of the diatonic octave strings of the *Timaeus* (*cont.*)

pattern STSTTTT. In the tenth octave the d.Hypolydian/a.Mixolydian pattern also disappears, leaving only the d.Lydian/a.Dorian pattern. The string disappears as a string of complete octaval sequences upon the completion of eleven octaves. There are, all together in CF2, nine (9) d.Hypolydian/a.Mixolydian diatonic octaves; ten (10) d.Lydian/a.Dorian diatonic octaves; and six (6) unaccustomed patterns of the type STSTTTT.

CNF2: This string, beginning from 59049, exhibits, in its first octave the d.Hypolydian/a.Mixolydian pattern (STTSTTT) characteristic of the ζ-string and two unaccustomed patterns STSTTTT and SSTTTTT; so, it is even more ambiguous in its beginnings than CF2 and CNF1. The string begins degenerating in its fourth octave with the loss of the unaccustomed pattern SSTTTTT. In the seventh octave, the unaccustomed pattern STSTTTT drops out, leaving only the d.Hypolydian/a.Mixolydian pattern. Considering that CNF2 is the tenth and last string of octaves, beyond the primary string, allowed by the numbers of the *Timaeus* table, the result is oddly fitting. As the string completing a decad of additional strings, CNF2 is a monad of a new order. Like the γ-string, the original monadic string (see the text below), it admits of only one standard possibility. This circumstance is a strong indication that all chromatic numbers of the *Timaeus* set, whether or not they are factors of 1719926784, are necessary to the collection. CNF2 disappears as a string of complete octaval sequences after nine octaves. There are, all together, in CNF2 nine (9) d.Hypolydian/a.Mixolydian diatonic octaves; six (6) unaccustomed patterns of the type STSTTTT; and three (3) unaccustomed patterns of the type SSTTTTT.

A few observations are in order about the phenomena exhibited by the eleven octave chains. Note first the inventory of standard octave patterns among the strings beginning with model scale numbers, as well their quantitative stability across the strings in which they occur. There are fifty (50) d.Dorian/a.Lydian diatonic octaves, ten (10) each in Δ-string, α-string, β-string, δ-string, and ε-string; forty-five (45) d.Mixolydian/a.Hypolydian diatonic octaves, nine (9) each in Δ-string, α-string, γ-string, δ-string, and ε-string; forty (40) d.Phrygian/a.Phrygian octaves, ten (10) each in α-string, β-string, ε-string, and ζ-string; forty-five (45) d.Hypodorian/a.Hypophrygian diatonic octaves, nine (9) each in α-string, β-string, δ-string, ε-string, and ζ-string; twenty-seven (27) d.Hypophrygian/a.Hypodorian diatonic octaves, nine (9) each in β-string, ε-string, and ζ-string; twenty (20) d.Lydian/ascending Dorian diatonic octaves ten (10) each in β-string and ζ-string; and nine (9) d.Hypolydian/a.Mixolydian diatonic octaves,

all occurring in ζ-string. The total number of standard octave patterns in the octave strings beginning with model scale numbers is 236.

There are in the chromatic strings thirty-six (36) d.Hypolydian/a.Mixolydian diatonic octaves, nine (9) each in CF1, CF2, CNF1 and CNF2; thirty (30) d.Lydian/a.Dorian diatonic octaves, ten (10) each in CF1, CF2, and CNF1; eighteen (18) d.Hypophrygian/a.Hypodorian diatonic octaves, nine (9) each in CF1 and CF2; and ten (10) d.Phrygian/a.Phrygian diatonic octaves all in CF1. These octave patterns are clearly also remarkable in their quantitative stability across the strings in which they occur.

The total inventory of standard diatonic octaves in the entire *Timaeus* set is, then, fifty (50) d.Dorian/a.Lydian octaves; forty-five (45) d.Mixolydian/a.Hypolydian octaves; fifty (50) d.Phrygian/a.Phrygian octaves; forty-five (45) d.Hypodorian/a.Hypophrygian octaves; forty-five (45) d.Hypophrygian/a.Hypodorian octaves; fifty (50) d.Lydian/a.Dorian octaves; and forty-five (45) d.Hypolydian/a.Mixolydian octaves. There are then one hundred **(100)** octaves, in all, focused in some way on Dorian/Lydian patterns, ninety **(90)** focused on Mixolydian/Hypolydian patterns; fifty **(50)** on Phrygian patterns; and ninety **(90)** on Hypophrygian/Hypodorian patterns. The Dorian/Lydian phenomena have clear prominence, with ascending and descending varieties of each scale, apparently equal in importance. There are **330** standard diatonic octave patterns, in all, that can be interpreted, equally as ascending or descending octave scales. If one counts ascending and descending patterns, separately, since they exist contemporaneously, there are **660** diatonic octave scales, in all.

All eleven octave strings also exhibit patterns not recognized as standard for the arrangement of the octave. These patterns are called "unaccustomed" patterns in this study. Their instances are fewer in number than instances of the standard patterns; but they are not inconsiderable in number. The unaccustomed patterns also display remarkable quantitative stability across the strings in which they occur. All unaccustomed diatonic patterns are charted in Table 39 below. "M" designates octave chains originating from model scale numbers. "C" designates octave chains arising from chromatic numbers.

The total number of diatonic octave patterns in the *Timaeus* set, combining standard and unaccustomed arrangements, is 484, when one does not count ascending and descending patterns separately. It is 968, if one counts ascending and descending scales separately.

Among the nonstandard patterns, there are some that allow a clear division of the octave into two fourths and an independent tone. Others do not and so would not, from the outset, accord with the Philolaic expectations for basal octave construction that the whole ancient Greek world accepted and passed on as tradition. The patterns allowing plausible division are the following:

TABLE 39 Unaccustomed diatonic octave patterns ("UDOP")

UDOP	Δ	α	β	γ	δ	ε	ζ	#M	CF1	CF2	CNF1	CNF2	#C	Total
TTTTSTS	6	0	0	6	6	0	0	18	0	0	0	0	0	18
TSTTTTS	0	7	7	0	0	7	0	21	0	0	0	0	0	21
TTTSTST	0	6	0	0	6	6	0	18	0	0	0	0	0	18
TTTTTSS	0	0	0	3	0	0	0	3	0	0	0	0	0	3
TTTTSST	0	0	0	0	3	0	0	3	0	0	0	0	0	3
TTSTSTT	0	0	6	0	0	6	6	18	0	0	0	0	0	18
TTTSSTT	0	0	0	0	0	3	0	3	0	0	0	0	0	3
TSTSTTT	0	0	0	0	0	0	6	6	6	6	0	0	12	18
STTTTST	0	0	7	0	0	0	7	14	7	0	0	0	7	21
TTSSTTT	0	0	0	0	0	0	3	3	0	0	0	0	0	3
STTTTTS	0	0	4	0	0	0	0	4	0	0	0	0	0	4
STSTTTT	0	0	0	0	0	0	0	0	0	6	6	6	18	18
TSSTTTT	0	0	0	0	0	0	0	0	0	3	0	0	3	3
SSTTTTT	0	0	0	0	0	0	0	0	0	0	0	3	3	3
Totals	6	13	24	9	15	22	22	111	13	15	6	9	43	154

TST/T/TTS (note that one can experiment with the division, but one cannot achieve two identical fourths); T/TTS/TST (same observation as for previous pattern); TST/STT/T (same observation as for previous patterns); TTS/T/STT (same observation as for previous patterns; observe also that the patterns of the two fourths run opposite each other); T/TTS/STT (same observation as for previous pattern); STT/TTS/T (same observation as for two previous patterns); TTS/STT/T (same observation as for two previous patterns). Because of the lack of symmetry between the two fourths of the unaccustomed patterns, ancient Greeks would probably have considered them to be poor choices as bases for an octave scale.

Compare the following divisions of the standard patterns, all of which produce identical fourths: (a) d.Mixolydian/a.Hypolydian: T/TTS/TTS; (b) d.Lydian/a.Dorian: STT/T/STT; (c) d.Phrygian/a.Phrygian: TST/T/TST; (d) d.Hypophrygian/a.Hypodorian: T/STT/STT; (e) d.Dorian/a.Lydian: TTS/T/TTS; (f) d.Hypolydian/a.Mixolydian: STT/STT/T; (g) d.Hypodorian/a.Hypophrygian: T/TST/TST. The most aesthetically appealing of these patterns are, unsurprisingly, those that allow for an independent whole tone in the middle, between the two fourths: the Lydian/Dorian or Phrygian options.

The relationships among the eleven octave strings are as interesting as the distribution and number of octave types within them. The seven model bands

are separated from one another by intervals of fifths as follows: (a) the beginning tone of every octave in the γ-string makes a fifth with the beginning tone of a parallel octave in the Δ-string, so that the γ-string and the Δ-string (model octave string) are separated by the sesquialter interval; (b) the beginning tones of the octaves in the Δ-string make fifths with the beginning tones of parallel octaves in γ-string and δ-string, so that the Δ-string and the δ-string and the Δ-string and γ-string are separated by the sesquialter interval; (c) the beginning tones of every octave in the δ-string make fifths with the beginning tones of parallel octaves in the Δ-string and α-string, so that the δ-string and the α-string and δ-string and Δ-string are separated by the sesquialter interval; (d) the beginning tones of every octave in the α-string make fifths with parallel octaves in the δ-string and ε-string, so that the α-string and the δ-string and the α-string and ε-string are separated by the sesquialter interval; (e) the beginning tones of every octave in the ε-string make fifths with the beginning tones of parallel octave in the α-string and β-string, so that the ε-string and the α-string and the ε-string and β-string are separated by the sesquialter interval; (f) the beginning tones of every octave in the β-string make fifths with the beginning tones of a parallel octaves of the ε-string and the ζ-string, so that the β-string and the ε-string and the β-string and the ζ-string are separated by the sesquialter interval; (g) the ζ-string is somewhat unique, as the beginning tone of every octave of the ζ-string makes a fifth with parallel octaves of the β-string and the CF1-string, based on chromatic factors of 1719926784. The relationship of ζ-string to the first chromatic string makes ζ-string a punctuation mark indicating the limit of the primary doubles phenomena associated with the elements of the model octave scale.

The doubles based on chromatic factors and chromatic nonfactors of 1719926784, complete the phenomena associated with the separation of bands by fifths. The beginning tones of the first chain of octaves based on chromatic factors of 1719926784 (CF1) are separated from the beginning tones of parallel octaves in ζ-string and CF2 by fifths, so that CF1 is separated from both the latter chains by the sesquialter interval. The beginning tones of the octaves of CF2, the second chain based on chromatic factors of 1719926784, are a fifth removed from the beginning tones of parallel octaves in CF1 and also in CNF1, the first string based on chromatic nonfactors of 1719926784, so that CF2 is separated from each of these chains by the sesquialter interval. The beginning tones of octaves in CNF1 are a fifth removed from beginning tones of parallel octaves in CF2 and CNF2; thus CNF1 is removed from each of the latter chains by the sesquialter interval. The beginning tones of CNF2 are removed by fifths from the beginning tones of parallel octaves in CNF1, so that CNF2 is removed from CNF1 by the sesquialter interval. CNF2, like γ-string, has a sesquialter relation-

ship to only one other chain. It marks the end of the progression of chains, just as γ-string (rather than Δ-string) marks the beginning. The special relationship of γ-string to the monad and Δ-string to the triad will become apparent, below.

The seven strings of doubles, based on elements of the model octave and the four chromatic strings of doubles, also produce important triple relationships with each other in the same order regulating their sesquialter relationships; however, the triple relationships involve not parallel octaves, but octaves removed up or down related chains by one from a parallel octave.

Note that, like the model octave chain, all of the other octave chains founded upon it begin to degenerate after the introduction of four chromatic elements, although some of them take longer to degenerate than others. It is impossible to compare the degeneration of the chromatic chains in the same way because they are wholly independent of the model octave. Taking into account both the original accustomed patterns and all variations possible for each octave chain beyond its original pattern, it is clear that that the eleven octave chains are not uniform in length. The model octave string (Δ-string) is eleven octaves long, including such variations. The α-string is fourteen octaves long. The β-string is sixteen octaves long. The γ-string is nine octaves long. The δ-string is twelve octaves long. The ε-string is fifteen octaves long. The ζ-string is fifteen octaves long. Among the strings based on chromatic numbers CF1 is fourteen octaves long; CF2 is twelve octaves long; CNF1 is eleven octaves long; and CNF2 is nine octaves long.

The shortest chains of those grounded in the model octave, γ-string and Δ-string, are also the stablest chains, i.e., the chains least susceptible to pattern proliferation due to the invasion of chromatic elements. Their relative purity may have something to do with the special relationships that they bear, respectively, to the monad and triad. One can discern the monad/triad relationship of γ-string and Δ-string upon a simple inspection of Figure 14, "Monad/Triad Relationship of γ -string and Δ-string."

In Figure 14 the relations among the seven diatonic octave strings based on elements of the original model octave sequence are manifest. Figure 14 is furnished with exemplifying numbers to render the representation less abstract. Numbers along left-oriented diagonals are doubles of each other, marking the starting points of contiguous octaves. Numbers along right-oriented diagonals are triples of each other. A number in any horizontal row × 3/2 yields the next number in the row to the right. The labeling of the elements, as belonging to particular octave strings, makes evident the intervals between continuously overlapping octave strings beginning from different elements of the original model octave scale. The numbers set forth do not by any means exhaust those included in the contiguous octave bands α–ζ and Δ. The number underlined is a chromatic factor.

128 (pre-γ)

256(pre-γ) 384(Δ)

512 (γ) 768 (Δ) 1152 (δ)

1024(γ) 1536(Δ) 2304(δ) 3456(α)

2048(γ) 3072 (Δ) 4608 (δ) 6912(α) 10368 (ε)

4096 (γ) 6144 (Δ) 9216 (δ) 13824 (α) 20736 (ε) 31104 (β)

8192 (γ) 12288 (Δ) 18432 (δ) 27648 (α) 41472 (ε) 62208 (β) 93312 (ζ)

16384 (γ) 24576 (Δ) 36864 (δ) 55296 (α) 82944 (ε) 124416 (β) 186624 (ζ) 279976

FIGURE 14 Monad/triad relationship of γ-string and Δ-string

Note that the proto elements of γ-string arise before the elements of any string. The Δ-string is the first string for which an element actually emerges, but that element arises in a triadic relationship to the proto-elements of γ-string. The γ-string, then has a monadic character, vis-a-vis all other strings, but especially in relation to Δ-string; and the Δ-string has a triadic character. The relationships represented above are also discernible from Table 24 entitled "Horizontal Chart of Numbers Filling in the Rows of the Timaeus Crantor Matrix."

Among the chromatic strings, CNF1 and CNF2 bear the same relationship to one another, in terms of length, as Δ-string and γ-string bear to one another. They are also comparably stable in their patterns and much less ambiguous than all strings except for Δ-string and γ-string. Such observations underscore the notion that the ultimate result of the chromatic invasion of the primary octave string is a new monad/triad pair, but in inverse order, such that CNF2 is the monad and CNF1 is the triad. It is fitting punctuation to mark the proper end of the *Timaeus* number matrix. One could possibly read the *Timaeus* number set backward, so that selected model scale numbers played the role of "chromatic" elements from the standpoint of a primary octave disdiapason based on CNF1. If one started with that primary octave and took each step in that octave as the basis for a chain of its own, calculating octaves, in reverse order (because they anticipate preceding numbers of the set), then one could, perhaps, find all of the chains in Table 39 in reverse order. This study has not undertaken the latter exercise, as it would become unduly lengthy. Readers are invited to follow through to see whether or not the hunch is correct.

Observe that, just as one can calculate strings of doubles based on model scale numbers and chromatic and nonchromatic factors of 1719926784, respec-

TABLE 40 Patterns of chromatic invasion of diatonic strings

Δ-string (11)	**α-string (14)**	**β-string (16)**	**γ-string (9)**
0:0	0:0	0:0	0:0
1:1	1:2	1:2	1:2
2:3	2:3	2:3	2:3
3:4	3:4	4:4	4:4
4:4	4:4	4:4	4:__
4:__	4:4	4:4	
	4:4	4:4	
		4:4	
δ-string (12)	**ε-string (15)**	**ζ-string (15)**	
0:1	0:1	0:1	
1:2	1:2	1:2	
2:3	3:3	3:3	
4:4	4:4	4:4	
4:4	4:4	4:4	
4:4	4:4	4:4	
	4:4	4:4	
	4:__	4:__	

tively, one can also calculate strings of triples (musical twelfths). Close study of the triples is not germane to unpacking the remainder of Plato's text from 35 A–36 D; so they are not treated here. Other scholars are invited to assume the task.

It is useful to examine the pattern of chromatic invasion of complete disdiapasonic intervals of the nonchromatic strings of doubles as a means of continuing to flesh out their interrelationships for possible relevance to Plato's later text. In Table 40, above, the octaves that can be completely articulated within each of the strings are paired in disdiapasons and separated by colons with the number of chromatic elements appearing in each octave on either side of the colon. Octaves counted include all completely articulable octaves, namely: (a) octaves articulable only in the primary octave for the chain; (b) ambiguous octaves articulable both in the primary pattern and one or more additional patterns made possible by the creep of chromatic elements into the chain; and, also, octaves, both ambiguous and not, that are articulable in some

octave pattern enabled by chromatic invasion, after the primary sequence has disappeared.[2]

Aside from string lengths, which vary, the octave strings, fall into two obvious subsets, comprising a tetrad and a triad. The four strings of the tetrad, i.e., Δ-string, α-string, β-string, and γ-string, each begin with a disdiapason, wholly undistorted by chromatic elements. An internal differentiation, calling to mind by the relationship of the strings, both the triple ratio (3:1) and the sesquitertian ratio (4:3), occurs in the circumstance that, thereafter, only α-string, β-string, and γ-string share the same pattern of invasion through the third complete disdiapason. A further distinction of α-string from β-string and γ-string, calling to mind by the relationship of the strings, both the double ratio (2:1) and the sesquialter ratio (3:2), occurs in the circumstance that β-string, and γ-string, alone, share the same pattern of invasion through a fourth disdiapason. None of the strings of the tetrad is, in the end, identical to any other of the strings in its whole pattern of invasion.

The three strings of the triad, i.e., δ-string, ε-string, and ζ-string each begin with a disdiapason having one perfect octave and a second octave invaded by one chromatic element. They share the same pattern of chromatic invasion through the next disdiapason; and, then, δ-string differentiates, while ε-string, and ζ-string continue to share the same pattern; nonetheless, δ-string, though shorter than the other two strings, is, on the whole, closely similar to ε-string and ζ-string, in its pattern of invasion. The internal distinction calls to mind, by the relationship of the strings, the double ratio (2:1) and sesquialter ratio (3:2).

One might make a preliminary hypothesis that the division into clear subsets of three and four among the nonchromatic strings on the basis of their patterns of chromatic invasion might have something to do with the subsets of bands that Plato creates at 36 D, but this would be an error, as the study will make plain below. The theory must be mentioned just so that it can later be discounted. The entire issue of chromatic distortion will, in fact, be moot by 36 D, because, as the study will show below, Plato separated all of the chromatic elements from the rest of the fabric of numbers, at 36 B, and then isolated them at 36 C and D. The part of the fabric containing the chains based on the model octave becomes the internal circle of difference that Plato divided at 36 D to get seven circles.

Having run ahead of itself a bit, by necessity, to give the reader a glimpse of how the character of the octave chains is pertinent to Plato's further analysis,

2 For example, there are eleven octaves counted for Δ-string, rather than ten, because the eleventh octave is not coincident with an ascending Lydian or descending Dorian pattern. An ascending Hypolydian or descending Mixolydian diatonic scale is still possible.

TABLE 41 Descending patterns of diatonic and chromatic octave species

Octave species	Diatonic	Chromatic
Mixolydian	TTTSTTS	T [TS]SS [TS]SS
Lydian	STTTSTT	S T [TS]SS [TS]S
Phrygian	TSTTTST	SS T [TS]SS [TS]
Dorian	TTSTTTS	[TS]SS T [TS]SS
Hypolydian	STTSTTT	S [TS]SS T [TS]S
Hypophrygian	TSTTSTT	SS [TS]SS T [TS]
Hypodorian	TTSTTST	[TS]SS [TS]SS T

this study can turn to the relevance of the *Timaeus* numbers for deriving chromatic and enharmonic scales in the seven *tonoi* prominent in the ancient Greek music of Plato's day. Interested readers cognizant of the chromatic and enharmonic scale data may wish to experiment with designing alternative UPS systems using the method demonstrated in Chapter 5 for derivation of the *Timaeus* UPS.

2 The Many Chromatic *Timaeus* Scales Hidden in the Fabric

The patterns for the chromatic scales of ancient Greek music in the seven major *tonoi* are set forth in Table 41, above. They are displayed alongside their diatonic counterparts, so that the reader may understand the manner in which the two genera are related. [TS] indicates an undivided interval of a tone plus a *leimma* of size 256/243.

To discover the chromatic scales and patterns in all seven major *tonoi* among the numbers of the *Timaeus* set, the analyst need only find the first number (note that this first number will not itself be a chromatic number) allowing him to construct a scale having a chromatic pattern. The sequence 3456–6912 is the first instance of a chromatic octave among the *Timaeus* numbers. It occurs within α-string, already analyzed for its diatonic sequences in the immediately preceding section of this chapter. It is a descending Dorian octave with no corresponding ascending reciprocal. Indeed, unlike the descending diatonic scales, none of the descending chromatic scale patterns has an ascending reciprocal and *vice versa*. To find all chromatic scales one must separately calculate descending and ascending patterns.

Once he has discovered the first chromatic sequence running between 3456 and 6912, the analyst should take the double of the first element in the scale and determine whether or not he can find a chromatic pattern or chromatic patterns continuing from that starting point among the *Timaeus* numbers in a chain contiguous with the first octave. He can then proceed to follow the line of doubles progressing from the first element to make the same determination for each of them until chromatic scales are no longer possible. Then he can go to the second element of the very first chromatic scale and follow through the chain of doubles proceeding from it to determine whether or not any of those numbers can serve as the first element of a chromatic sequence. He should continue until he has exhausted the procedure for the last element of the very first chromatic scale. He will find, by following the procedure suggested, that he will have successively found all chromatic octave chains running in α-string, β-string, γ-string, δ-string, ε-string, and ζ-string, and Δ-string. The Δ-string, in other words, is the last string not originating from chromatic elements to include chromatic octave chains.

The analyst should next proceed to consider chromatic scales starting from chromatic numbers. Beginning with 2187, he should determine whether or not a chromatic pattern or patterns can originate from it. Then he should take the double of 2187 until the chain of doubles proceeding from 2187 is exhausted as a set of possibilities for first elements of a chromatic octave in a contiguous chain (or chains) of such octaves. He should repeat the same procedure for 6561 and then for 19683 and 59049. Appendix 8 to this study, *Chromatic Scale Tables*, sets forth the derivation of the chromatic chains of the *Timaeus*, manifesting ascending and descending chromatic octave patterns in all seven *tonoi*. It constitutes the basis for the synopses of chromatic phenomena provided in Table 42, below.

Table 42 below summarizes the chromatic scale behavior for α-ζ strings, Δ-string, and CF1-CNF2, already analyzed in the immediately preceding section of this chapter for their diatonic octave chains.

TABLE 42 Chromatic octave chains of the *Timaeus* in the order of their emergence

α-string chromatic behavior: After an exclusive descending Dorian disdiapason starting from 3456 (fourth octave of the diatonic sequences of α-string [432 × 2 × 2 × 2 = 3456]), the third chromatic octave introduces a d.Mixolydian and a.Hypophrygian sequence, retaining previous chromatic patterns. The fourth chromatic octave has the same range of sequences as the third. The fifth chromatic sequence introduces a.Mixolydian and Lydian chromatic octaves, retaining previous chromatic patterns. The sixth and seventh chromatic octaves have the same possibilities as the fifth. In the

TABLE 42 Chromatic octave chains of the *Timaeus* in the order of their emergence (*cont.*)

eighth chromatic octave, the d.Dorian and a.Hypophrygian patterns give out, while the other patterns mentioned remain. In the ninth octave, the d./a.Mixolydian patterns die, leaving only the d.Hypophrygian and a.Lydian patterns. The tenth octave has the same structure as the ninth. It is the last chromatic octave of the string. Thus, chromatic α-string is ten octaves long, ending at 3538944; the diatonic possibilities for the string continue for one more octave before being exhausted What began as a d.Dorian pattern ends as a d.Hypophrygian or a.Lydian pattern. There are thirty-four chromatic octaves (34) in α-string, in all, nineteen (19) of which are descending and fifteen (15) of which are ascending as follows: six (6) d.Mixolydian octaves, seven (7) d.Dorian octaves, six (6) d.Hypophrygian octaves, four (4) a.Mixolydian octaves, six (6) a.Lydian octaves, and five (5) a.Hypophrygian octaves.

β-string chromatic behavior: After an exclusive a.Dorian disdiapason starting from 3888 (fourth octave of the diatonic sequences of β-string [$486 \times 2 \times 2 \times 2 = 3888$]), the third chromatic octave introduces d.Phrygian, d.Hypodorian, and a.Hypodorian sequences, while retaining the a.Dorian pattern. The fourth chromatic octave introduces a.Phrygian and d.Dorian sequences, while retaining earlier patterns. The fifth, sixth, and seventh chromatic octaves allow the same possibilities as the fourth. In the eighth chromatic octave, the d.Phrygian and a.Dorian patterns give out. In the ninth chromatic octave, the d.Hypodorian, a.Phrygian, and a.Hypodorian patterns give out, leaving only the d.Dorian pattern. The tenth octave, like the ninth, allows only a d.Dorian pattern. There are no chromatic octaves beyond it. Thus, chromatic β-string is ten octaves long, ending with 3981312, three octaves before β-string diatonic possibilities are exhausted. What began as an a.Dorian pattern ends as a d.Dorian pattern. There are thirty-six (36) chromatic octaves in β-string, eighteen (18) of which are descending and eighteen (18) of which are ascending as follows: five (5) d.Phrygian octaves, seven (7) d.Dorian octaves, six (6) d.Hypodorian octaves, five (5) a.Phrygian octaves, seven (7) a.Dorian octaves, and six (6) a.Hypodorian octaves.

γ-string chromatic behavior: The first chromatic octave, beginning from 4096 (fourth octave of the diatonic sequences of γ-string [$512 \times 2 \times 2 \times 2 = 4096$]), exhibits d.Phrygian and a.Hypolydian patterns. The second chromatic octave is like the first. The third chromatic octave adds d.Lydian and a.Hypodorian patterns to the former possibilities. The fourth, fifth, and sixth chromatic octaves are like the third. The chromatic octave chain ends abruptly with completion of the sixth chromatic octave. The chromatic octave chain of γ-string is, thus, six octaves long, ending at 262144, along with diatonic possibilities of the string. The γ-string is paired with CNF2-string in having the shortest chromatic octave string. The γ-string is a relatively stable string, retaining its original

TABLE 42 Chromatic octave chains of the *Timaeus* in the order of their emergence (*cont.*)

patterns over its entire length, even as it adds additional ones, with the two beginning patterns sharing equal dominance in the string. There are twenty (20) chromatic octaves in γ-string, ten (10) of which are descending and ten (10) of which are ascending as follows: four (4) d.Lydian octaves, six (6) d.Phrygian octaves, six (6) a.Hypolydian octaves and four (4) a.Hypodorian octaves.

δ-string chromatic behavior: After an exclusive d.Mixolydian chromatic disdiapason beginning from 4608 (fourth octave of the diatonic chain of the δ-string [576 × 2 × 2 × 2 =4608]), the third chromatic octave adds d.Hypophrygian, a.Mixolydian, and a.Lydian patterns, while retaining the d.Mixolydian pattern. The fourth octave adds d.Phrygian, d.Hypolydian, a.Dorian, and a.Hypolydian possibilities, while retaining all previous patterns. The fifth and sixth chromatic octaves are like the fourth. In the seventh chromatic octave, the d./a.Mixolydian patterns drop out, but all other patterns remain. The eighth chromatic octave is like the seventh. In the ninth chromatic octave, the d.Hypolydian, d.Hypophrygian, a.Lydian, and a.Dorian patterns drop out, leaving only the d.Phrygian and a.Hypolydian patterns. The ninth chromatic octave is the last one. The δ-string chromatic chain is, then, nine octaves long, ending with 2359296, concurrently with the diatonic possibilities for δ-string. What began as a d.Mixolydian pattern ends ambiguously in d.Phrygian and a.Hypolydian patterns. There are forty-four (44) chromatic octaves in δ-string, twenty-three (23) of which are descending and twenty-one (21) of which are ascending, as follows: six (6) d.Mixolydian octaves, six (6) d.Phrygian octaves, five (5) d.Hypolydian octaves, six (6) d.Hypophrygian octaves, four (4) a.Mixolydian octaves, six (6) a.Lydian octaves, five (5) a.Dorian octaves, and six (6) a.Hypolydian octaves.

ε-string chromatic behavior: After d./a.Hypodorian patterns beginning from 5184 in the first chromatic octave (fourth octave of the diatonic sequences of ε-string [648 × 2 × 2 × 2 = 5184]), the second chromatic octave introduces d.Dorian and a.Phrygian patterns, while retaining the original sequences. The third chromatic octave is like the second. The fourth chromatic octave introduces d.Mixolydian and a.Hypophrygian patterns and retains all previous sequences. The fifth and sixth chromatic octaves are like the fourth. The seventh chromatic octave loses the d.Hypodorian, a.Hypodorian, and a.Phrygian patterns. The eighth chromatic octave is like the seventh. The ninth chromatic octave loses the d.Dorian and a.Hypophrygian patterns, leaving only the d.Mixolydian pattern. There are no chromatic octaves after the ninth; thus, the chromatic octave chain of ε-string is nine octaves long, ending at 2654208 (three octaves before the end of the ε-string diatonic chain). What began ambiguously in d. and a.Hypodorian chromatic patterns ends as a d.Mixolydian chromatic pattern. There are

TABLE 42 Chromatic octave chains of the *Timaeus* in the order of their emergence (*cont.*)

thirty-five (35) chromatic octaves in ε-string, nineteen (19) of which are descending and sixteen (16) of which are ascending, as follows: six (6) d.Mixolydian octaves, seven (7) d.Dorian octaves, six (6) d.Hypodorian octaves, five (5) a.Phrygian octaves, five (5) a.Hypophrygian octaves, and six (6) a.Hypodorian octaves.

ζ-string chromatic behavior: After an exclusive a.Mixolydian pattern in the first chromatic octave starting from 5832 (fourth octave of the diatonic sequences of ζ-string [729 × 2 × 2 × 2 = 5832]), the second chromatic octave introduces d.Hypophrygian and a.Dorian patterns, while retaining the original a.Mixolydian pattern. The third chromatic octave is like the second. The fourth chromatic octave introduces d.Phrygian, d.Hypodorian, and a.Hypodorian patterns, while retaining all previous patterns. The fifth and sixth chromatic octaves are like the fourth. The seventh chromatic octave loses the d.Hypophrygian and a.Mixolydian patterns. The eighth chromatic octave is like the seventh. The ninth chromatic octave loses the d.Phrygian and a.Dorian patterns, leaving only the d./a.Hypodorian patterns. There are no chromatic octaves after the ninth one; the ζ-string chromatic octave chain is, thus, nine octaves long, ending at 2985984, three octaves before the diatonic possibilities for ζ-string are exhausted. What began as an a.Mixolydian pattern ends ambiguously in the d./a.Hypodorian patterns. There are thirty-five (35) chromatic octaves in ζ-string, sixteen (16) of which are descending and nineteen (19) of which are ascending, as follows: five (5) d.Phrygian octaves, five (5) d.Hypophrygian octaves , six (6) d.Hypodorian octaves, six (6) a.Mixolydian octaves, seven (7) a.Dorian octaves, and six (6) a.Hypodorian octaves.

Δ-string chromatic behavior: After a first chromatic octave starting from 6144 (fourth octave of the diatonic sequences of Δ-string [768 × 2 × 2 × 2 = 6144]), that displays d.Hypophrygian and a.Lydian patterns, the second chromatic octave introduces d.Phrygian, d.Hypolydian and a.Hypolydian patterns, while retaining the original patterns. The third chromatic octave adds an a.Dorian pattern, while retaining all of the previous patterns. The fourth chromatic octave introduces d.Lydian and a.Hypodorian patterns, while retaining all previous possibilities. The fifth and sixth chromatic octaves are like the fourth one. The seventh chromatic octave loses the d.Hypolydian, d.Hypophrygian, a.Lydian and a.Dorian patterns, leaving the d.Lydian, d.Phrygian, a.Hypolydian, and a.Hypodorian patterns. There are no chromatic octaves beyond the seventh one. The Δ-string chromatic chain is, thus, just seven octaves long, ending at 786432 concurrently with the diatonic possibilities of Δ-string. It is one of the shorter chromatic octave chains. What began ambiguously in d.Hypophrygian and a.Lydian patterns ends even more ambiguously in d.Lydian, d.Phrygian, a.Hypolydian, and a.Hypodorian patterns. There are forty-one (41) chromatic octaves in Δ-string, twenty-one (21) of which are

TABLE 42 Chromatic octave chains of the *Timaeus* in the order of their emergence (*cont.*)

descending and twenty (20) of which are ascending, as follows: four (4) d.Lydian octaves, six (6) d.Phrygian octaves, five (5) d.Hypolydian octaves, six (6) d.Hypophrygian octaves, six (6) a.Lydian octaves, four (4) a.Dorian octaves, six (6) a.Hypolydian octaves, four (4) a.Hypodorian octaves. Observe that of all the strings beginning from model scale numbers, Δ-string is the latest to give rise to chromatic octaves.

CF1 chromatic string behavior: After a disdiapason beginning from 4374 (second octave of the CF1 diatonic chain), exhibiting in both octaves d.Lydian and a.Hypophrygian patterns, the third chromatic octave introduces d./a.Mixolydian patterns, while retaining the previous patterns. The fourth chromatic octave introduces d.Hypophrygian and a.Dorian chromatic patterns, while retaining all previous patterns. The fifth and sixth chromatic octaves are like the fourth. The seventh chromatic octave loses the d.Mixolydian, d.Lydian, and a.Hypophrygian patterns. The eighth chromatic octave loses the a.Mixolydian pattern. The ninth chromatic octave loses the d.Hypophrygian pattern, retaining only the a.Dorian pattern. The tenth chromatic octave is like the ninth. There are no chromatic octaves after the tenth one. Chromatic CF1 is, thus ten octaves long, ending at 4478976, three octaves before the diatonic possibilities for CF1 are exhausted. What began ambiguously in d.Lydian and a.Hypophrygian patterns ends unambiguously in an a.Dorian pattern. There are thirty-four (34) chromatic octaves in the CF1-string, fifteen (15) of which are descending and nineteen (19) of which are ascending, as follows: four (4) d.Mixolydian octaves, six (6) d.Lydian octaves, five (5) d.Hypophrygian octaves, six (6) a.Mixolydian octaves, seven (7) a.Dorian octaves, and six (6) a.Hypophrygian octaves.

CF2 chromatic string behavior: After a first chromatic octave running from 6561 (concurrently with the first diatonic octave of CF2) that displays d.Hypolydian and a.Phrygian patterns, a second chromatic octave introduces d.Lydian, d.Dorian, a.Hypophrygian, and a.Hypolydian chromatic patterns, while retaining the original patterns. The third chromatic octave is like the second. The fourth chromatic octave introduces d./a.Mixolydian patterns. The fifth and sixth chromatic octaves are like the fourth. The seventh chromatic octave loses the d.Dorian, d.Hypolydian, a.Mixolydian, and a.Hypophrygian patterns. The eighth chromatic octave loses the d.Mixolydian, d.Lydian, and a.Hypophrygian patterns, retaining only the a.Mixolydian pattern. The ninth chromatic octave is like the eighth. There are no chromatic octaves beyond the ninth. The chromatic CF2 string is, thus, nine octaves long, ending at 3359232 (three octaves prior to the end of the diatonic possibilities for CF2 string). What began ambiguously in d.Hypolydian and a.Phrygian patterns ends unambiguously in an a.Mixolydian pattern. There are forty-four (44) chromatic octaves in CF2-string, twenty-one (21) of which are

TABLE 42 Chromatic octave chains of the *Timaeus* in the order of their emergence (*cont.*)

descending and twenty-three (23) of which are ascending, as follows: four (4) d.Mixolydian octaves, six (6) d.Lydian octaves, five (5) d.Dorian octaves, six (6) d.Hypolydian octaves, six (6) a.Mixolydian octaves, six (6) a.Phrygian octaves, five (5) a.Hypolydian octaves, and six (6) a.Hypophrygian octaves.

CNF1-string chromatic behavior: After a first chromatic octave beginning from 19683 (concurrently with the diatonic possibilities for CNF1-string) that displays d.Hypolydian, d.Hypodorian, a.Phrygian, and a.Lydian chromatic patterns, a second chromatic octave introduces d.Lydian, d.Dorian, a.Hypolydian, and a.Hypophrygian patterns, while retaining all previous possibilities. The third and fourth chromatic octaves are like the second. The fifth chromatic octaves loses the d.Hypodorian and a.Lydian patterns. The sixth chromatic octave is like the fifth. The seventh chromatic octave loses the d.Dorian, d.Hypolydian, d.Hypophrygian, ascending Phrygian, and a.Hypolydian chromatic patterns, leaving only the d.Lydian and a.Hypophrygian patterns. There are no chromatic octaves after the seventh one. Chromatic CNF1-string is, thus, only seven octaves long, making it like Δ-string, in being one of the shorter chromatic octave chains. It ends at 2519424, three octaves prior to the exhaustion of the diatonic possibilities for CNF1 string. What began ambiguously in d.Hypolydian, d.Hypodorian, a.Phrygian, and a.Lydian chromatic patterns ends somewhat less ambiguously in the d.Lydian and a.Hypophrygian patterns. There are forty-two (42) chromatic octaves in CNF1-string, twenty-one (21) of which are descending and twenty-one (21) of which are ascending, as follows: six (6) d.Lydian octaves, five (5) d.Dorian octaves, six (6) d.Hypolydian octaves, four (4) d.Hypodorian octaves, four (4) a.Lydian octaves, six (6) a.Phrygian octaves, five (5) a.Hypolydian octaves, and six (6) a.Hypophrygian octaves.

CNF2-string chromatic behavior: After four chromatic octaves beginning from 59049 (concurrently with the diatonic octave possibilities) that display d.Hypodorian, d.Hypolydian, a.Lydian, and a.Phrygian chromatic patterns, a fifth chromatic octave loses the d.Hypodorian and a.Lydian patterns, while retaining the d.Hypolydian and a.Phrygian patterns. A sixth chromatic octave is like the fifth. There are no chromatic scale possibilities past the sixth octave. The CNF2-string chromatic octaves extend, then, for only six octaves, making the CNF2-string, along with γ-string, the shortest among the chromatic strings. It ends at 3779136, three octaves prior to the end of the diatonic possibilities for CNF2-string. There are twenty (20) chromatic octaves in the CNF2-string, ten (10) of which are ascending and ten (10) of which are descending as follows: six (6) d.Hypolydian octaves, four (4) d.Hypodorian octaves, four (4) a.Lydian octaves, and six (6) a.Phrygian octaves.

Note that the first descending Phrygian scale to emerge begins from 4096 in γ-string. The first descending Lydian scale to emerge begins from 4374 in CF1. The first descending Mixolydian scale to emerge begins from 4608 in δ-string. The first descending Hypodorian scale to emerge begins from 5184 in ε-string. The first descending Hypophrygian scale begins from 6144 in Δ-string. The first descending Hypolydian scale to emerge begins from 6561 in CF2.

The first ascending Dorian scale begins from 3888 in β-string. The first ascending Phrygian scale begins from 6561 in CF2. The first ascending Lydian scale emerges from 6144 in Δ-string. The first ascending Mixolydian scale emerges from 5832 in ζ-string. The first ascending Hypodorian scale to emerge begins from 5184 in ε-string. The first ascending Hypophrygian scale begins from 4374 in string CF1. The first ascending Hypolydian scale to emerge begins from 4096 in γ-string.

There are, in total, 385 chromatic octaves in the *Timaeus* set, many fewer than the total diatonic possibilities, as follows: twenty-six (26) ascending and twenty-six (26) descending Mixolydian octaves, for a total of fifty-two (52) Mixolydian octaves; twenty-six (26) ascending and twenty-six (26) descending Lydian octaves for a total of fifty-two (52) Lydian octaves; twenty-eight (28) ascending and twenty-eight (28) descending Phrygian octaves for a total of fifty-six (56) Phrygian octaves; thirty (30) ascending and thirty-one (31) descending Dorian octaves for a total of sixty-one (61) Dorian octaves; twenty-eight (28) ascending Hypolydian and twenty-eight (28) descending Hypolydian octaves for a total of fifty-six (56) Hypolydian octaves; twenty-eight (28) ascending and twenty-eight (28) descending Hypophrygian octaves for a total of fifty-six (56) Hypophrygian octaves; and twenty-six (26) ascending and twenty-six (26) descending Hypodorian octaves for a total of fifty-two (52) Hypodorian octaves. Clearly, the most prevalent octave type is the Dorian, followed in co-equal importance by the Phrygian, Hypolydian, Hypophrygian, and Hypodorian octave varieties and, then, by the slightly less prevalent Mixolydian and Lydian octaves in equal number. The comparative quantitative data can be summarized in tabular form, as in Table 43.

One might observe by inspecting Table 26, the "Fully Annotated Table of *Timaeus* Numbers Arranged in Order from Least to Greatest," that some of the TS sequences relevant to constructing chromatic scales among *Timaeus* numbers introduce some interesting possibilities for coloratura within the scales. They can actually be divided into trihemitones, that is three semitones consisting of some combination of S and S′ intervals, representing, respectively, the *leimma* (256/243) and *apotomē* (2187/2048).[3] The intervals that can be thus divided are listed in Table 44 below.

3 Levin, *Manual*, 174 (chromatic fourth is comprised by an "incomposite" trihemitone and two

If one examines these intervals, using Table 26, one finds that four types of trihemitone sequences occur among the *Timaeus* numbers: SS′S, S′SS′, S′SS, and SSS′. The *apotomē* (S′) of size 2187/2048, slightly larger than the *leimma* (S) of size 256/243, is precisely the interval that emerges when chromatic numbers split a whole tone between elements native to the repeating descending Dorian or ascending Lydian diatonic octave scale primary to the *Timaeus*.[4] The S′ *apotomē* is important because it is the key to deriving the chromatic genus of scale from the *Timaeus* numbers. All *tonoi* of the chromatic genus employ semitones of both sizes as Gustav Reese appears to have recognized.[5] Further, three of the four trihemitone types (not S′S S′ because it is larger than the TS component of the chromatic scale) are accounted for among the chromatic scales of the *Timaeus*, although not all of the specific intervals listed in Table 44, below, find a place in a chromatic scale articulated by *Timaeus* numbers. A catalogue of all chromatic scales articulated by the *Timaeus* numbers that use trihemitone sequences is provided in Appendix 9, "Specification of Trihemitones and Chromatic Scales in Which They Manifest."

Two hundred fifty-six (256) of three hundred eighty-five (385) chromatic octaves in the *Timaeus* number set, in total, use trihemitones. One hundred twenty-nine (129) do not. Of octaves using trihemitones, forty-six (46) use two. All scales using two trihemitones exhibit only the S S′ S variety. Note that the

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semitones); Proclus *in Timaeum* 3.2.168.19–20 (chromatic fourth is comprised by a trihemitone [minor third] and two semitones), 180.1–5 (and n. 268), 180.26–181.1, 188.20–26, 189.10–11, 190.26–27; cf. Reese, *Music*, 23 (refers to a sesquitone, rather than a trihemitone, though it is the same composite interval space, plus two semitones, "more or less," as comprising the chromatic fourth).

Note that Levin's characterization of the structure of the chromatic fourth as an "incomposite trihemitone" plus two semitones is not inaccurate, so much as incomplete. This study shows that some of the "trihemitone" intervals can be broken down into three actual components allowing a possibility of coloratura that would not otherwise be possible. Reese's preference for the term "sesquitone," without a qualification whether or not it is composite or incomposite, allows for the greatest flexibility. Sometimes the element under discussion can be broken down, as has just been noted, into three components, and the remainder of the time, as this study also proves, into two, a tone and a *leimma*, permitting a different species of coloratura. The existence of these two kinds of variations, in addition simply to considering the interval in an undivided way, certainly makes the chromatic genus much more interesting than it would otherwise be. The current study also underscores Reese's "more or less" characterization of the two semitones following the "sesquitone" in the chromatic fourth, since it shows that they are not the same size, but consist in the *leimma* and the *apotomē*.

4 See Proclus *in Timaeum* 3.2.180.1–5 (and n. 268), 180.26–181.1, 188.20–26 for confirmation that the divided whole tone (*epogdos*) is comprised of the *leimma* (256/243 ratio) and the *apotomē* (2187/2048 ratio).

5 Reese, *Music*, 23.

TABLE 43 Inventory of *Timaeus* chromatic scales

Octave type ⇒	Mixolyd.		Lyd.		Phryg.		Dor.		Hypo-lyd.		Hypo-phryg.		Hypo-dor.		String totals d./a.		String totals combined	
String ⇓	d.	a.	d.	a.	d.	a.	d.	a.	d.	a.	d.	a.	d.	a.	d.	a.	No.	Dominant octave type in string
Δ			4	6	6			4	5	6	6			4	21	20	41	Hypolyd.
α	6	4		6			7				6	5			19	15	34	Hypophryg.
β					5	5	7	7					6	6	18	18	36	Dorian
γ			4		6					6				4	10	10	20	Phryg./Hypolyd.
δ	6	4		6	6			5	5	6	6				23	21	44	Hypolyd.
ε	6					5	7					5	6	6	19	16	35	Hypodor.
ζ		6			5			7			5		6	6	16	19	35	Hypodor.
CF1	4	6	6					7			5	6			15	19	34	Hypophryg.
CF2	4	6	6			6	5		6	5		6			21	23	44	Hypolyd.
CNF1			6	4		6	5		6	5		6	4		21	21	42	Hypolyd.
CNF2				4		6			6				4		10	10	20	Phryg./Hypolyd.
d.totals	26		26		28		31		28		28		26		193			
a.totals		26		26		28		30		28		28		26		192		
Comb.	52		52		56		61		56		56		52				385	
Octave type most commonly dominating individual strings																		**Hypolydian**
Overall dominant octave type																		**Dorian**

proportion of octaves using trihemitones to those that do not, 256/129, approximates the double ratio; the proportion of total chromatic octaves to those using trihemitones 385/256 approximates the 3/2 ratio; and the proportion of total chromatic octaves to those not using trihemitones 385/129 approximates the triple ratio. See Appendix 9 for additional detail concerning trihemitones.

TABLE 44 List of all intervals divisible into trihemitones

Column A: 1–24		Column B: 25–48	
1.	1944–2304	25.	118098–139968
2.	3888–4608	26.	124416–147456
3.	5832–6912	27.	131072–157464
4.	7776–9216	28.	139968–165888
5.	11664–13824	29.	186624–221184
6.	15552–18432	30.	196608–236196
7.	16384–19683	31.	209952–248832
8.	17496–20736	32.	221184–262144
9.	23328–27648	33.	236196–279936
10.	31104–36864	34.	248832–294912
11.	32768–39366	35.	262144–314928
12.	34992–41472	36.	279936–331776
13.	46656–55296	37.	373248–442368
14.	49152–59049	38.	393216–472392
15.	52488–62208	39.	419904–497664
16.	55296–65536	40.	559872–663552
17.	59049–69984	41.	746496–884736
18.	62208–73728	42.	786432–944784
19.	65536–78732	43.	839808–995328
20.	69984–82944	44.	1119744–1327104
21.	93312–110592	45.	1679616–1990656
22.	98304–118098	46.	2239488–2654208
23.	104976–124416	47.	3359232–3981312
24.	110592–131072	48.	6718464–7962624

3 The Many Enharmonic *Timaeus* Scales Hidden in the Fabric

3.1 *Preliminary Observations*

The patterns for the enharmonic scales of ancient Greek music in the seven major *tonoi* are set forth in Table 45 below. They are displayed alongside their diatonic and chromatic counterparts, so that the reader may understand the manner in which the three genera are related.

Enharmonic scales of all octave species can be derived from the *Timaeus* set, as Appendix 10 to this study, "Enharmonic Scale Tables," illustrates. The

TABLE 45 List of descending patterns for the seven octave species within the three genera

Octave species	Diatonic	Chromatic	Quarter tone enharmonic	Semitone enharmonic
Mixolydian	TTTSTTS	T[TS]SS[TS]SS	T[TT]QQ[TT]QQ	T[TT]S[TT]S
Lydian	STTTSTT	ST[TS]SS[TS]S	QT[TT]QQ[TT]Q	Not applicable
Phrygian	TSTTTST	SST[TS]SS[TS]	QQT[TT]QQ[TT]	ST[TT]S[TT]
Dorian	TTSTTTS	[TS]SST[TS]SS	[TT]QQT[TT]QQ	[TT]ST[TT]S
Hypolydian	STTSTTT	S[TS]SST[TS]S	Q[TT]QQT[TT]Q	Not applicable
Hypophrygian	TSTTSTT	SS[TS]SST[TS]	QQ[TT]QQT[TT]	S[TT]ST[TT]
Hypodorian	TTSTTST	[TS]SS[TS]SST	[TT]QQ[TT]QQT	[TT]S[TT]ST

derivation of the quarter tone enharmonic scales integrally depends, in the vast majority of cases, upon a kind of split of the 256/243 semitone interval. All semitone enharmonic scales of the *Timaeus* use only the latter semitone; and the quarter tone enharmonic scales of the *Timaeus*, except for those of the Lydian and Hypolydian variety, are derived solely from the semitone enharmonic scales using the 256/243 semitone.

The method for deriving the quarter tones of *Timaean* enharmonic scales is evident upon consideration of the following sequence derivable from the *Timaeus* set.

768 TT 972 QT 998 QT 1024 T 1152 TT 1458 QT 1497 QT 1536

The above sequence, wherein TT represents "ditone" expresses a descending Dorian quarter tone enharmonic scale. The numbers in orange do not occur in the *Timaeus* set but are calculated from numbers in the *Timaeus* set. They are bounds of quarter tone intervals. The quarter tones are achieved by two splits, one of the 256/243 semitone interval between 972 and 1024 and the other of the semitone interval between 1458 and 1536. One adds the two numbers in each case and divides the sum in half. The splits result in quarter tones of two different sizes. One quarter tone is approximately represented by the decimal 1.026748971 (998/972 and 1497/1458) and the other is approximately represented by the decimal 1.026052104 (1024/998 and 1536/1497).

Note from Table 45, that the ascending and descending Lydian and Hypolydian enharmonic scales come only in the quarter tone variety, as a quarter tone both begins and ends each of these scales. For the most part, beginning and

ending quarter tones are reached, in the case of these scale varieties, by the very method articulated in the previous paragraph. The semitone split for the initiating quarter tone is the 256/243 interval preceding the scale following the quarter tone; and the semitone split for the ending quarter tone is the 256/243 interval following the element preceding the quarter tone. The first quarter tone (following increasing numerical order) is of the (approximately) 1.026052104 variety. The second is of the (approximately) 1.026748971 variety. The two quarter tones in the middle of each of these scales comprise one of each type. Although the two quarter tones vary in size, they are identical to the thousandths place. Whole number ratios approaching the same approximate values are 1764734496/1719926784 (1.026052104—so, accurate to the ninth place following the decimal only) and 1765933056/1719926784 (1.02674897119342—accurate thus, to the fourteenth place following the decimal). Observe that 1719926784 is the last element of the *Timaeus* set.

In rare instances, the ending quarter tones for scales of the Lydian and Hypolydian type are computed in a manner different from the one described above. Sometimes, at the end of an enharmonic octave string, a tone, rather than a semitone is split to reach the appropriate quarter tone value. The size of this quarter tone is 1.03125. It is computed by adding the endpoints of the relevant tone, splitting the sum, adding the result to the endpoint with the lower value, splitting the sum, and computing the ratio of the result of the split to the endpoint with the lower value. A whole number ratio expressing the size of this quarter tone is 1773674496/1719926784.

Although the theoretical possibility exists of reaching quarter tone values by the split of a tone, rather than a 256/243 semitone interval, at the beginning, as well as at the end, of some Lydian and Hypolydian enharmonic sequences of the *Timaeus* set, the necessity of such a split never actually presents itself, in the number matrix, for those sequences. It is interesting, however, to do the math to find the value that would result, if such an instance actually did occur. In the case of an initial quarter tone, the quarter tone would be of the size $1.0\overline{285714}$. It would be computed by adding the endpoints of the relevant tone, splitting the sum, adding the result to the endpoint with the higher value, splitting that sum, and computing the ratio of the higher endpoint to the result of the split. A whole number ratio expressing this value accurately to the ninth digit following the decimal point is 1769067549/1719926784 (1.028571428).

Two additional quarter tones are theoretically possible based on splits of the *apotomē*; but they do not occur in the enharmonic scales of the *Timaeus* set, as Appendix 10 makes abundantly clear. These theoretical quarter tones are also uniform and slightly different in size. The split value of the *apotomē* makes a ratio of approximately **1.033935547** with its lower boundary and approximately

1.032821724 with its higher boundary. For example, consider the 2187/2048 *apotomē* interval between 73728 and 78732. Splitting the interval leads to the sequence:

73728 76230 78732

The ratio of 76230/73728 is approximately represented by the decimal 1.033935547 and the ratio of 78732/76230 is approximately represented by the decimal 1.032821724. Whole number ratios approximating theoretical quarter tones deriving from splits of the *apotomē* are, respectively, 1778293438/1719926784 (1.03393554—accurate to the eighth digit following the decimal) and 1776377744/1719926784 (1.03282172—accurate to the eighth digit following the decimal point).

To recapitulate, only the *leimma* of interval size 256/243 and the tone of size 9/8 figure into finding the quarter tones of the quarter tone enharmonic scales of the *Timaeus*. In the case of the "split" of the *leimma*, the relevant quarter tone sizes are 1.026748971 and 1.026052104. In the rare cases, sometimes relevant to Lydian and Hypolydian sequences, in which the tone is the basis for computing a quarter tone, the size of the relevant quarter tone is 1.03125. The only semitone interval occurring in *Timaean* semitone enharmonic scales is the *leimma*, a circumstance that should be unsurprising since the enharmonic scales are derived from diatonic sequences.

3.2 *A Note on the Obvious*

Before presenting the enharmonic data, a note on the obvious seems to be in order. Clearly the quarter tones produced by "splitting" semitones in the manner exemplified above for the enharmonic scales of the *Timaeus* are not true quarter tones. A true split of either the ancient *leimma* or *apotomē* is, in fact, mathematically inexpressible as a ratio of whole numbers, as, in fact, is the split of the ancient whole tone into semitones or the octave into six equal parts.

The sixth root of two, an irrational number, expresses a true sixth part of an octave because, when raised to the sixth power, it equals two. Where a sixth part of an octave meets this measure, the octave truly does have no more and no fewer than six exactly equal parts of the measure indicated. A very close decimal approximation to the sixth root of two, the size of a true whole tone, that can be expressed as a ratio of whole numbers (1930552540/1719926784) is 1.122462048. Whether ancient thinkers ever appreciated this particular ratio is doubtful. In any case, raised to the sixth power, the value 1.122462048 yields a value extremely close to two.

The ratio 9/8, the value that the ancients assigned to the whole tone, raised to the sixth power, is actually somewhat larger than an octave. Modern methods verify that the calculation $9/8 \times 9/8 \times 9/8 \times 9/8 \times 9/8 \times 9/8$ is greater than two. A close decimal approximation to the calculation is 2.02728653. Unsurprisingly, then, the measure of the true whole tone is somewhat smaller that the value 1.125 corresponding to 9/8.

Because the sixth root of two, properly expresses the size of a true whole tone, the square root of the sixth root of two, another irrational number, expresses the exact size of each of the two exactly equal semitones of the whole tone and the square root of the square root of the sixth root of two, that is, the fourth root of the sixth root of two, expresses the exact size of the two exactly equal true quarter tones of the whole tone.

A very close decimal approximation to the irrational square root of the sixth root of two (approximately represented by the value 1.122462048), is 1.059463094. This approximate value, too, can be expressed as a ratio of whole numbers, utilizing 1719926784: 1822198952/1719926784. This very close approximation of the semitone of the true whole tone is somewhat larger than 1.053497942, the approximate decimal value approaching 256/243, the ancient measure for the semitone of the diatonic scale. It is substantially smaller than 1.067871094, the approximate decimal value approaching 2187/2048, the other semitone, the *apotomē*, relevant to ancient Greek music. The *apotomē* occurs in the *Timaeus* set whenever an interval of the size 256/243 falls between two numbers separated from each other by an interval of the size 9/8.

The square root of 9/8, the size of a semitone that would properly correspond to the erroneous notion that 9/8 is the size of a true whole tone, is approximately represented by the decimal value, 1.060660172. The latter value is somewhat smaller than the ratio 2187/2048 (1.067871094) and substantially larger than the ratio 256/243 (1.053497942). The size of a quarter tone based on the value of the ancient 9/8 tone would be the fourth root of the sixth root of $(9/8)^6$ or simply the fourth root of 9/8 or approximately 1.02988357195356. This value is approximately expressed by the whole number ratio 1771324340/1719926784 (1.02988357—accurate to the eighth digit following the decimal). This value does not, of course, express a true quarter tone. Twenty-four of these quarter tones exceed the size of the octave by a considerable amount as they compute to 2.02728652954102. Nonetheless, this computation comes closer to the value of the true semitone than any semitone that actually occurs in the enharmonic scales of Appendix 10.

The square root of the square root of the sixth root of two, in other words, the fourth root of the sixth root of two, is the irrational number expressing

the exact size of a true quarter tone of a true whole tone of an octave. It is only approximately represented by the decimal value **1.029302237**. A whole number ratio approaching this value is 1770324486/1719926784. The value indicated is larger than either of the two "quarter tones" achieved by the method of splitting the *leimma* (256/243) illustrated above and smaller than either of the two theoretical "quarter tones" achieved by the method of splitting the *apotomē*.

It is interesting that the last element of the *Timaeus* set can be, as noted above, the denominator of whole number ratios, closely approximating (a) the exact size of a true quarter tone (square root of the square root of the sixth root of two, approximately 1.029302237), namely, 1770324486/1719926784; (b) the true split of the ancient *leimma* (square root of 256/243 or approximately 1.026400479), namely 1765333675/1719926784; (c) the true split of the ancient *apotomē* (square root of 2187/2048 or, approximately, 1.033378485), namely, 1777335335/1719926784; (d) the two ancient quarter tones associated with the *leimma* actually used in the construction of enharmonic scales (approximately represented by the decimals 1.026748971 and 1.026052104), namely, 1765933056/1719926784 and 1764734495/1719926784; and (e) the two ancient quarter tones associated with the *apotomē* (approximately represented by the decimals 1.033935547 and 1.032821724), namely, 1778293440/1719926784 and 1776377746/1719926784.

It is also the denominator of whole number ratios closely expressing all relevant differences between semitone values that can be compared. For example, the value 1719926784 can be the denominator of a whole number ratio closely approaching the approximate difference between the size of a true semitone and the ancient value for the semitone, 256/243 (1.005662234 because 1.005662234 × 1.053497942 = 1.059463094). The calculation 1729665412/1719926784 = 1.005662234. The latter whole number ratio can be further reduced to 864832706/859963392 and 432416353/429981696. Similarly, the difference between the size of the true semitone and the size of the *apotomē* (2187/2048) (.99212639 because .99212639 × 1.067871094 = 1.059463094) is closely approximated by the whole number ratio 1706384751/1719926784.

The number 1719926784 can even be a foundation to express the difference, known as the *comma*, between the two ancient semitone values of *leimma* and *apotomē* (1.013643265 because 1.013643265 × 1.053497942 = 1.067871094). This difference is closely approximated by the whole number ratio 1743392201/1719926784, a number that does not appear to be reducible. Note that the ancients themselves had an independent whole number ratio 531441/524288, using smaller whole numbers, that expresses the *comma* just as accurately as the ratio just offered. They claimed that it represented the *comma* in the low-

est terms.[6] Neither the numerator nor the denominator is an element of the *Timaeus* number set proposed by this study.

The number 1719926784 is, in fact, the first and only number of the *Timaeus* set that allows the construction of whole number ratios closely expressing both the true whole tone, semitone, and quarter tone measures, as well as all of the differences between and among the ancient and *bona fide* values for the whole tone and semitone. The mathematical reason, though the ancient Greeks would not have appreciated the point, may be that it is the first number within the set possessing a number of digits equal to the number of digits required for decimal values closely approximating the measures sought. It could not, in any case, be an accident that the last number of the *Timaeus* set is the key to the solution of ancient *conundrums* associated with the sizes of various intervals in the musical scale. Plato's *Timaeus* offers, *inter alia*, a means of asymptotically approaching irrational values, within relevant ranges, to seekers sufficiently persistent and astute to find them.

The sizes of the whole numbers in the ratios needed to achieve close approximations to actually irrational whole tone, semitone, and quarter tone measures should elucidate why the ancients were unable to construct practically feasible musical scales using them. Even if they had the mathematical sophistication to arrive at such whole number ratios (as their own approximation to the size of the *comma* indicates they might have had), the values were too large to be practically adapted to the production of music. The string lengths needed to express the values for a true whole tone and true semitone were potentially infinitely large, truly cosmic in scope, depending upon the degree of accuracy one sought, or else infinitesimally divisible from an ancient perspective. Perhaps an insight that cosmic lengths would be required to solve the problem of the equal division of the octave is really at the bottom of the ancient usage of the diatonic scale to symbolize cosmic extension. A similar problem attended the expression of the size of a true quarter tone; so the ancients took a more practical approach than the construction of cosmic lengths to reach quarter tone values. They achieved a rough, but acceptable measure for their own purposes by "splitting" the semitone in the modes suggested.

3.3 *The Enharmonic Phenomena*

As in the case of the chromatic scales, there are no reciprocal ascending and descending enharmonic scales. Ascending and descending patterns must be separately calculated to make a complete inventory of enharmonic scales. Only fully semitone and fully quarter tone enharmonic scales are counted in Tables

6 Proclus *in Timaeum* 3.2.184.2.

TABLE 46 Basic inventory of semitone and quartertone enharmonic types

String ⇒ Scale type ⇓	Δ	α	β	γ	δ	ε	ζ	CF1	CF2	CNF1	CNF2	Totals
ST	38	47	58	18	47	56	56	47	47	38	18	470
QT	34	42	52	16	42	50	50	42	42	34	16	420
Assoc. QT	34	34	34	18	32	32	34	34	34	32	16	334
Total	106	123	144	52	121	138	140	123	123	104	50	1224

46, 47, and 48, summarizing the enharmonic data, not mixed varieties in which only one semitone can be split. A hybrid enharmonic using a mixed scale of this kind would be purely speculative.

The Lydian and Hypolydian quarter tone enharmonic scale varieties begin a quarter tone before or after the starting tone numbers for the diatonic octaves forming the basis for the enharmonic scales. They do not, therefore, cover the same octave range exactly. They are called "associated scales," in Tables 46, above, and 47 and 48, below, in connection with those closely related octaves. See Appendix 10 for further detail.

There are 1224 enharmonic scales running concurrently with diatonic octaves in the eleven octave strings, Δ-CNF2, of the *Timaeus*. They include ascending and descending enharmonic varieties of all seven relevant octave species. Table 46, above, classifies them, initially, just by semitone, quarter tone, or associated quarter tone (always either Lydian or Hypolydian) enharmonic scale type. A further breakdown of string detail according to the octave species represented is set forth, below, in Table 47.

Overall, the Dorian and Phrygian scale types predominate at 190 apiece. Each exhibits ninety-five (95) descending and ninety-five (95) ascending scales. These are followed by the Mixolydian, Hypodorian, and Hypophrygian species at 170 scales apiece. Each of these types displays eighty-five (85) ascending and eighty-five (85) descending scales. The Lydian and Hypolydian quarter tone enharmonic scales are fewest at 167 scales each, but all of these scales are quarter tone scales, and none of them is a semitone enharmonic. The Lydian displays a moderate preponderance of descending scales (80a./87d.), while the Hypolydian displays a moderate preponderance of ascending scales (87a/80d). The total number of ascending and descending scales is equal at 612 apiece.

Table 48 summarizes the enharmonic scale behavior of the eleven octave strings Δ-ζ and CF1-CNF2. In Table 48 and the catalogue of scales in Appendix 10, ST indicates "semitone" and QT indicates "quarter tone."

TABLE 47 Breakdown of octave species represented in the enharmonic scales of each string

String ⇒ Scale type ⇓	Δ	α	β	γ	δ	ε	ζ	CF1	CF2	CNF1	CNF2	Totals
a.Mixo. ST							9	9	9	9	9	45
a.Mixo. QT							8	8	8	8	8	40
d.Mixo. ST	9	9		9	9	9						45
d.Mixo. QT	8	8		8	8	8						40
Subtotals:	17	17		17	17	17	17	17	17	17	17	170
a.Lyd. ST												
a.Lyd. QT	8	8	8		8	8	8	8	8	8	8	80
d.Lyd. ST												
d.Lyd. QT	9	9	9	9	8	8	9	9	9	8		87
Subtotals:	17	17	17	9	16	16	17	17	17	16	8	167
a.Dor. ST			10				10	10	10	10		50
a.Dor. QT			9				9	9	9	9		45
d.Dor. ST	10	10	10		10	10						50
d.Dor. QT	9	9	9		9	9						45
Subtotals:	19	19	38		19	19	19	19	19	19		190
a.Hypolyd. ST												
a.Hypolyd. QT	9	9	9	9	8	8	9	9	9	8		87
d.Hypolyd. ST												
d.Hypolyd. QT	8	8	8		8	8	8	8	8	8	8	80
Subtotals:	17	17	17	9	16	16	17	17	17	16	8	167
a.Hypodor. ST			9			9	9	9	9			45
a.Hypodor. QT			8			8	8	8	8			40
d.Hypodor. ST		9	9		9	9	9					45
d.Hypodor. QT		8	8		8	8	8					40
Subtotals:		17	34		17	34	34	17	17			170
a.Phryg. ST	10	10	10		10	10						50
a.Phryg. QT	9	9	9		9	9						45
d.Phryg. ST			10				10	10	10	10		50
d.Phryg. QT			9				9	9	9	9		45
Subtotals:	19	19	38		19	19	19	19	19	19		190
a.Hypophryg. ST	9	9		9	9	9						45
a.Hypophryg. QT	8	8		8	8	8						40
d.Hypophryg. ST							9	9	9	9	9	45
d.Hypophryg. QT							8	8	8	8	8	40
Subtotals:	17	17		17	17	17	17	17	17	17	17	170
String totals	106	123	144	52	121	138	140	123	123	104	50	1224

TABLE 48 Enharmonic scale behavior of the eleven octave strings

Δ-string enharmonic behavior: The first enharmonic octaves in Δ-string begin from 384, comprising d.Dorian and a.Phrygian ST enharmonic patterns. The second enharmonic octave allows the first d.Dorian and a.Phrygian QT enharmonic patterns, while retaining all previously existing patterns. The third enharmonic octave adds the first a.Hypophrygian and d.Mixolydian ST enharmonic patterns and the first associated d.Hypolydian QT and a.Lydian QT enharmonic patterns, also retaining all preexisting patterns. The fourth enharmonic octave contributes the first d.Mixolydian and a.Hypophrygian QT enharmonic patterns, as well as the first associated d.Lydian QT and a.Hypolydian QT enharmonic patterns, while retaining all preexisting patterns, bringing Δ-string to its full complement of enharmonic octave types. The fifth through tenth enharmonic octaves are identical in their patterns to the fourth enharmonic octave. The d.Dorian, a.Phrygian, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns vanish from the eleventh enharmonic octave, leaving all other patterns intact. The twelfth enharmonic octave loses the d.Mixolydian and a.Hypophrygian enharmonic patterns, leaving only the associated d.Lydian QT and a.Hypolydian QT enharmonic patterns. The twelfth enharmonic octave ends with the 1492992 QT 1539648 quarter tone, extending approximately one octave beyond the end of the diatonic possibilities for Δ-string. No further enharmonic octaves are possible in Δ-string beyond this point. The enharmonic Δ-string is, thus, twelve octaves long. What began in d.Dorian and a.Phrygian ST patterns ends in the associated d.Lydian QT and a.Hypolydian QT enharmonic patterns. The conclusion of the enharmonic string in the associated patterns makes sense, since the diatonic scales grounding the enharmonic possibilities are no longer possible in the same octave.

In Δ-string, Phrygian (all ascending) and Dorian scales (all descending) predominate at nineteen (19) each, followed by seventeen (17) each of the Mixolydian (all descending), Lydian (8a./9d.), Hypolydian (9a./8d.) and Hypophrygian (all ascending) scale types. There is an equal number of ascending and descending scales, in all, at fifty-three (53) apiece, for a total of 106 scales. All patterns, ST, QT, and associated QT, are accommodated by this inventory. There are no Hypodorian scales.

α-string enharmonic behavior: The first enharmonic octave in α-string is a d.Hypodorian ST enharmonic octave beginning from 1728, coincident with the third α-string diatonic octave. The second enharmonic octave adds the d.Hypodorian QT enharmonic pattern and the d.Dorian and a.Phrygian ST enharmonic patterns, while retaining previously existing patterns. The third enharmonic octave adds d.Dorian and a.Phrygian QT enharmonic patterns, retaining preexisting patterns. The fourth enharmonic octave adds the d.Mixolydian and a.Hypophrygian ST enharmonic patterns, as

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

well as the first associated d.Hypolydian QT and a.Lydian QT enharmonic patterns, keeping all prior patterns intact. The fifth enharmonic octave adds the d.Mixolydian and a.Hypophrygian QT enharmonic patterns, as well as the first associated d.Lydian QT and a.Hypolydian QT enharmonic patterns. All other patterns remain. The sixth through ninth enharmonic octaves are like the fifth. The tenth enharmonic octave loses the d.Hypodorian scale types but maintains all other patterns. The eleventh enharmonic octave is like the tenth. The twelfth enharmonic octave loses the d.Dorian, a.Phrygian, associated d.Hypolydian QT, and associated a.Lydian QT scale types. The thirteenth enharmonic octave loses the d.Mixolydian and a.Hypophrygian scale types, retaining only the associated d.Lydian QT and associated a.Hypolydian QT scale types. The thirteenth enharmonic octave ends with the 13436928 QT 13856832 quarter tone, extending approximately one octave beyond the end of the diatonic possibilities for α-string. No further enharmonic octaves are possible in α-string beyond this point. The enharmonic α-string is, thus, thirteen octaves long. What began in a d.Hypodorian ST pattern ends in the associated d.Lydian QT and associated a.Hypolydian QT patterns. The conclusion of the enharmonic string in the associated patterns makes sense, since the diatonic scales grounding the enharmonic possibilities are no longer possible in the same octave.

In α-string, there is also a predominance of Dorian (all descending) and Phrygian scales (all ascending) at nineteen (19) each. The Lydian (9d./8a.), Hypolydian (8d./9a), Mixolydian (all descending), Hypodorian (all descending), and Hypophrygian (all ascending) scale types follow at seventeen (17) apiece. There is a total of seventy (70) descending and fifty-three (53) ascending scales, for a total of **123** scales. All octave species are represented in this string.

β-string enharmonic behavior: The first enharmonic octaves in β-string, d.Phrygian and a.Dorian ST enharmonic types, begin from 486, concurrently with the string's first diatonic octave. The second enharmonic octave introduces the first d.Phrygian, a.Dorian, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic octave patterns, retaining the earlier patterns. The third enharmonic octave introduces the a.Hypodorian ST enharmonic scale type, while preserving all previously existing patterns. The fourth enharmonic octave adds the a.Hypodorian QT enharmonic pattern to the set of occurrent patterns. The fifth enharmonic octave is like the fourth. The sixth enharmonic octave introduces the d.Hypodorian ST enharmonic scale type, retaining all previously occurring patterns. The seventh enharmonic octave adds the first d.Hypodorian QT, a.Phrygian ST, and d.Dorian ST enharmonic patterns, retaining all previously occurring scale types. The eighth enharmonic octave adds the first a.Phry-

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

gian QT and d.Dorian QT enharmonic patterns, allowing all others previously mentioned, as well. The ninth enharmonic octave introduces the first associated a.Lydian QT and d.Hypolydian QT enharmonic patterns while retaining all previous patterns. The tenth enharmonic octave is like the ninth. The eleventh enharmonic octave loses all d.Phrygian, a.Dorian, associated d.Lydian QT, and associated a.Hypolydian QT scale types. The twelfth enharmonic octave loses the a.Hypodorian enharmonic patterns. The thirteenth and fourteenth enharmonic octaves are like the twelfth. The d.Hypodorian enharmonic patterns vanish in the fifteenth enharmonic octave. The sixteenth enharmonic octave is like the fifteenth. The last number in an enharmonic pattern in the sixteenth enharmonic octave is 31850496, belonging to the occurrent d.Dorian and a.Phrygian patterns. The d.Hypolydian and a.Lydian enharmonic octaves associated with the sixteenth enharmonic octave end with the 30233088 QT 31041792 quarter tone. No further enharmonic octaves are possible in β-string beyond the end of the d.Dorian and a.Phrygian patterns in the sixteenth enharmonic octave. The enharmonic β-string is, thus, sixteen octaves long, ending concurrently with the β-string diatonic octaves. What began in d.Phrygian and a.Dorian ST enharmonic patterns ends in d.Dorian ST and QT, a.Phrygian ST and QT, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns.

In β-string, there is a preponderance of Dorian (19d./19a.) and Phrygian (19d./19a.) scales at thirty-eight (38) each, followed closely by the Hypodorian variety at thirty-four (34) (17d./17a.). The Lydian (9d./8a.) and Hypolydian (8d./9a.) varieties are much less numerous at seventeen (17) apiece. There is an equal number of descending and ascending scales at seventy-two (72) apiece for a total of 144 scales. There are no Mixolydian or Hypophrygian varieties.

γ-string enharmonic behavior: The first enharmonic octaves in γ-string, d.Mixolydian and a.Hypophrygian ST enharmonic types, begin from 512, concurrently with the string's first diatonic octave. The second enharmonic octave adds d.Mixolydian, a.Hypophrygian, associated d.Lydian QT and associated a.Hypolydian QT enharmonic scale types. The third through ninth enharmonic octaves are like the second enharmonic octave. In the tenth enharmonic octave the d.Mixolydian and a.Hypophrygian scale types vanish, leaving only the associated scales. The tenth enharmonic octave ends with the 497664 QT 513216 quarter tone, extending about an octave beyond the end of the γ-string diatonic chain. No further enharmonic octaves are possible in γ-string beyond this point. The enharmonic γ-string is, thus, ten octaves long. What began in d.Mixolydian and a.Hypophrygian ST enharmonic patterns ends in the associated d.Lydian QT and associated a.Hypolydian QT patterns. The conclusion of the enhar-

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

monic string in the associated patterns makes sense, since the diatonic scales grounding the enharmonic possibilities are no longer possible in the same octave.

In γ-string, there is a preponderance of Mixolydian (descending) and Hypophrygian (all ascending) varieties at seventeen (17) each, followed by the much less numerous Lydian (all descending) and Hypolydian (all ascending) varieties at nine (9) each. There is an equal number of ascending and descending scales at twenty-six (26) apiece for a total of fifty-two (52) scales. There are no Dorian, Hypodorian, or Phrygian scales.

δ-string enharmonic behavior: The first enharmonic octave in δ-string, a d.Hypodorian ST enharmonic type, begins from 576, concurrently with the string's first diatonic octave. The second enharmonic octave adds the d.Dorian ST, a.Phrygian ST, and d.Hypodorian QT enharmonic patterns, while retaining the original pattern. The third enharmonic octave introduces the d.Dorian and a.Phrygian QT enharmonic types, while preserving all previous patterns. The fourth enharmonic octave adds the d.Mixolydian ST, a.Hypophrygian ST, associated d.Hypolydian QT and associated a.Lydian QT enharmonic patterns, retaining all other previously occurring patterns. The fifth enharmonic octave adds the d.Mixolydian, a.Hypophrygian, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic patterns. The sixth through ninth enharmonic octaves are like the fifth enharmonic octave. The tenth enharmonic octave loses the d.Hypodorian enharmonic patterns. The eleventh enharmonic octave is like the tenth. The twelfth enharmonic octave loses the d.Dorian, a.Phrygian, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic scale types, leaving only the d.Mixolydian QT, a.Hypophrygian QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic scale types. The last number in an enharmonic pattern of the twelfth enharmonic octave is 2359296, belonging to the occurrent d.Mixolydian QT and a.Hypophrygian QT scale types. The d.Lydian and a.Hypolydian QT enharmonic octaves associated with the twelfth enharmonic octave end with the 2239488 QT 2299392 quarter tone. No further enharmonic octaves are possible in δ-string beyond the end of the d.Mixolydian and a.Hypophrygian patterns in the twelfth enharmonic octave. The enharmonic δ-string is, thus, twelve octaves long, ending concurrently with the δ-string diatonic octaves. What began in a d.Hypodorian ST pattern ends in d.Mixolydian ST and QT, a.Hypophrygian ST and QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic patterns.

In δ-string, there is a preponderance of Dorian (all descending) and Phrygian (all ascending) scales at nineteen (19) apiece, followed by the Mixolydian (all descending), Hypodorian (all descending), and Hypophrygian (all ascending) varieties at seventeen

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

(17) apiece, and then the Lydian (8d./8a.), Hypolydian (8d./8a.) at 16 apiece. There are sixty-nine (69) descending scales and fifty-two (52) ascending scales for a total of 121 scales. As in α-string, all octave species are represented.

ε-string enharmonic behavior: The first enharmonic octave in ε-string, an a.Hypodorian ST enharmonic type, begins from 648, concurrently with the string's first diatonic octave. The second enharmonic octave adds the a.Hypodorian QT enharmonic scale type, while retaining the original pattern. The third enharmonic octave is like the second. The fourth enharmonic octave adds the d.Hypodorian ST enharmonic pattern, retaining all previous patterns, while the fifth introduces the d.Hypodorian QT, the d.Dorian ST, and the a.Phrygian ST enharmonic types. The sixth enharmonic octave adds the first d.Dorian and a.Phrygian QT enharmonic types, retaining all previous patterns. The seventh enharmonic octave adds d.Mixolydian ST, a.Hypophrygian ST, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic scales to the existing ongoing repertoire of ε-string patterns. The eighth enharmonic octave adds the d.Mixolydian QT, a.Hypophrygian QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic patterns. The ninth enharmonic octave is like the eighth. The tenth enharmonic octave loses the a.Hypodorian enharmonic scale type, retaining all other scale types. The eleventh and twelfth enharmonic octaves are like the tenth. The thirteenth enharmonic octave loses the d.Hypodorian enharmonic scale type, retaining all others that have not previously dropped out. The fourteenth enharmonic octave is like the thirteenth. The fifteenth enharmonic octave loses the d.Dorian, a.Phrygian, associated d.Hypolydian QT, and associated a.Lydian QT scale types, retaining only the d.Mixolydian, a.Hypophrygian, associated d.Lydian QT, and associated a.Hypolydian QT scale types. No further enharmonic octaves are possible in ε-string beyond the end of the d.Mixolydian and a.Hypophrygian patterns in the fifteenth enharmonic octave. The enharmonic ε-string is, thus, fifteen octaves long, ending concurrently with the ε-string diatonic octaves. What began in an a.Hypodorian ST pattern ends in d.Mixolydian ST and QT, a.Hypophrygian ST and QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic patterns.

In ε-string, Hypodorian (17d./17a.) scales preponderate at thirty-four (34), followed by the Dorian (all descending) and Phrygian (all ascending) scale varieties at nineteen (19) each, the Mixolydian (all descending) and Hypophrygian (all ascending) at seventeen (17) each, and the Lydian (8d./8a.) and Hypolydian (8d./8a.) at sixteen (16) each. There is an equal number of descending and ascending scales at sixty-nine (69) each for a total of 138. All octave species are represented.

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

ζ-string enharmonic behavior: The first enharmonic octaves in ζ-string, a.Mixolydian and d.Hypophrygian ST enharmonic types, begin from 729, concurrently with the string's first diatonic octave. The second enharmonic octave introduces a.Mixolydian QT, d.Hypophrygian QT, d.Phrygian ST, a.Dorian ST, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic scale types, retaining all previous patterns. The third enharmonic octave adds the d.Phrygian QT, a.Dorian QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic patterns, while preserving all previous patterns. The fourth enharmonic octave adds the a.Hypodorian ST enharmonic pattern. The fifth enharmonic octave introduces the a.Hypodorian QT enharmonic scale type, while retaining all previous patterns. The sixth enharmonic octave is like the fifth. The seventh enharmonic octave adds the d.Hypodorian ST enharmonic scale type, retaining all previous patterns, while the eighth, also retaining all previous patterns, introduces the first d.Hypodorian QT scale type. The ninth enharmonic octave is like the eighth. The tenth enharmonic octave loses the d.Hypophrygian, a.Mixolydian, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns, retaining all others. The eleventh enharmonic octave is like the tenth. The twelfth enharmonic octave loses the d.Phrygian, a.Dorian, associated d.Lydian QT, and associated a.Hypolydian QT scale types, retaining all that have not dropped out to that point. The thirteenth enharmonic octave loses the a.Hypodorian enharmonic pattern, retaining only the d.Hypodorian pattern. The fourteenth and fifteenth enharmonic octaves are like the thirteenth. No enharmonic octaves are possible beyond 23887872, the end of the fifteenth octave. Accordingly, the ζ-string enharmonic octave chain is fifteen octaves long, ending concurrently with the diatonic possibilities for ζ-string. What began in a.Mixolydian and d.Hypophrygian ST enharmonic types ends in the d.Hypodorian ST and QT patterns.

In ζ-string, Hypodorian (17d./17a.) scales preponderate at thirty-four (34), followed by Dorian (all ascending) and Phrygian (all descending) scales at nineteen (19) each, and then Mixolydian (all ascending), Lydian (9d./8a.) Hypolydian (8d./9a.), and Hypophrygian (all descending) at seventeen (17) each. There is an equal number of descending and ascending scales at seventy (70) each for a total of 140 scales. All octave species are represented.

CF1-string enharmonic behavior: The first enharmonic octaves in CF1-string, d.Hypophrygian and a.Mixolydian ST enharmonic types, begin from 2187, concurrently with the string's first diatonic octave. The second enharmonic octave introduces the d.Hypophrygian QT, a.Mixolydian QT, d.Phrygian ST, a.Dorian ST, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns, while retaining the original ones.

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

The third enharmonic octave adds the a.Phrygian QT, d.Dorian QT, associated d.Lydian QT, and associated a.Hypolydian QT, while preserving all previous patterns. The fourth enharmonic octave adds the a.Hypodorian ST enharmonic pattern, while retaining all previously occurring patterns. The fifth enharmonic octave adds the first a.Hypodorian QT enharmonic pattern, preserving all others. The sixth through ninth enharmonic octaves are like the fifth. The tenth enharmonic octave loses the d.Hypophrygian, a.Mixolydian, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic scale types, while preserving all other patterns. The eleventh enharmonic octave is like the tenth. The twelfth enharmonic octave loses the d.Phrygian, a.Dorian, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic scale types, leaving only the a.Hypodorian ST and QT patterns. No enharmonic octaves are possible beyond 8957952, the end of the twelfth octave. The CF1-string enharmonic octave chain is, thus, twelve octaves longs, ending one octave prior to the cessation of diatonic possibilities for the string. What begin in d.Hypophrygian and a.Mixolydian ST enharmonic octaves ends in the a.Hypodorian ST and QT enharmonic patterns.

In CF1-string, Dorian (all ascending) and Phrygian (all descending) scales predominate at nineteen (19) apiece, followed by the Mixolydian (all ascending), Lydian (9d./8a.), Hypolydian (8d./9a.), Hypodorian (all ascending), and Hypophrygian (all descending) scale varieties at seventeen (17) apiece. There are fifty-three (53) descending scales and seventy (70) ascending scales for a total of 123. All octave species are represented. Note that the inventory is opposite α-string vis-à-vis the proportion of ascending and descending scales.

CF2-string enharmonic behavior: The first enharmonic octaves in CF2-string, d.Hypophrygian and a.Mixolydian ST enharmonic types, begin from 6561, concurrently with the string's first diatonic octave. The second enharmonic octave introduces the d.Hypophrygian QT, a.Mixolydian QT, d.Phrygian ST, a.Dorian ST, associated d.Hypolydian QT, associated a.Lydian QT enharmonic patterns, while retaining the original scale types. The third enharmonic octave adds the d.Phrygian QT, a.Dorian QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic patterns, retaining all previous patterns. The fourth enharmonic octave introduces the a.Hypodorian ST enharmonic pattern, while preserving all previously occurring patterns. The fifth enharmonic octave adds the a.Hypodorian QT enharmonic scale type, while preserving all previous patterns. The sixth through ninth enharmonic octaves are like the fifth. The tenth enharmonic octave loses the d.Hypophrygian, a.Mixolydian, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic scale types. The eleventh enharmonic octave is like the tenth. The twelfth enharmonic octave loses the associated d.Lydian

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

QT, associated a.Hypolydian QT, d.Phrygian, and a.Dorian enharmonic scale types, leaving only the a.Hypodorian ST and QT enharmonic patterns. No enharmonic octaves are possible beyond 26873856, the end of the twelfth octave. The CF2-string enharmonic octave chain is, thus, twelve octaves longs, ending concurrently with the diatonic possibilities for the string. What begin in d.Hypophrygian and a.Mixolydian ST enharmonic octaves ends in the a.Hypodorian ST and QT enharmonic patterns.

In CF2-string, Dorian (all ascending) and Phrygian (all descending) scale varieties predominate at (nineteen) 19 each, followed by the Mixolydian (all ascending), Lydian (9d./8a.), Hypolydian (8d./9a.), Hypodorian (all ascending), and Hypophrygian (all descending) varieties at (seventeen) 17 each. There are fifty-three (53) descending and seventy (70) ascending scales for a total of **123**. CF2-string mirrors CF1-string in this way and, like CF1-string, stands in contrast to α-string vis-à-vis the proportion of ascending and descending scales. All octave species are represented.

CNF1-string enharmonic behavior: The first enharmonic octaves in CNF1-string, d.Hypophrygian and a.Mixolydian ST enharmonic types, begin from 19683, concurrently with the string's first diatonic octave. The second enharmonic octave introduces the d.Hypophrygian QT, a.Mixolydian QT, d.Phrygian ST, a.Dorian ST, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns, while retaining the original patterns. The third enharmonic octave adds the d.Phrygian QT, a.Dorian QT, associated d.Lydian QT, and associated a.Hypolydian QT enharmonic scale types while retaining all previous patterns. The fourth through ninth enharmonic octaves are like the third. The tenth enharmonic octave loses the d.Hypophrygian, a.Mixolydian, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns, while retaining all other possibilities. The eleventh enharmonic octave loses the associated d.Lydian QT and associated a.Hypolydian QT enharmonic scale types, leaving only the d.Phrygian and a.Dorian QT and ST enharmonic patterns. No enharmonic octaves are possible beyond 40310784, the end of the eleventh octave. The CNF1-string enharmonic octave chain is, thus, eleven octaves longs, ending concurrently with the diatonic possibilities for the string. What begin in d.Hypophrygian and a.Mixolydian ST enharmonic octave patterns ends in the d.Phrygian and a.Dorian QT and ST enharmonic octave patterns.

In CNF1-string, the Dorian (all ascending) and Phrygian (all descending) scale varieties predominate at nineteen (19) apiece, followed by the Mixolydian (all ascending) and Hypophrygian (all descending) scale types at seventeen (17) apiece, and then the Lydian (8d./8a.) and Hypolydian (8d./8a.) scale varieties at sixteen (16) apiece. There is

TABLE 48 Enharmonic scale behavior of the eleven octave strings (*cont.*)

an equal number of ascending and descending scales at fifty-two (52) each, for a total of one hundred four (104) scales. There are no Hypodorian scale varieties.

CNF2-string enharmonic behavior: The first enharmonic octaves in CNF2-string, d.Hypophrygian and a.Mixolydian ST enharmonic types, begin from 59049, concurrently with the string's first diatonic octave. The second enharmonic octave introduces the d.Hypophrygian QT, a.Mixolydian QT, associated d.Hypolydian QT, and associated a.Lydian QT enharmonic patterns. The third through the ninth enharmonic octaves are like the second. No enharmonic octaves are possible beyond the 30233088 QT 31041792 quarter tone at the end of the ninth octave. The CNF2-string enharmonic octave chain is, thus, nine octaves longs, ending one quarter tone beyond the end of the diatonic possibilities for the string. What begin in d.Hypophrygian and a.Mixolydian ST enharmonic octave patterns ends in d.Hypophrygian and a.Mixolydian ST and QT enharmonic octave patterns, together with associated d.Hypolydian QT and a.Lydian QT enharmonic octave types.

In CNF2-string, Mixolydian (all ascending) and Hypophrygian (all descending) scale varieties predominate at seventeen (17) each, followed by the Lydian (all ascending) and Hypolydian (all descending) varieties at (eight) 8 each. There is an equal number of ascending and descending scales at twenty-five (25) each for a total of fifty (50) scales. There are no Dorian, Hypolydian, Hypodorian, or Phrygian, enharmonic varieties.

With the description of the enharmonic strings of the *Timaeus* set, the derivation of all relevant octave scale phenomena allowed by the number matrix is complete. This study has demonstrated that 660 diatonic octaves, 385 chromatic octaves, and 1224 enharmonic octaves can be articulated from the *Timaeus* number set. Despite the unexpected proliferation of enharmonic octaves, these cannot be understood as primary because they are possible only upon the foundation of the diatonic octaves. Further, if one counts only the semitone enharmonic scales as truly native to the set, then the number of enharmonic octaves is many fewer than the diatonic octaves at 470; and there are no Lydian or Hypolydian types. The chromatic octaves are not primary, not only because they are less numerous than the other types, but because they arise as distortions of diatonic types. The diatonic octave is most fairly understood as the prominent octave type of the *Timaeus*.

Ascending and descending scales exist for the most part in almost equal prominence for all genera. They are equal for the diatonic and enharmonic

types. There is one more descending scale than ascending scale among the chromatic octaves. The Dorian chromatic octaves are imbalanced at 30a./31d. All of the other chromatic octave types exhibit equal numbers of descending and ascending scales. Only on the very narrow basis offered by the additional descending Dorian chromatic scale can one claim that the descending scale has any prominence. A fair assessment requires one to give equal weight to ascending and descending octave types.

Based on sheer numbers of articulable octave types, one can draw some conclusions about their prominence for the *Timaeus*. Among the diatonic patterns, there are, as noted above, one hundred (100) octaves focused on reciprocal Dorian/Lydian patterns; ninety (90) on reciprocal Mixolydian/Hypolydian patterns; (90) on reciprocal Hypophrygian/Hypodorian patterns; and fifty (50) on the self-reciprocal Phrygian pattern. One could legitimately multiply the Phrygian number by two to count the ascending and descending patterns, separately, instead of together. In view of the numbers, it is not possible truly to count the Dorian as the most prominent type particularly vis-à-vis the Lydian. The Phrygian is also a contender for first place, but the other types are not far behind.

Among chromatic scales, there is a slight, but clear dominance of the Dorian octave type over others. There are sixty-one (61) Dorian octaves, followed, as noted above, by the Phrygian, Hypolydian, and Hypophrygian scale types at fifty-six (56) each, and, then, the Mixolydian, Lydian, and Hypodorian at fifty-two (52) apiece.

Among the enharmonic scales Dorian and Phrygian phenomena are equally prominent, as there are 190 of each, followed by the Mixolydian, Hypodorian, and Hypophrygian at 170 each, and then the Lydian and Hypolydian at 167 each. The Dorian emerges with a slight edge over the Phrygian, it appears, when one considers the total picture. The entire derivation of the study begins, of course, with the d.Dorian/a.Lydian diatonic octave, as it is the first octave of any kind to emerge in the *Timaeus* set. It begins more specifically with the d.Dorian/a.Lydian disdiapason, also the first disdiapason in the set. The study has shown, in Chapter 5, that a unique UPS system centered on this disdiapason can be derived from the *Timaeus* set. It has prominence for the *Timaeus* only because the disdiapason on which it is based emerges first. However, what came to be known as the standard UPS system also emerges from the *Timaeus* set; and the derivation and description of the many secondary octave strings of the *Timaeus* suggests that many alternative UPS systems have a foundation in the number matrix.

This study has derived all of the scale phenomena it describes as a set of circular permutations on the first diatonic octave to emerge, the d.Dorian/a.Lydi-

an diatonic octave, beginning from 384. It thus gives credibility to the idea that the whole system of ancient Greek music can be generated from the basic d.Dorian pattern. However, it appears that one could make a similar derivation of the system as a function of circular permutations of any of the other octave patterns of the set. Just as there is no absolute starting point to follow the circumference of a circle, so there is no absolute starting point for the derivation of the possibilities for ancient Greek music.

It should be clear that each of the octave strings of the *Timaeus*, Δ-CNF2, has many dimensions. None of them is truly just one octave string, but many octave strings. Each is actually more like a fiber optic cable. The study presents each of these "cables" as though it contained overlapping fibers of continuous homogeneous elements, e.g. as though there were a discrete Dorian diatonic fiber coexisting with a discrete diatonic fiber of another type and a chromatic fiber of a discrete type and an enharmonic fiber of a discrete type. However, one could easily adopt another perspective. One could surely construct fibers within each of the cables that would string diatonic octaves of different kinds together or diatonic octaves followed by chromatic and then enharmonic octaves. No fiber that one could build from within possibilities native to any of the cables would ever be free from ambiguity, however, just because all of the octaves in each really are running all together in one flow with no definite markers except the cessation of certain types of possibilities at different points of the flow. The permutations are manifold. Imagine the instrument that could play one line of the music actually constituting the harmonic flow in any of these "cables." It would be no earthly instrument; and the music would, indeed, be either the music of the gods or a descent into chaos.

On this note, it is time to consider the relationship of the musical data to the cutting of the fabric, the making of the χ, and the construction of the cosmic orbits. These are the preoccupations of the very next segments of Plato's text, *Timaeus* 36 B–D and so are ours, in Chapter 7 of the study, as well.

CHAPTER 7

The Musical Data of the *Timaeus* Vis-à-vis the Cutting of the Fabric, the Making of the "Chi," and the Cosmic Orbits

Timaeus 36 B through 36 D reads as follows, in its entirety:

> ***Timaeus*** 36 B (con't) – D
>
> [36 B] … Hence he then cut the whole of this composition according to length, and produced two from one; and adapted middle to middle, like the form of the letter X. [C] Afterwards he bent them into a circle, connecting them, both with themselves and with each other, in such a manner that their extremities might be combined in one directly opposite to the point of their mutual intersection; and externally comprehended them in a motion revolving according to sameness, and in that which is perpetually the same. And besides this, he made one of the circles external, but the other internal; and denominated the local motion of the exterior circle, the motion of that nature which subsists according to *sameness*; but that of the interior one, the motion of the nature subsisting according to *difference*. He likewise caused the circle partaking of *sameness* to revolve laterally towards the right hand; but that which partakes of *difference* diametrically towards the left. But he conferred dominion on the circulation of that which is *same* and *similar*: for he suffered this alone [D] to remain undivided. But as to the interior circle, when he had divided it six times, and had produced seven unequal circles, each according to the interval of the double and triple; as each of them are three, he ordered the circles to proceed in a course contrary to each other:—and three of the seven interior circles he commanded to revolve with a similar swiftness; but the remaining four with a motion dissimilar to each other, and to the former three; yet so as not to desert order and proportion in their circulations.[1]

1 Plato *Tim.* (T. Taylor) 36 B–36 D.

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The passage indicates the following sequence of operations: (1) the division of the "entire compound in two along its length"; (2) the joinder of the two resulting parts to form a figure like a χ through the superimposition of the center of one part on the center of the other; (3) having formed the χ, the bending of each arm thereof back on itself (a) to connect the two ends of each arm both to each other and to ends of the other arm, at the same time involving the joined extremities in the motion of sameness and positioning them directly opposite the point of mutual intersection of the arms; (4) the inclusion of the entire construction in a motion revolving in the same place without variation; (5) the separation of the arms of the former χ into an outer circle and an inner circle; (6) the separation and definition of two different motions, an outer motion of Same proceeding "laterally towards the right hand," belonging to the exterior circle, and an inner motion of Different proceeding "diametrically towards the left," belonging to the inner circle; (7) the elevation of the movement of the Same to a uniform, dominant, and undivided status; (8) the sixfold division of the inner movement of the Different among seven unequal circles according to double and triple intervals of which there were three each, as in the original set of *Timaeus* numbers; and (9) the setting of the seven circles in contrary directions, such that a subset of three of them moved at similar speeds and a subset of four moved at speeds differing both from those of the subset of three and from those of other members of the subset of four, nonetheless preserving order and proportion in their circulations. The analysis herein follows the above steps, in the order given.

1 Division of the Material

The fabric or material to cut consists in the Crantor style matrix of *Timaeus* numbers derived in the course of this study. The matrix is presented below, in Figure 15, row by row, left-justified for convenience, with symbols substituted for the numbers to convey their special significance.

In Figure 15, one sees candidate starting tone numbers for diatonic octaves of Δ-ζ and CF1-CNF2 strings lined up in parallel columns in the fabric. As Appendices 5–7 show, complete octaves are not possible for all of these potential starting tone numbers. The ∫ symbols designate candidate starting tone numbers for diatonic octaves of CF1, the § symbols designate the same for CF2, the ∫ symbols designate potential STNs for diatonic octaves of CNF1, and the ∫ symbols designate potential STNs for diatonic octaves of CNF2. The elements of these parallel columns, as will become clear below, also have a particular relationship

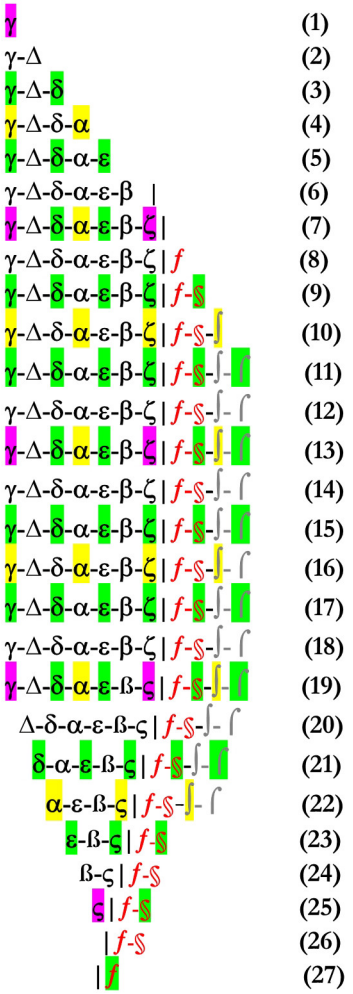

FIGURE 15 Fabric to cut

to the normative scale for the set, namely, the descending Dorian/ascending Lydian diatonic scale beginning from 384.

In Figure 15 perfect squares are highlighted in green, perfect cubes are highlighted in yellow and numbers that are both perfect squares and perfect cubes are highlighted in purple. The cut-line of the fabric is represented, in the above extension, by the succession of "|" symbols. It separates all numbers in the matrix properly belonging to an octave or fifth or other interval, in relation to the normative descending Dorian/ascending Lydian diatonic scale (including the elements marking the rise to and decline from the normative scale, as the scale repeats), from all numbers in the matrix comprising nonperiodic ele-

ments in relation to that *ordo*. It marks, in other words, the division between chromatic elements and elements bearing relation to the normative scale. No other cut-line makes such clear and obvious sense. The elements relating to the normative descending Dorian/ascending Lydian diatonic scale or to intervals in the rise to or decline from that scale are bound into a unit by numbers that are both perfect squares and perfect cubes.

It is a remarkable feature of the matrix that the scale related and chromatic elements vis-à-vis the normative scale occur in two distinct, sharply defined areas. One may verify the relations indicated by consulting and comparing, *in tandem*, the following tables provided above: (a) Table 24, the "Horizontal Chart of Numbers Filling in the Rows of the *Timaeus* Crantor Matrix"; (b) Table 26, the "Fully Annotated Table of Timaeus Numbers Arranged in Numerical Order from Least to Greatest"; and (c) Table 27, "Rise of the Diatonic Scale in the Set of Timaeus Numbers."

The matrix has yet another remarkable feature. All rows of numbers to the left of the cut-line have distinct correlations to specific steps in the normative octave scale. The first two γ symbols in Figure 15, above, represent the principles of number generating the triad from which the normative scale rises and, thereafter, all of the *mesai* of the scale, assuming an ascending Lydian order.

The **Δ**'s, arising next, represent the successive octave STN'S of the *Timaeus* number set for the normative scale (*hypate* and, as appropriate, *nete* of the next repeating octave, assuming an ascending Lydian scale) in their proper positions. They stand for octave periodicity.

The δ's represent all of the *paramesai* in the *Timaeus* number set (again assuming an ascending Lydian scale). The α's and ε's, coming next, represent, respectively, all of the *parhypates* and *trites* of the *Timaeus* number set in their proper positions, assuming an ascending Lydian scale.

The ζ's, represent all of the *paranetes* of the *Timaeus* set prior to the dominance of fifth periodicity, assuming an ascending Lydian scale, while the β's represent all of the *lichanos* elements of the *Timaeus* set before that occurrence. The ς's represent the *paranetes* after the onset of "fifth" periodicity, while the ß's, represent the *lichanos* elements after that point, assuming an ascending Lydian scale.

If one prefers to adopt the perspective of the descending Dorian, each Δ is *nete* and *hypate*, each α is *paranete*, each β[ß] is *trite*, each γ is *paramese*, each δ is *mese*, each ε is *lichanos*, and each ζ[ς] is *parhypate*. This remarkable correspondence does not mean that one can find diatonic octaves of the normative scale simply by circling through the horizontal rows of Figure 15 and Table 24, horizontally.

For example, to construct the first normative octave beginning from 384, using the number matrix comprising Figure 15 and Table 24, rather than the numbers given in increasing order, as set forth in Table 26, one begins in the second column of horizontal row number (9) on Table 24 (and Figure 15), the Δ column of Figure 15, goes up one number to horizontal row (8) and over two numbers to 432 (the second number) in the fourth column of Table 24 corresponding to the α column on Figure 15. One repeats the procedure to find 486 in the sixth column of horizontal row (7) of Table 24 (and Figure 15), corresponding to the β column of Figure 15. One then circles back to the first column of the tenth row, the γ column of Figure 15, to find 512, making the 256/243 proportion with 486.

Note that there is always a 256/243 proportion starting with horizontal rows (6) and (9) between the first element of γ column and the last element of β column three rows above. This relation obtains for rows (7) and (10) in the cases of 512 and 486. From 512, one repeats the procedure of up and over two to find 576 in δ column of horizontal row (9). One repeats it again to find 648, the next element of the scale, in ε-column of horizontal row (8) and once more to find 729 in ζ-column of horizontal row (7). One finds 768, the ending 256/243 interval, by proceeding from the last element of ζ-column in the seventh horizontal row to find 768 in the second column, the Δ-column of horizontal row (10).

Just as in the cases of the β and γ columns, a special relation obtains between the Δ and ζ columns, vis-à-vis the 256/243 ratio. The second element of Δ column, beginning with row ten, makes a 256/243 ratio with the last element of ζ column three rows above it. Thus, one finds diatonic octaves in the fabric by circling through the horizontal rows on a kind of diagonal, going up and toward the right, back down again and up toward the right and back down. It takes a span of four horizontal rows to complete a circuit for the normative octave.

In any event, in the case of either an ascending Lydian or descending Dorian normative scale, the ƒ, §, ∫, and ſsymbols represent all of the chromatic elements in the *Timaeus* set vis-à-vis the normative scale. In particular, ƒ and § symbols represent chromatic elements with numerical indices that are factors of 1719926784; so, as mentioned above, the two columns of these symbols represent the potential STNs for the diatonic (and other types of) octaves of CF1 and CF2-strings, respectively, in order from left to right. The ∫ and ſ symbols represent the chromatic elements with numerical indices that are not factors of 1719926784; and so, the columns of these symbols, as noted, represent potential STNs for diatonic octaves (and other types of octaves) of CNF1 and CNF2 strings, respectively, in order from left to right.

To connect any of the octave elements belonging to any of the chains left of the cut-line, on diagonals, across the horizontal rows of the fabric, in a manner similar to that illustrated for the normative scale (having its STN in Δ-string), so that they can run continuously with themselves and coincidently with others, while eliminating the possibilities that arise with chromatic distortion, specific operations must be performed on the fabric. It must be cut, certain of its edges must be brought together to conform to a spherical shape, and the whole construction must be made to move in particular ways, since only moving numbers make music.

Note that the cutting operation limits the set of octave chains, constructible from the STNs of α-ζ strings and Δ-strings, to those that are free of chromatic distortion and the ambiguities it introduces. It operates, in effect, to purge them of chromatic elements and to ensure that each of the seven diatonic strings, α-ζ and Δ, comprise sources only for their respective primary diatonic octave types. Those types are as follows: the STNs of Δ-string pertain solely to pure d.Dorian/a.Lydian diatonic chain extending for ten octaves; those of α-string to only a pure d.Phrygian/a.Phrygian diatonic chain perduring for ten octaves; those of β-string to only a pure d.Lydian/a.Dorian diatonic chain ten octaves long; those of γ-string to only a pure d.Mixolydian/a.Hypolydian diatonic chain of nine octaves; those of δ-string to only a pure d.Hypodorian/a.Hypophrygian diatonic chain nine octaves long; those of ε-string to only a pure d.Hypophrygian/a.Hypodorian diatonic chain extending for nine octaves; and ζ-string to only a pure d.Hypolydian/a.Mixolydian diatonic chain consisting of nine octaves. Enharmonic possibilities can only be derivative of these pure chains; and chromatic possibilities are eliminated all together.

The cutting operation nixes the potential of the chromatic part of the fabric for use in scales of any kind. Clearly none of the octave chains CF1-CNF2 set forth in Appendices 5, 6, 8, and 10 contains scales consisting only in chromatic elements. Separated from α-ζ and Δ-strings, diagonal circles through horizontal rows of Figure 15 cannot be completed to yield those scales. The chromatic part of the fabric, therefore, remains undivided as the analysis continues.

Certainly, the arms resulting after the cut are not equal in size. The fabric cannot, in fact, be cut, so that each part will have an equal number of vertical columns; there is an odd number of columns. In addition, Plato provides no clues that would support even a nearly equal division. The clues he does provide run exactly to the contrary. The boundary of numbers comprising both perfect cubes and perfect squares that surround the seven left-most vertical columns of the fabric, together with the striking musical fact, noted above, that all of the chromatic elements vis-à-vis the normative diatonic scale of the *Timaeus* are to the right of the cut-line proposed, support the division made.

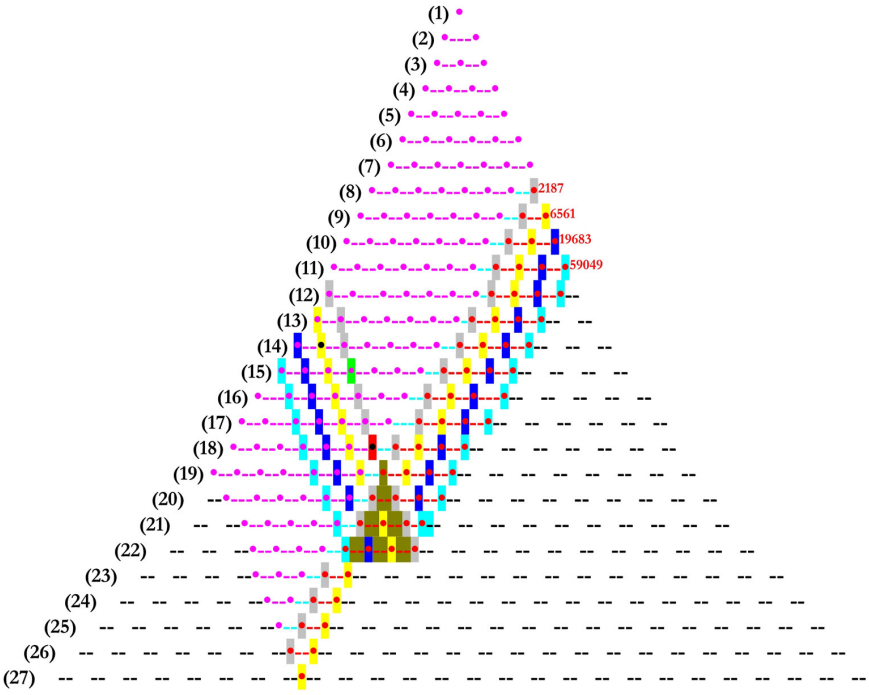

FIGURE 16 Dot chart for χ operation showing pattern of matrix emerging from the *Timaeus*

Plato's further text also indicates that he did not intend an equality of parts after the cut. The study has made the correct division of the fabric and now proceeds to make the χ.

2 Forming the χ Figure

For the purposes of the χ operation, it appears that Plato wanted his reader to view the number matrix as a geometric object. From this perspective, the numbers are like dots separated by uniform unit spaces on the object, although in their mathematical relations they represent different intervals. The formation of the χ figure presents a conundrum because it is not, at first, clear how one is to lay "middle" upon "middle" of the two parts of the fabric to achieve the χ construction. Any number of different possibilities present themselves; but one might expect the correct one to be suggested by some clue or clues hidden in the number matrix. There are, indeed, clues, as an examination of Figure 16, above, the annotated "Dot Chart For χ Operation Showing Pattern of Matrix Emerging from the Timaeus" will reveal.

The numbers corresponding to the shaded dots along the extreme left edge of the matrix in rows (12), (13), (14), and (15) are, from top to bottom 2048, 4096, 8192, and 16384. They are important to note.

Observe, comparing Table 24, that the line of doubles, descending from 2187 (4374, 8748, 17496, 34992, 69984,139968, 279936, 559872, 1119744, 2239488, 4478976), marking one edge of the narrow part of the fabric following the cut, intersects the line of triples, in the fat part of the fabric, descending from 2048 (6144, 18432, 55296, 165888, 497664, 1492992, 4478976). The number marking the intersection is 4478976, the eighth number from the left in horizontal row (19) of Table 24, corresponding exactly to the representation in Figure 16 as the element highlighted in olive in that row. The latter circumstance is not insignificant for one seeking clues about how to proceed because 2187/2048 is the size of the *apotomē* making the chromatic scales possible. Indeed, if one adopts the working hypothesis that the latter ratio is the key to forming the χ properly, then other clues soon manifest.

Note also, comparing Table 24, that the line of doubles descending from 6561 (13122, 26244, 52488, 104976, 209952, 419904, 839808, 1679616, 3359232, 6718464, 13436928, **26873856**) intersects the line of triples descending from 4096 (12288, 36864, 110592, 331776, 995328, 2985984, 8957952, **26873856**). The number marking the intersection is 26873856, the seventh number from the left in horizontal row (21) of Table 24, corresponding exactly to the representation in Figure 16 as the element highlighted in yellow in that row.

Observe that the line of doubles descending from 19683 (39366, 78732, 157464, 314928, 629856, 1259712, 2519424, 5038848, 10077696, 20155392, 40310784, 80621568) does not intersect with the line of triples descending from 8192 (24576, 73728, 221184, 663552 1990656, 5971968, 17915904, 53747712). Instead, these lines both end at row (22) of Table 24 in numbers that sit side by side: 53747712, the sixth number from the left in horizontal row (22) and 80621568, the seventh number from the left in horizontal row (22). Figure 16 represents this relationship between the dots highlighted, respectively, in dark blue (53747712) and yellow (80621568) in horizontal row (22).

The line of doubles descending from 59049 (118098, 236196, 472392, 944784, 1889568, 3779136, 7558272, 15116544, 30233088, 60466176, **120932352**), likewise, does not intersect the line of triples descending from 16384 (49152, 147456, 442368, 1327104, 3981312, 11943936, 35831808, 107495424). Instead, the line of doubles descending from 59049 ends in horizontal row (22) immediately to the right of 80621568, at 120932352, the eighth number from the left of horizontal row (22). The line of triples descending from 16384 actually ends in horizontal row (23); however, its element in horizontal row (22), 35831808, the fifth number from the left in that row, is immediately to the left of 53747712. This

4478976

8957952 13436928

171915904 26873856 403010784

35831808 53747712 80621568 120932352

FIGURE 17 Triangle of convergence

1

2 3

4 6 9

8 12 18 27

FIGURE 18
Earliest analogous triangle in the Crantor matrix

number completes a triangle of numbers, displayed in Figure 17, above, the apex of which is marked by the point of intersection between the line of doubles descending from 2187 and the line of triples descending from 2048, i.e., 4478976 (2187 × 2048). The numbers 35831808 and 120932352, the points marking the ends of its base are represented, in Figure 16, exactly in accord with the description in this paragraph as the light blue (35831808) and gray (120932352) highlighted dots in horizontal row (22).

Note that the proportion between the two points of intersection, of the lines that do intersect, 26873856: 4478976, is equal to six. This relation provides the foundation for building exactly the triangle of numbers, indicated in Figure 17, in relation to the making of the χ. The set of numbers enclosed within the triangle of convergence of the lines of doubles descending from 2187, 6561, 19682, and 59049 and the lines of triples descending from 2048, 4096, 8192, and 16384 is the subject matter of Figure 17.

The triangular region of Figure 17 corresponds precisely to the triangle of dots manifested on Figure 16 between rows (19) and (22). It contains all of the numbers in the *Timaeus* set that have a particular set of other numbers, namely, 186624 (432^2), 2187, 2048, 256, 243, and 4478976 (2187 × 2048) as common factors. The last number of the triangle is twenty-seven times the number at the apex of the triangle, as one might expect. The same proportions exist, therefore, among this set as exist in Figure 18 above.

In contrast to the extension of numbers in Figure 18, the triangular subset of Figure 17 takes as a monad, a number, 4478976 (2187 × 2048), that is clearly associated with chromatic phenomena in relation to the normative scale of the *Timaeus*. The circumstance seems particularly fitting.

Note that the base of the Triangle of Convergence of Figure 17 sits just above the tail formed by the chromatic factors of 1719926784 where, beginning with the number 107495424, in the line of triples descending from 16384, they outnumber the chromatic nonfactors of 1719926784. The convenient location of the triangle is a sign that the imbalance in the chromatic set has a real function in relation to solving Plato's riddle. It operates precisely to aid the visibility of the Triangle of Convergence.

Considered together with the intersecting lines that meet or converge within it, the triangular area of Figure 16 suggests a χ, but the shape is clearly incomplete. The area is, nonetheless, key to discerning how one must fit the two parts of the cut fabric "middle to middle" to get a well-formed χ. The following operation achieves the correct results for reasons that will become clear.

One achieves a χ, after cutting the fabric, that makes sense in view of the "middle to middle" direction, by sliding the narrow band up four rows and over to the left by one place to position the lower left point of the triangular area, corresponding to 35831808 (fifth number in horizontal row (22)), over the only point, in Figure 16, with a red background, corresponding to 1492992 (the seventh number from the left in horizontal row (18) of the matrix) on the right edge of the fat part of the cut fabric. Using the later point as a pivot, one rotates the narrow band counterclockwise until the apex of the triangle, 4478976 in the narrow band, is directly over 55296 (fourth position from the left in the fifteenth row of the matrix), the only number in Figure 16 with a green background, in the wide band. Figure 19, below, illustrates the operation.

The original position of the knife-shaped narrow band, in Figure 19, is indicated by the red dots. The numbers superimposed transversely across the broad band and exceeding its two edges indicate the position of the narrow band after being cut, slid up, and rotated in accord with the directions provided just above in the text. Some yellow highlighting has been provided, where possible, to draw attention to the numbered area representing the narrow band. The numbers themselves indicate the rows and positions of the narrow band that are superimposed upon the broad band after the slide and rotation. They make it possible to identify the paired numbers of the two bands after the operation. The green highlighting where the narrow band emerges at top left from the broad band is provided to emphasize that boundary.

Observe the triangle of rotation, first emphasized in Figure 16, in its original position. In Figure 19, this original position is marked as follows: the green highlighted narrow band dot in row (22) is 35831808, the left bottom element of the triangular area emphasized on Figure 16. The light blue highlighted narrow band position in row (19), marked by "23," in Figure 19, is 4478976, the apex of the indicated triangular area, in its unrotated placement. After the slide and

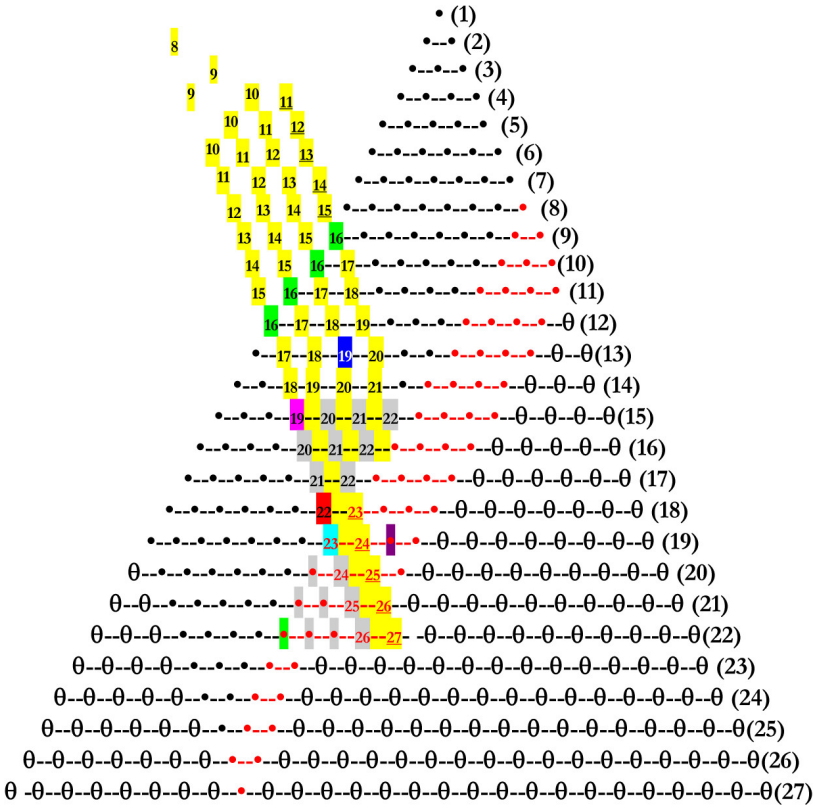

FIGURE 19 Cut, slide, and rotation operation

rotation, the number 71663616 occupies this position. The rest of the triangular area of Figure 16, in its unrotated placement, is marked, in Figure 19, by dots highlighted in gray.

When the narrow band is rotated, the triangle has an altered position as follows. The orange highlighted broad band position, marked by "22," in row (18), in Figure 19, is 1492992. The number 35831808 from row (22) of the narrow band is superimposed upon it in the slide and rotation. The broad band position highlighted in purple in row (15) and marked by "19," in Figure 19, is 55296. The number 4478976 from row (19) of the narrow band is superimposed upon it after the slide and the rotation. The numbers highlighted in gray represent the rest of the original triangular area of interest from Figure 16 after the slide and rotation.

Note that the rotation turns the Triangle of Convergence on its side, so that the triples proceeding from 4478976 to 120932352 run with numbers of the broad band, in row (15), that make sesquialter (3/2) intervals with each other.

The other triples of the narrow band obey the same rule because of the rotation. The directionality of the narrow band triples after the rotation from 4478976 to 120932352 is clear from Figure 19 upon an examination of the row numbers specified for the narrow band in its overlay of particular positions of row (15) of the broad band.

Because of this relationship between the narrow band triples and the broad band sesquialter intervals, both the triples and the doubles of the broad band cut across the triples of the narrow band after the sliding and pivot operation. The doubles of the narrow band, meanwhile, run in the same direction as the triples of the broad band and cut across its doubles. The broad band triples, in turn, cut across the narrow band sesquialter intervals, while the doubles of the broad band run with them.

The blue highlighted broad band position, marked by "19," in row (13) of Figure 19 is 13824. The number 10077696 of row (19) of the narrow band is superimposed upon it after the slide and rotation. The brown highlighted dot in row (19) of the narrow band is 10077696 in its unrotated placement. The claim of this study is that these two numbers are properly the "middles" of the broad and narrow bands for the purpose of making the χ.

The argument for 13824, in the case of the broad band, is straightforward. The wide band has twenty-five rows, extending from row one to row twenty-five of the matrix. The thirteenth row is the middle row. It contains seven positions. The middle position is the fourth from either the right or the left. The number occupying that position is 13824. There is, in addition, an odd number of terms in the wide band; 13824 is the sixty-seventh term in the wide band, counting either in increasing numerical order from the top of the wide band or, in the opposite direction, in decreasing numerical order from the bottom or, from an alternative positional perspective, counting from left to right from the top and right to left from the bottom.

The middle of the wide band can be geometrically determined, as well. Note that the numbers 1 and 191102976 mark the extreme ends of the wide band, measured length to length. The midpoint on the diagonal between them is the square root of 191102976 [$1/x = x/191102976$, so that $x^2 = 191102976$], i.e., 13824. The midpoint can also be found *via* a calculation performed on the other set of extremes of the band, that is, 729, in the seventh row, and 262144 in the nineteenth row: $262144/x = x/729$, so that $262144 \times 729 = x^2$. The product of the numbers on the left hand of the equation is 191102976, with the result that $x = 13824$. The number 13824 is exactly in the middle of the matrix for the wide band from any relevant point of view.

It is more difficult to find the proper middle of the narrow band because the latter has twenty rows with, in general, but not always, four positions per

row. This knife-shaped object (amusing in view of the need to cut the fabric) has neither a middle row nor a middle position that can be determined, in any row, because rows and positions are even. The middle cannot, accordingly, be geometrically determined in a manner analogous to the determination on the wide band.

What one really has to look for, in the narrow band, is a kind of center of gravity which, when placed over 13824 in the broad band, will preserve an appropriate symmetry between the broad and narrow bands, once crossed. Note that, however one positions the narrow band with respect to the broad band, symmetry demands that the narrow tail of numbers at the bottom of the narrow band be outside the crossover area of the bands in forming the χ. This tails consists in two short strings of doubles, one originating from 71663616 in the sequence 71663616, 143327232, 286654464, 573308928 and the other originating from 107495424 in the sequence 107495424, 214990848, 429981696, 859963392, 1719926784. The longer string, forming the right-most edge of the narrow band emerging at the bottom of the broad band, has five elements.[2]

Because symmetry demands that the "tail" be so accommodated, all numbers above horizontal row (16) of the narrow band automatically fall outside of the crossover of the wide and narrow bands. Note that the short side of the narrow band exiting the wide band at the top right, has, as a result, five numbers along that edge namely, 59049, 118098, 236196, 472392, and 944784, before the band narrows and bends. These lengths of five of the narrow band, emerging below and above the broad band, at the lower right and the upper right sides of the narrow band, create a symmetry between the parts of the narrow band, below and above, exceeding the crossover of the wide band.

The exclusions noted that are demanded by symmetry leave rows (16) through (22) of the narrow band as the area of possible overlap of the narrow band with the broad band. Figure 20 below shows that this area is shaped like a parallelogram and, indeed, it has just such an appearance in the rotated narrow band on Figure 19, too.

2 The number of elements in the two strings of the "tail" vis-à-vis one another form the 5/4 (1.25) corresponding to a major third. Jamie James, *The Music of the Spheres: Music, Science, and the Natural Order of the Universe* (New York: Copernicus/Springer-Verlag, 1995), 88, 96. Although this ratio was not recognized as a harmonic ratio by Pythagoreans and is not otherwise suggested by the *Timaeus* number set, it roughly corresponds to the size of the ditone (9/8 × 9/8 = 1.265625; sixth root of two squared is approximately equal to 1.25992105), important to building enharmonic scales, and eventually became recognized as a significant harmonic ratio in the later history of Western music in dialogue with Pythagorean thought. Ibid., 88–89, 96, 150.

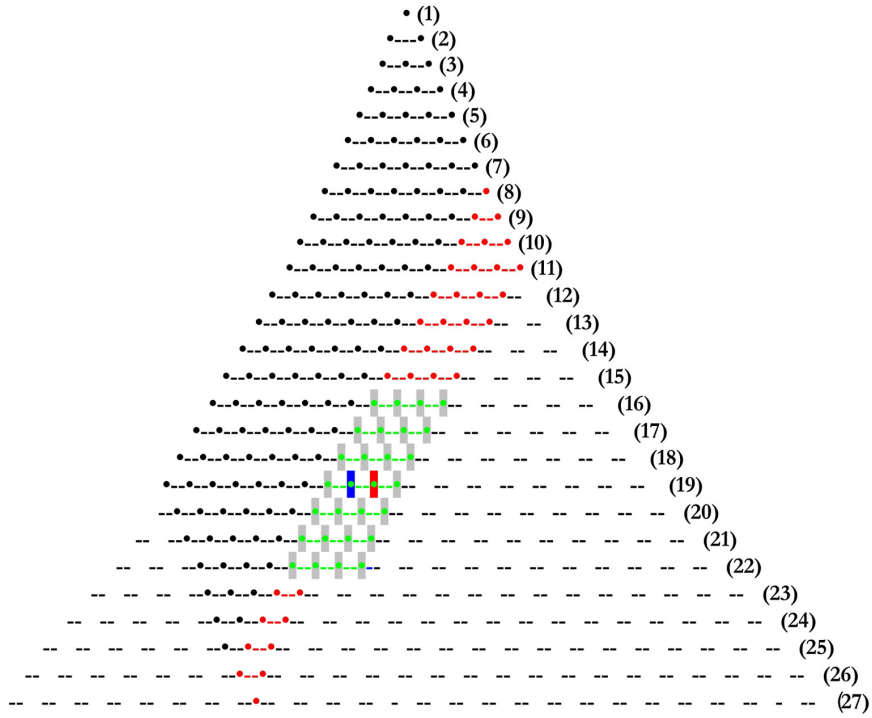

FIGURE 20 Finding the "center" of the narrow band

In Figure 20 the red and green dots represent the numbers in the narrow band. The region of green dots, highlighted in gray, blue, and red, is the parallelogram shaped area between rows sixteen and twenty-two. There are seven rows in this parallelogram shaped area. Row nineteen is in the middle and there are two middle dots in that row. The dot highlighted in red is 10077696 and the other candidate for the middle, highlighted in blue, is 6718464, as a quick inspection of Table 24 shows.

The clue that 10077696 is the center of interest is its special relationship with 13824. Both numbers are perfect cubes. The cube root of 10077696 is 216, a number that is also a cube root, with root 6. The cube root of 13824 is 24. The ratio 10077696/13824 is 729, itself a cube root, with root nine, a number having square root of 3. The ratio of 216/24 is also nine, having square root 3.

The number 6718464 is not a perfect cube, but a perfect square, with root 2592, a number that is neither a perfect cube nor a perfect square. The ratio of 6718464 to 13824 is 486, also a number that is neither a perfect cube nor a perfect square. The cutting, sliding, and rotating operation that this study proposes perfectly aligns 13824 and 10077696, as Table 49 below shows. If 6718464 were

to be taken as the center of the narrow band parallelogram, one would simply slide the pivot point of the narrow band, 35831808, in row (22) of the narrow band, up one more row and over one place to 746496 in row (17) of the broad band, do the pivot, achieving an alignment of 4478976, the apex of the triangle of pivot with 27648 in the fourth position of horizontal row (14) on the broad band, and also achieve an alignment of 6718464 and 13824. While this operation is possible, it is not preferable.

In the case of the option presented in Figure 19, the pivot point of the narrow band is moved up exactly four rows and over to the left by one place to be superimposed upon a number in the broad band. In the case of the alternative under consideration, one must move the pivot point up five places and over one. Plato does not concentrate on fivefold movements in the *Timaeus*. Plato's clear preference for fourfold movements and schemes of generation suggest that the choice presented in Figure 19 is correct.

While it is possible that Plato meant for the reader to find a center in the narrow band between 6718464 and 10077696, it is unlikely. The reader would have to split the fifth interval between them. In no other instance in the *Timaeus* does Plato express any interest in splitting intervals of the 3/2 variety. To split the interval in a mathematically correct way one would have to multiply 6718464 by the square root of 3/2 and divide 10077696 by the same number. The number at issue is not a whole number. If one split the interval in the same manner that this study has split intervals to achieve quarter tone enharmonic scales, then the number immediately between 6718464 and 10077696 is 8398080. This number makes a 5/4 ratio with 6718464 and a 6/5 ratio with 10077696. Neither ratio is of any interest to Plato in the *Timaeus*.

Therefore, to make the χ figure it is most likely that one places 10077696 on the narrow band over 13824 on the broad band, in line with the slide and rotation operation presented in Figure 19. Table 49 below provides the correspondences of overlapping numbers on the broad and narrow bands that are commensurate with the operations presented in Figure 19. Observe that rows (8)–(15) at the top end of the narrow band and rows 23–27 at the bottom end of the narrow band, are completely outside of the area of overlap of the broad and narrow bands. Rows (16)–(22) of the narrow band overlap rows (9)–(18) of the broad band, as Table 49 indicates. The designations in parentheses, in Table 49, indicate the band and band row: (16N) means row 16 on the narrow band.

Once the cutting, sliding and rotation is complete one has a χ figure, truly laid middle to middle, with very definite parameters, namely the following. The dots in lavender, on Figure 21, represent the numbers on the broad band; and the lavender numbers in parentheses represent the rows of the broad band.

TABLE 49 Area of overlap between the broad and narrow bands in the formation of the χ

1889568 (16N)/ 256 (9B)	3779136 (17N)/ 768 (10B)	7558272 (18N)/ 2304 (11B)	15116544 (19N)/ 6912(12B)	30233088 (20N)/ 20736 (13B)	60466176 (21N)/ 62208 (14B)	120932352 (22N)/ 186624 (15B)
1259712 (16N)/ 512 (10B)	2519424 (17N)/ 1536 (11B)	5038848 (18N)/ 4608 (12B)	10077696 (19N)/ 13824 (13B)	20155392 (20N)/ 41472 (14B)	40310784 (21N)/ 124416 (15B)	80621568 (22N)/ 373248 (16B)
839808 (16N)/ 1024 (11B)	1679616 (17N)/ 3072 (12B)	3359232 (18N)/ 9216 (13B)	6718464 (19N)/ 27648 (14B)	13436928 (20N)/ 82944 (15B)	26873856 (21N)/ 248832 (16B)	53747712 (22N)/ 746496 (17B)
559872 (16N)/ 2048 (12B)	1119744 (17N)/ 6144 (13B)	2239488 (18N)/ 18432 (14B)	4478976 (19N)/ 55296 (15B)	8957952 (20N)/ 165888 (16B)	17915904 (21N)/ 497664 (17B)	35831808 (22N)/ 1492992 (18B)

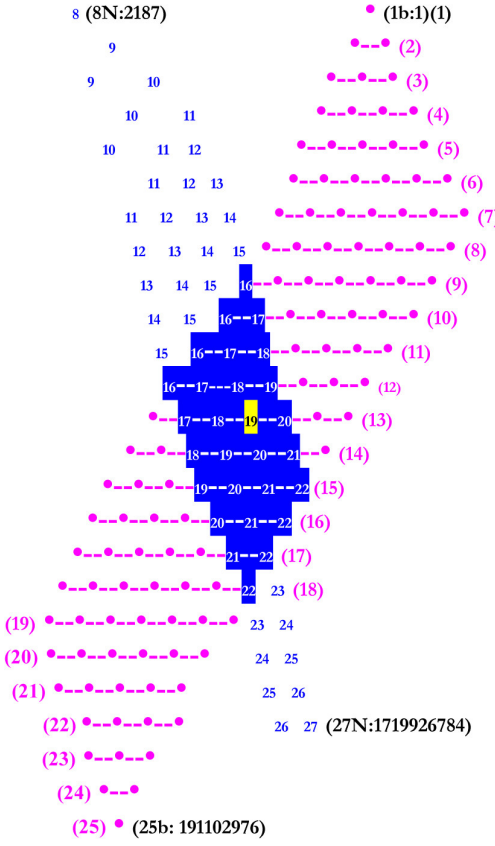

FIGURE 21
The χ figure

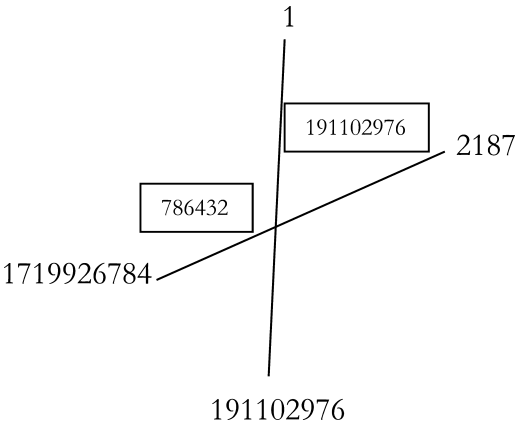

FIGURE 22 Proportions among endpoints of the arms upon their joinder

The numbers transversing the fabric represent the numbers on the narrow band, providing information about their rows and positions on that band. A comparison with Table 24 identifies all numbers in Figure 21, including the area of intersection, already specified in Table 49. The dot highlighted in yellow represents the superimposed middles of the broad and narrow bands, namely 13824 in row (13) of the broad band and 10077696 in row 19 of the narrow band.

3 Bending the Arms to Form Circular Shapes

Once the analyst has made the χ, he can bend the arms to form the circular shapes that Plato directed at 36 C. For the purpose of making circles of the arms, Plato indicated that each arm is to be joined to itself and also to the other arm at a point opposite the locus at which the two arms had first been laid together. In other words, the extreme points, represented for the wide band by 1 and 191102976, and for the narrow band, by 2187 and 1719926784, are to be brought together end to end opposite the locus of joinder. The locus of joinder of the two arms middle to middle is actually the set of loci marked by the middle points constituting the centers of the wide and narrow bands as calculated above, i.e., 13824 and 10077696.

The proportions among the joined ends of the two arms yield the numbers set forth in Figure 22. Figure 22 shows that when the opposite endpoints of each arm come together they make proportions of 191102976:1 and 1719926784/2187, respectively, so that when all four come together, the proportion 191102976: 786432 reducing to **243**, defines the relation among the endpoints. This number

is significant in its relationship to the numbers occurring at the polar opposite of the soul sphere, once the χ arm endpoints are joined. The proportion defining the opposite pole is 10077696/13824, reducing to **729.** The relation between the two poles, then, is marked by the triple proportion 729:243, i.e., 3. This result is very neat and clean, because it brings the analyst back full circle to a triad. Just as the original soul stuff emerges from a triad, so does the cosmos from the world soul.

Since the final ratio of the numbers at the poles of the soul sphere is 3:1, the analyst may use it to measure the soul sphere's diameter. The measure it makes is not in absolute mathematical units of size and distance, such as those commonly used by people in the affairs of ordinary life to measure, for example, the edge of a table, but in the relative units appropriate to the measurement of a universe that, in absolute and objective terms, may be either infinitesimally small or infinitely large, depending upon the observer's perspective. These relative units are harmonic units, as one might guess, since the matrix emerging from Plato's *Timaeus* is a harmonic expansion.

The harmonic expansion relevant to reaching the size of the cosmic soul sphere diameter is clearly a triple interval, comprising an octave plus a fifth (but subject to other interpretations in its internal divisions): [(TTSTTTS)(TTST)]. One can define this unit, the triple, for the sake of simplicity, as the model harmonic unit and rest content that the size of the cosmic diameter is one harmonic unit. Note that the unit is a dimensionless (pixel like) number.

Having determined the size of the diameter of the soul sphere, one can find the size of its radius, and then, just for fun, calculate both the volume and surface area of the sphere, as a function of harmonic units, using Egyptian π, 256/81, in the relevant calculations, as it is the only measure for π represented by the *Timaeus* numbers.[3] If the size of the diameter is one harmonic unit, then the radius is .5 harmonic units. Further, since a harmonic unit is a triple expansion, the number, 3, raised to the exponential power of .5 should yield the size of the harmonic expansion marking the radius. The value $3^{1/2}$, (the square root of three) is approximately equal to 1.7320508075688 8 (accurate to the fourteenth

3 Turnbull wrote:

> "Recent investigations of the Rhind Papyrus, the Moscow Papyrus of the Twelfth Egyptian Dynasty, and the Strassburg Cuneiform texts have greatly added to the prestige of Egyptian and Babylonian mathematics. While no general proof has yet been found among these sources, many remarkable ad hoc formulae have come to light, such as the Babylonian solution of complicated quadratic equations dating from 2000 B.C., which O. Neugebauer published in 1929, and an Egyptian approximation to the area of a sphere (equivalent to reckoning π = 256/81)." (Turnbull, "The Great Mathematicians," 87.)

digit after the decimal point), indicating a harmonic expansion significantly less than an octave.

One might get a more intuitive understanding of the size of the harmonic expansion constituting the radius by comparing its formulaically achieved mathematical value to the mathematical value found by directly considering the composition of a harmonic unit. Such a method would be akin to ancient methods, generally. The unit consists of an octave and diapente, expressed in the order of a descending Dorian or ascending Lydian scale.

[(TTSTTTS)(TTST)] = 3

The above expansion can be reexpressed as [(TTST) (TTS) (TTST)]. Its middle segment is highlighted in yellow and its center is the center of the whole expansion. This center cannot determined without splitting the central tone of the fourth into S and S′, so that the TTS sequence becomes TSS′S:

[(TTST TSS′S)(TTST)]

Such a split, puts the S highlighted in blue at the center of the system between block sequences in yellow, each comprising a tone. The center is, thus, a fifth and a tone from either end of the whole expanse. The sequence, then, to be computed to the center, in a manner analogous to ancient styles, is (TTST TS), i.e. [9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]. The product $\mathbf{1.\overline{7}}$ is expressible as the whole number ratio 16/9. It is an approximation to the square root of three and as close an expression of the mathematical value of the harmonic expansion to the center as this ancient style of procedure can reach.

The square root of three itself (approximately 1.732050807568877) can be expressed approximately as a ratio of whole numbers. The ratio 2979000575/1719926784 is accurate to the ninth digit (1.732050807) beyond the decimal. It is clearly less than the value taking 16/9, as the center, but 16/9 is not a bad approximation.

3.1 *Volume*

Using .5 harmonic units as the measure of the radius of the soul sphere and Egyptian π (256/81) as the most appropriate measure for π, since it is the only measure for π that can be based on the *Timaeus* numbers, one can calculate the volume of the soul sphere as a function of harmonic units.

$V = 4/3\pi r^3$

$4/3 \times 256/81$ (Egyptian π) $\times (.5)^3$ = (approximately) .52674897119342 harmonic units. This number is expressible in a whole number ratio as 905969664/1719926784 with accuracy to the fourteenth place beyond the decimal point. Since each harmonic unit is a triple expansion, the mathematical value of the harmonic expansion associated with the volume of the sphere is $3^{905969664/1719926784}$ or approximately 1.78370541213.

One might, for curiosity's sake, compare the above measure with the one achieved using a modern value for π, accurate to the fourteenth place following the decimal, and the value for the square root of three accurate to an equal number of digits. In that case, one achieves the following value for the volume of the soul sphere. $4/3 \times 3.14159265358979$ (modern π) $\times (.5)^3$ = (approximately) .5235987755983 harmonic units; and $3^{.5235987755983}$ or 1.7775429564881 1 is the mathematical value of the harmonic expansion to such a volume. Clearly the measure for the harmonic expansion associated with the volume hovers around a value near but somewhat larger than the square root of three.

3.2 *Surface Area*

Using .5 harmonic units as the measure of the radius of the soul sphere, one can also calculate its surface as a function of harmonic units.

$$SA = 4\pi r^2$$

$4 \times 256/81$ (Egyptian π) $\times (.5)^2 = 3.\overline{160493827}$ harmonic units. This number is expressible in a ratio of whole numbers as 5435817984/1719926784. Note that the number of harmonic units is the exact value of Egyptian π. The first and third terms cancel each other out ($4 \times ¼$). The mathematical value of the harmonic expansion associated with surface area of the sphere is $3^{5435817984/1719926784}$, i.e., approximately 32.2061477029295.

One can compare the measure one achieves using modern π just for curiosity's sake. $4 \times 3.14159265358979$ (modern π) $\times (.5)^2$ = approximately 3.1459265358979 harmonic units. Note that the number of harmonic units is the exact value of modern π, because, as explained above, the first and third terms cancel each other. This mathematical value of the harmonic expansion associated with the surface area of the sphere, in this case, is $3^{3.1459265358979}$, i.e., 31.5442807001975. Both means of calculating the value of the harmonic expansion associated with the surface area of the sphere produce values hovering crudely around 2^5.

Observe, in the case of either value for π, that the ratio, in harmonic units, of the surface area of the sphere to its volume is, for all intents and purposes, 6:1; so, in the case of either measure of π, it is clear that a sphere with a diameter of

one harmonic unit (dimensionless, remember) has a surface area six times the size of its volume. In addition, the surface area of such a sphere is equal in harmonic units to the value of whatever π one uses to calculate the surface area, while the size of the harmonic expansion associated with the volume of the sphere in harmonic units approximates the square root of three. The soul sphere seems interesting mathematically.

There is no evidence that any ancient musician, philosopher, or music theoretician known to scholarship ever tried to calculate the volume or surface area of Plato's soul sphere as a function of harmonic units in the manner set forth above or in any other manner. The exercise is purely academic in that regard. Note, however, that since a "harmonic unit" is elastic with respect to actual physical size, the sphere defined is, potentially, indefinitely large or, even, alternatively, infinitesimally small.

4 The Uniform Motion of the Whole without Variation

After Plato joined the ends of the arms as indicated above to achieve a spherical shape, he involved both circles in a motion that revolved in the same place without variation (36 C). This operation is what makes a sphere of the circles. An inspection of the *Timaeus* set, as set forth in the number matrix, prior to the cutting of the fabric (see Figure 15) helps one to identify this single motion, involving the whole construction, as movement in the direction of the 3/2 ratio.

For every single line of the uncut fabric, the latter ratios progress laterally toward the right. Each number in any line in the fabric is 3/2 × the number immediately to its left. The lateral progression is suited to the idea of revolving in the same place without variation. It befits the notion of a uniform motion of the whole. In addition, the end status of fifth periodicity as the major pattern persisting musically in the whole uncut fabric reinforces the fittingness of assigning the 3/2 ratio to the uniform motion. Mark the necessity, as well, of this motion to the musical potential of the broad band of the fabric.

Only when the broad band of the fabric is set in motion in the uniform direction of the 3/2 ratio, can each of the distinctive pure octave chains grounded in the seven strings of starting tone numbers of the broad band emerge to sound its harmony, according to the circling procedure outlined above in this chapter. One has to move up the doubles of the broad band and, then, laterally to the right in the direction of the 3/2 ratio, as well as make the turn twice through a 256/243 ratio, possible only on the basis of the triple motion, to make an

octave scale. The 3/2 motion, when initiated, implies activation of the double and triple motions, in the fabric, equally necessary to the circular movement; for it does not exist except as the mediator of those motions. Accordingly, when Plato formed the sphere by joining the ends of the two arms of the χ, he made the entire construction revolve to the right in step with the 3/2 ratio of the broad band of the χ.

Note that the narrow band is not capacitated for music by its involvement in the uniform motion. This uniform movement, following the 3/2 ratios in the broad band, actually involves the narrow band in movement following its own triple ratios, since the process of χ formation caused its triple ratios to run in the same direction as the 3/2 ratios of the broad band (see Figures 19 and 21). The kind of circling operation necessary to the emergence of diatonic scales does not, therefore, occur for the narrow band when it follows the uniform motion of the whole. The elements of the narrow band, considered as an isolated set, are insufficient anyway, as previously mentioned, to allow the construction of complete scales of any kind.

5 Separation of the Arms into an Outer and Inner Circle

Upon having involved the whole construction in the uniform motion following the 3/2 ratios of the broad band, Plato complicated the picture. He made one of the circles an "outer" circle and the other an "inner" circle. He necessarily separated them in the process, it seems. The "outer" circle is the narrow band of numbers of the original χ (36 C), containing all of the chromatic elements vis-à-vis the normative scale of the *Timaeus*. The inner circle is the wide band of numbers containing all of the elements relevant to articulating both the primary ascending Lydian/descending Dorian diatonic scale, defined by the *Timaeus* number set, and selected secondary diatonic chains, independent of chromatic influence for their emergence.

6 Separation and Definition of the Motions of Same and Different

6.1 *The Outer Band's Motion of the Same*

Plato defined an outer and inner motion corresponding to the outer and inner circles (36 C). The outer motion he called "the same." It is the same as the uniform motion, already discussed, that moves laterally toward the right in step with the 3/2 ratios of the broad band. However, for the narrow band, as indicated above, this movement follows its own line of triples.

The designation of "same" for the narrow band makes sense for at least three reasons. In the first place, it is suitable in view of the narrow band's movement following its own triple ratios, even as it participates in the uniform motion of the whole. In Plato's original construction of the soul stuff, the triad emerging with the triple ratio, represents "sameness in difference" in the *ordo* of generation. Because of this character of the triple ratio, the character of "same" properly marks the narrow band of chromatic numbers.

"Same" is also properly assigned to the narrow band because of the "sameness" in the potentiality of each of its strings to generate octave chains, prior to being isolated from the rest of the fabric. The "sameness" of these potentialities is evident in the fact that the descending Hypolydian/ascending Mixolydian pattern was the lead pattern, in the uncut fabric, for diatonic octaves starting from STNs of any of the four chromatic number strings. The chromatic scale possibilities are also similar starting from STNs within the respective strings. Three of the four strings of narrow band STNs include among their chromatic scale possibilities the descending Lydian and ascending Hypophrygian scale types (see Appendix 8). The one that does not leads primary descending Hypolydian and ascending Phrygian chromatic scale patterns that are also lead patterns for two of the other strings of chromatic STNs (see Appendix 8). All four strings, in addition, inaugurate similar enharmonic possibilities (see Appendix 10). The STNs of all four string lead descending Hypophrygian and ascending Mixolydian enharmonic patterns.

The close similarity of patterns among diatonic, chromatic, and enharmonic possibilities in the chains of octaves beginning from chromatic numbers in the narrow band strings of the uncut fabric entails that they have a capacity to move *in tandem* with each other in a way that is not possible for the octave starting tone strings in the wide portion of the fabric. It is appropriate, therefore, that the chromatic numbers be grouped together, separated, and assigned the character of same.

The narrow band is also properly regarded as the band of "same" because of the unique incapacity of its elements to generate complete scales of any kind in isolation. The chromatic numbers are the "same" in the barrenness of their undivided state, even resembling, in their sterility, the indivisible and unchangeable being of 35 A. This, too, lacked capacity for generation unless mixed with changeable natures.

6.2 *The Inner Band's Motion of the Different*

Plato assigned the motion of the "different" to the inner band, i.e., the broad band of the fabric. He described this motion as proceeding toward the left by way of the diagonal. An inspection of the uncut number matrix suggests that

this motion is the octave movement. It is fitting that the octave movement be the motion of the different because it is this repeating movement that becomes subject to variation toward a new order of periodicity in the *Timaeus* set.

In addition, each of the octave chains stemming from the seven different strings of starting tone numbers in the wide portion of the fabric are different from one another in the dominant diatonic patterns that they exhibit, when purified from chromatic interference. Note that, even before being severed from the narrow band, each of the starting tone strings of the broad band exhibited its own distinctive generative order of diatonic, chromatic, and enharmonic possibilities.

6.3 *The Fitting Relationship of the Outer and Inner Bands*

Observe that the ratio between the motions of the inner and outer bands is 3/2. The *ordo* of soul sphere genesis thus mirrors Plato's original *ordo* of generation of the soul stuff. It effects a separation of doubles from triples, so to speak, akin to the first separation of doubles from triples at 35 B. Accordingly, although Plato did not draw his reader's attention to it, a sesquialter harmonic operates between the motions of the inner and outer circles. This is fitting, since the octave periodicity of the broad band degenerates to fifth periodicity in the end. In addition it accords with the sesquialter ratio that separated the ζ-string of STNs from CF1 in the uncut fabric.

7 Elevation of the Motion of the Same to Primacy

Plato made the motion called "same," dominant (36 C). The result makes sense because, following the triple ratios of the narrow band, the motion of same recalls the original triad of "sameness in difference" from which the soul stuff was generated. Plato also left this motion undivided (36 C–36 D), causing it to resemble the stability of the indivisible and unchangeable element at 35 A. Surely what is stable and undivided has primacy from a Platonic standpoint.

8 Sixfold Split of the Inner Movement of the Different, i.e., the Octave Movement

The other motion, the inner octave motion, Plato did divide (36 D). Having formed the sphere by slipping the narrow band over the wide one, creating an outer and inner circle and defining an outer and inner movement, Plato decided to split the inner movement, i.e., the octave movement defined by the

relations of the starting tone numbers in each of the seven strings of the broad band, once they had been set in uniform motion. He divided it in six places or six times to produce seven unequal circles, according to intervals of doubles and the triples (36 D), each of which he says were three. Again he conformed to the standard *ordo* of generation by division into doubles and triples.

It is not, at first, completely clear how one is to make the division in a way that uses all of the fabric of the broad band, as Plato did not instruct his reader that there would be any remainder. One might be tempted simply to make the sixfold division along the lines of the separate strings of STNs, in their bent condition as they form the inner circle. After all, exactly six divisions would sever the seven lines of STNs pertaining to distinct octave chains. However, such a division would fail to accommodate Plato's very clear direction that the division to be made is according to three intervals of doubles and three intervals of triples (presumably proceeding in the alternative fashion familiar since 35 C). The easy division proposed would require speculative definitions of the further motions that Plato specified for the two subsets of separated bands and has the disadvantage of not being in line with the tradition of interpretation of the text. The most authoritative translators of Plato's text, mindful of the commentary tradition, seem uniformly to interpret the direction at 36 D as requiring a set of bands exhibiting triple ratios and a set of bands exhibiting double ratios.[4]

The division one must make, then, it seems, in a principled way, is the simplest sixfold division, proceeding alternately by way of doubles and triples, that will account for all of the fabric of the broad band. Figure 23 presents the proposed division of this study, laying the strings out flat, just for the sake of illustration. The actual split is made from the circular form of the broad band.

Figure 23, derived from Figure 15, is a representation solely of the broad band of the cut fabric. It represents the following sixfold division. The first division consists in cutting off the line of doubles designated as γ-string, highlighted in light blue in Figure 23. It makes sense to segregate this line for two reasons.

(A) The γ-string consists in the STNs of a d.Mixolydian/a.Hypolydian chain of octaves that is singular among scale types in beginning with an independent tone just preceding the two tones of the first fourth, so that three tones begin the sequence. The d.Mixolydian/a.Hypolydian octave seems related, nonetheless, to the d.Dorian/a.Lydian scale for which the adjacent Δ-string contains the STNs, as the two scale types differ only in the

4 Plato *Tim.* [Zeyl] 36 D; *Tim.* [Jowett] 36 D; *Tim.* [T. Taylor] 36 D; *Tim.* [Bury] 36 D.

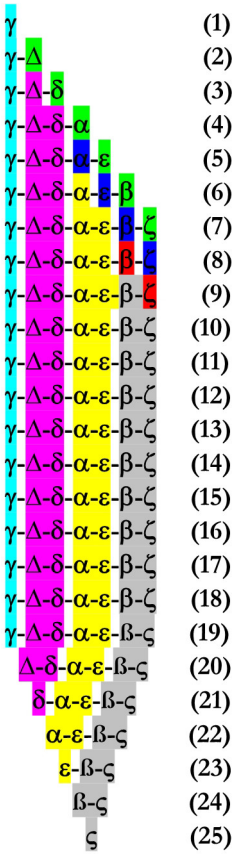

FIGURE 23
The sixfold division of the broad band: the band of difference

tone/semitone sequence following the first two tones. In the d.Mixolydian/a.Hypolydian octave, the sequence is TS and in the d.Dorian/a.Lydian octave, it is ST. One might wonder whether a pairing is justified, in view of the limited number of divisions permitted by Plato's recipe. One consideration rules out a pairing in support of the singularity of γ-string, as set forth in (B).

(B) The γ-string is the only string among the seven that is a line of doubles beginning from a double, 2. The other starting tone strings are lines of doubles proceeding from numbers in the triple series, 3, 9, 27, 81, 243, and 729, respectively.

The γ-string should be alone because of its singularity, especially.

The second division is the segregation of a single line of triples, highlighted in green in Figure 23, consisting in the numbers 3, 9, 27, 81, 243, and 729 (cf. Table 24). This is the simplest division of triples following the division of γ-string from the broad band and is, thus, methodologically justified.

The third division reverts to the doubles. This time a pair of strings, Δ-δ, as highlighted in lavender, is segregated from the broad band, beginning with the number 6 in Δ-string and 18 in δ-string, as these numbers are the first possible in those chains after the second division. Note that, unless pairs of doubles are the focus of division at this point in the process, a sixfold division will not consume the whole fabric but leave a remainder that Plato does not mention. While one might take more than a pair, there seems to be no justification for such a procedure because it is not the simplest next step. The most orderly way to proceed with the second division of doubles is to take the first pair remaining. The progression to a pair in relation to the singularity of the γ-string also suggests the progression of a monad to a duad.

The segregation of Δ-δ, as a twosome, in fact, makes sense. The Δ-string consists in the STNs of a d.Dorian/a.Lydian octave chain, and the δ-string consists in the STNs of a d.Hypodorian/a.Hypophrygian octave chain. These two scale types differ only in the last tone and semitone of their scales. In the d.Dorian/a.Lydian octave, TS is the ending sequence. In the d.Hypodorian/a.Hypophrygian octave, ST is the final sequence. They are, in other words, variants of each other.

The fourth division cuts off the next line of triples, highlighted in dark blue in Figure 23. It is methodologically justified as the simplest division of triples possible at this point in the process. Its numbers are 54, 162, 486, and 1458.

The fifth division cuts off the next pair of doubles chains, α-ε, highlighted in yellow, beginning from the number 108 in α-string and 324 in ε-string (cf. Table 24), as they are what remains following the earlier divisions of triples. The pairing makes sense. The α-string consists in the STNs of a d.Phrygian/a.Phrygian octave chain, while the ε-string consists in a d.Hypophrygian/a.Hypodorian octave chain. These two scale types differ only in the placement of a tone/semitone sequence in the last fourth of the scale. The last fourth of the d.Phrygian/a.Phrygian octave begins with a TS sequence, while the last fourth of the d.Hypophrygian/a.Hypodorian sequence begins with an ST sequence. The two scale types are, thus, variants of each other.

The sixth division cuts off the next, very short line of triples, highlighted in red, consisting in the numbers 972 and 2916 (cf. Table 24). The division is justified as the simplest division of triples possible at this point in the process. Note that the sixth division *de facto* releases the last pair of doubles strings, β-ζ from what had been the broad band of the fabric. This pairing makes sense in the same way as the other pairings of doubles made. The β-string consists in the STNs for d.Lydian/a.Dorian octave chain, and the ζ-string consists in the STNs for a d.Hypolydian/a.Mixolydian octave chain. The two types of octave are variants of one another, differing only in the *ordo* of the tone/semitone sequence

following the first fourth. In the d.Lydian/a.Dorian octave, the sequence is TS, and in the d.Hypolydian/a.Mixolydian octave it is ST.

The result of these divisions fits Plato's description. The seven circles are certainly unequal, as Plato required in 36 D. Further, three of the divisions are made according to double intervals; and three of the divisions are made according to triple intervals, also as required by 36 D.

After separating the strings, Plato made a subset of the four circles exhibiting double intervals and a separate subset of the three circles exhibiting triple intervals (36 D). He made them revolve in contrary directions (36 D), a move that makes perfect sense in view of the fact that the triples in the uncut fabric move in a direction contrary to the doubles. Thus the three circles consisting in triples move in the direction of the triples of the formerly uncut broad band; and the four circles consisting in doubles move in the direction of the doubles of the formerly uncut broad band.

Just as Plato required, the subset of three moves at *similar* speeds (36 D), as all of its elements share in the triple motion and are pure lines of triples. They do not move at identical speeds, however, because their different lengths produce different proportions in the numbers along their strings. For example, the most extreme proportion in the first line of triples is 729:3, resolving to 243:1. The most extreme proportion of the second band of triples is 1458:54, resolving to 27:1; and the most extreme proportion of the third band of triples is 2916:972, resolving to 3:1. The differences among these proportions are, however, all expressible in terms of powers of three, the term of the triple ratio; so they do not detract from the basic similarity in the triple speed of the strings.

The subset of four bands of doubles moves at dissimilar speeds, in the differing octave types of the double motions. The γ-string is also the only string exhibiting the double motion alone. The other three strings of doubles are not purely doubles strings but also exhibit the 3/2 ratio, internally, unlike γ-string (although all adjacent bands share a 3/2 movement vis-à-vis each other). The Δ-δ band includes all ratios to which the α-ε and β-ζ pairing give rise, but also exhibits some ratios *unique* to itself that cannot be reduced to the terms of the basic double ratio of the string, e.g., 98304:18, 98304:36, and 98304:72. Likewise, the α-ε pairing exhibits all ratios to which the β-ζ pairing gives rise, but includes some possibilities, shared with the Δ-δ pairing, that are not included in the set of possibilities for the β-ζ pairing and that are not reducible to the terms of the basic double ratio of the strings, e.g., 221184:324 and 98304:144. Therefore, all strings of doubles move at speeds dissimilar to one another but still in synchronization with the double movement, so as not to desert order and proportion in their circulation, as Plato required (36 D). All four also obviously move at

different speeds from the strings of the subset of three because they do not participate in the triple movement.

Note that all lateral movements within the subset of four bands participating in the double motion depend, for their occurrences, upon the influence of the oblique triple motion of the subset of three, reinforced by the oblique triple motion of band of same in the fixed celestial sphere. The 3/2 ratio does not come into play, for example, without the cross play of the triple motion; nor does any other lateral motion among the doubles bands, including the 256/243 motion mentioned, above, in the discussion concerning how one finds an octave in the matrix, rather than by means of Table 26.

In any case, in view of the proposed divisions, the planetary orbits align with the separated strings of the broad band of the fabric, as follows, compatibly with the tradition of Proclus' commentary on the text. The orbit of the sun is the string of triples extending from 3 to 729.[5] The orbit of Venus corresponds to the shorter string of triples extending from 54 to 1458. Mercury's orbit is the shortest orbit of all, aligning with the string of triples extending from 972 to 2916. The moon's orbit, in its singular proximity to the earth is represented by the γ-string of doubles. The orbit of Mars is the band consisting in the Δ-δ doubles strings pair; that of Jupiter consists in the band comprised of the α-ε doubles strings; and the orbit of Saturn consists in the β-ζ doubles strings pairs.[6]

At the end of Plato's construction, the soul sphere (represented in only two dimensions), looks something like Figure 24. The musical explanation of the text certainly fits with the astronomical interpretation that others have given it. Each of the seven "planets" for which Plato sought an account occupies one of the circles resulting from the divisions of the motion of different against the fixed celestial sphere, represented by the motion of same. The triple motion moves obliquely to the double motion among the bands of difference, just as it

5 See n. 10 to Chapter 4 regarding Plutarch's comment that the tradition of interpretation of the *Timaeus* associated the orbit of the sun with the number 729.

6 Proclus also believed that the sun, Mercury, and Venus belonged in the same grouping. In other words, he assigned them to the subset of three, rather than the subset of four. He put the moon with Mars, Jupiter and Saturn and assigned the latter four to the subset moving at dissimilar speeds. See Proclus *in Timaeum 3.2.264;* Wear, *Syrianus*, 146; see also, Plato *Tim.* [Jowett] 36 D: Plato *Tim.* [Zeyl n. 16] 36 D. However, he noted a disparity of interpretation concerning the proper groupings among interpreters of the text and expressed mild consternation at Plato's representation that the subsets of three and four planets moved in opposite manner to each other. He offered no explanation elucidating that part of Plato's text but spoke, instead, of the differences of interpretation on that point. Ibid.

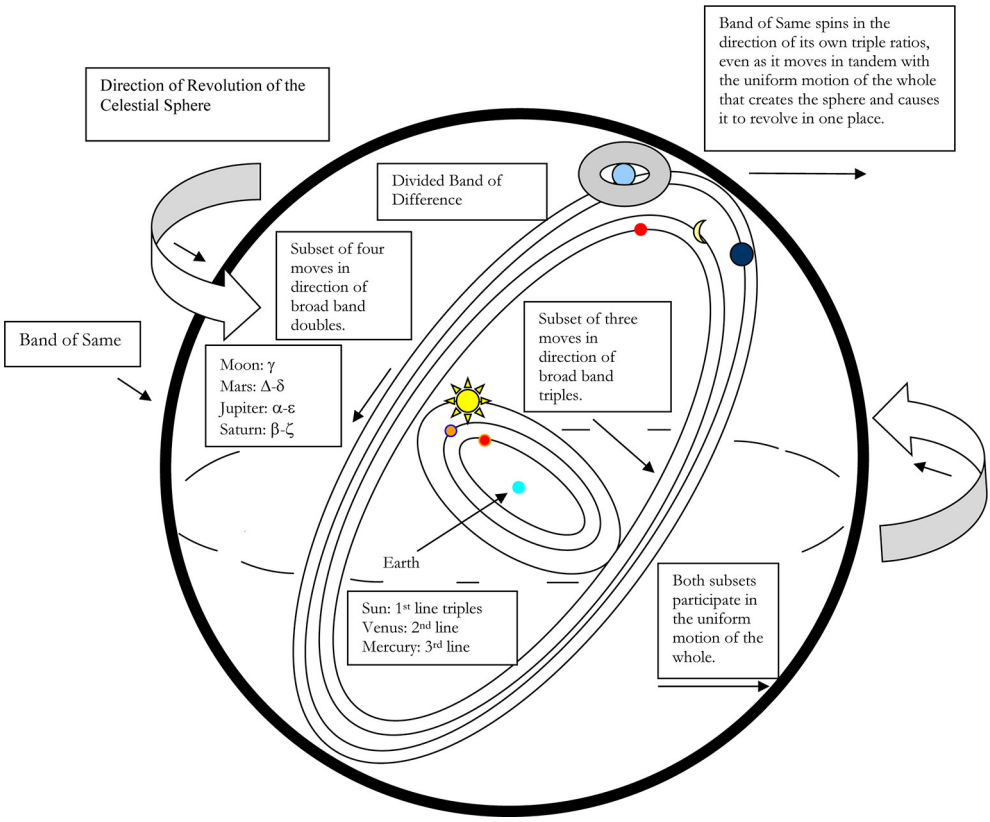

FIGURE 24 The (stylized) soul sphere

did in the uncut fabric. This cross movement, reinforced by the triple motion of the fixed celestial sphere, moving in *tandem* with it, is the movement of the 3/2 ratio. It is one with the uniform motion of the whole. In harmony with the double and triple motions, it allows the circling among the elements of the bands of difference that sound the harmony of the spheres, namely, the sounds of the different octave chains for which the numbers in γ-ζ are STNs.

Recall that the octave chains are pure monotypes since the removal of chromatic elements to the fixed celestial sphere has freed the doubles of chromatic interference. As the study has indicated, purified of such influence, the γ-string gives rise to nine (9) d.Mixolydian/a.Hypolydian diatonic patterns before octave periodicity disappears. The Δ-string gives rise to ten (10) d.Dorian/a.Lydian diatonic patterns. The δ-string gives rise to nine (9) d.Hypodorian/a.Hypophrygian diatonic octaves. The α-string gives rise to ten (10) d.Phrygian/a.Phrygian diatonic patterns. The ε-string gives rise to nine (9) d.Hypophrygian/a.Hypodorian diatonic octaves. The β-string gives rise to ten

(10) d.Lydian/a.Dorian diatonic octaves; and the ζ-string gives rise to nine (9) d.Hypolydian/a.Mixolydian diatonic octaves.

All seven of the possible octave chains sound, moving according to proportion among the bands of difference comprising the planetary orbits, to produce the cosmic *harmonia*. The musical model, then, becomes the symbol of the planetary order, in relation to the earth, giving new significance to the notion of a music of the spheres. The text is polyvalent, in reference to astronomy and music, in accord with the special genius of the ancients, because of the parallel relations among its different legitimate levels of interpretation.

The foregoing observations very nearly bring this study to a close—but not quite; for Plato did not restrict his consideration of the musical cosmology hidden in *Timaeus* 35 A–36 D to the *Timaeus* itself. He generalized his model in the *Laws*, as Chapter 8 shows, bearing witness to the intellectual fascination it held for him. There he considered its application at the microcosmic level and articulated a system of human social relations conforming to the *harmonia* in the image of the "All Perfect Animal."

CHAPTER 8

Plato's Generalization of the *Timaean Harmonia* in *Laws*

Timaeus 35 A–36 D are reaffirmed and generalized in a hidden manner in another set of famous related passages pertinent to the history of western music theory, namely *Laws* 737 E–738 E, 745 B–745 E, and 746 D–747 B.[1] In the *Laws, Plato* was concerned to erect an ideal of a *polis* optimally constituted, in its legislation, to foster conditions in which citizens would be directed to the common good. He sought in other words, to achieve a *harmonia* of political life.

One might suspect that number is not to be neglected as a matter of legislation where concerns about a political *harmonia* exist; and it, indeed, entered into Plato's reckoning in Book V of the Laws at 737 E–738 A, as he attempted to determine the ideal number of landholders or households in the *polis*. Plato settled upon 5040 as the appropriate number, styling it a convenient number, as follows:

> Let us assume—to take a convenient number—that we have five thousand and forty landholders, who can be armed to fight for their holdings, and that the territory and houses are likewise divided among the same number, so that there will be one man to one holding. Let this total be divided first by two, and then by three; in fact it will permit of division by four, five, and the successive integers up to ten. Of course anyone who is acting as a legislator must be at least familiar enough with figures to understand what number, or kind of number, will prove most useful in a given state. Accordingly we will select that which has the greatest number of immediately successive divisions. The whole integer series, of course, admits division by any number and with any quotient, while our five thousand forty can be divided, for purposes, of war, or to suit the engagements and combinations of peace, in the matter of taxes to be levied and public

1 The possibility of what the text, above, characterizes as a "generalization" suggests that the *Timaeus* predated the *Laws* in Plato's authorship. While it is possible that the relationship is reversed, it seems more natural that the less comprehensive effort of the *Timaeus* came first, providing a basis for later insight concerning a wider scheme.

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> distributions to be made, into fifty-nine quotients and no more, ten of them, from unity onward, being successive.[2]

While it is, of course, convenient that the number of households in the *polis* be able to be divided for different purposes by any of the first ten integers, that observation does not exhaust the significance of Plato's choice of 5040 households as the correct size for the community or his designation of the number as a convenient number. To unpack the mystery, one must dig a bit deeper.

Plato was well aware of the possibilities for religious conflict in the *polis* that he erected in *Laws*, as it gathered peoples of different customs and backgrounds; so he suggested at 738 B–D that a legislator must avoid interfering with the worship of the gods in the community and must provide, instead, for their reverence. Further, he tied the reasons for a legislator's legitimate concern with number to this issue, stating:

> These facts of number, then, must be thoroughly mastered at leisure by those whose business the law will make it to understand them—they will find them exactly as I have stated them—and they must be mentioned by the founder of a city, for the reason I shall now give. Whether a new foundation is to be created from the outset or an old one restored in the matter of gods and their sanctuaries—what temples must be founded in a given community, and to what gods or spirits they should be dedicated—no man of sense will presume to disturb convictions inspired from Delphi, Dodona, the oracle of Ammon, or by old traditions of any kind of divine appearances or reported divine revelations, when those convictions have led to the establishment of sacrifice and ritual—whether original and indigenous, or borrowed from Etruria Cyprus, or elsewhere—the consequent consecration by the tradition of oracles, statues, altars, and shrines, and the provision for each of these of its sacred precinct. A legislator should assign every district its patron god, or spirit, or hero, as the case may be, and his first step in the subdivision of a territory should be to assign to each of them his special precinct with all appertaining dues. His purpose in this will be that the convocations of the various sections at stated periods may provide opportunities for the satisfaction of their various needs, and that the festivities may give occasion for mutual

2 Plato *Laws* (trans. Trevor J. Saunders, in *Plato: Complete Works*, eds. John M. Cooper and D.S. Hutchinson [Indianapolis, Cambridge: Hackett Publishing Co., 1997]) 737 E–738 A.

> friendliness, familiarity, and acquaintance. There is indeed [E] no such boon for a society as this familiar knowledge of citizen by citizen.[3]

It is clear from the above passage that Plato understood the *harmonia* of the entire community to be dependent upon the absence, in it, of bickering about the gods; and this is best achieved if each relevant god is assigned a special precinct. One might expect the correct number of subdivisions to be twelve according to the accepted division of the Greek pantheon; and Plato bore out this expectation in *Laws* 745 B–745 E:

> Next, the founder of the city must see that his city is placed as nearly as possible at the center of the territory, after selecting a site possessed of the other favorable conditions for his purpose—it will not be difficult to discover or to state them. Then he must divide his city into twelve parts but first he should establish and enclose a sanctuary of Hestia, Zeus, and Athena—which he call the citadel—from which he will draw his twelve divisions of the city and its whole territory. Equality of the twelve regions should be secured by making those of good soil small and those of worse soil larger. He should then make a division into five thousand and forty allotments. Each of these, again, shall be bisected and two half sections, a nearer and a remoter, paired together to form an allotment, one which is contiguous to the city with one on the border, one in the next degree of proximity to the city with one next most nearly on the border, and so on in all cases. We should further practice in these half sections the already mentioned contrivance relative to the poverty or excellence of the soil and effect an equalization by the greater or lesser size of the divisions. Of course, the legislator must also divide the population into twelve sections, constructing these sections so as to be nearly as possible on an equality in respect of their other property, of the whole of which he will have made a careful record. Next he will be at pains to assign the twelve divisions to twelve gods, naming each section after the god to whom it has been allotted and consecrated, and calling it a tribe. Further, the twelve segments of the city must be made on the same lines as the division of the territory in general, and each citizen must have two houses, one nearer the center of the state and the other nearer the border. And this shall complete the business of settlement.[4]

3 Ibid., 738 B–D

4 Ibid., 745 B–E.

Note that, among its other convenient features the number 5040 can be divided into twelve equal parts of four hundred twenty, fitting Plato's requisite to assign an equal precinct in the community to each of the twelve gods. The assignment of one god each to these twelve parts leads the astute reader to suspect that there should be a special convenient number unique to each of the twelve parts, as well as a convenient number fitted to the whole; and further that Plato's references to more and less fertile soil have to do with a distribution of more or less convenient numbers. Accordingly, part of the effort to unpack how the political *harmonia* is connected to a more universal *harmonia* is a search for the principle whereby Plato calls a number convenient.

The hunch that one is to search for a universal *harmonia*, encompassing the political *harmonia* that Plato attempts to achieve, through number, in the matter of worship of the gods, is legitimated by a the further discourse of Laws 746 D–747 B. Plato wrote as follows:

> Our immediate concern, now that we have resolved on the division into twelve parts, must be precisely to see in what conspicuous fashion these twelve parts, admitting, as they do such a multitude of further divisions, with the subsequent groups which arise from them, down to the five thousand and forty individuals—this will give us our brotherhoods, wards, and parishes, as well as our divisions of battle and columns of route, not to mention our currency and measures of capacity, dry and liquid and of weight—to see I say how all these details must be legally determined so as to fit in and harmonize with each other. There is a further fear we must dismiss, apprehension for finicking pedantry if the law enacts that no utensil whatever in the possession of a citizen shall be of other than the standard size. The legislator must take it as a general principle that there is a universal usefulness in the subdivisions and complications of numbers whether these complications are exhibited in pure numbers, in lengths and depths or again in musical notes and motions, whether of rectilinear ascent and descent or of revolution. All must be kept in view by the legislator in his injunction to all citizens, never, so far as they can help it, to rest short of this numerical standardization. For alike in domestic and public life and in all the art and crafts there is no other single branch of education which has the same potent efficacy as the theory of numbers, but its greatest recommendation is that it rouses the naturally drowsy and dull, and makes him [747 B] quick, retentive, and shrewd—a miraculous improvement of cultivation upon his native parts ….[5]

5 Ibid., 746 D–747 B.

TABLE 50 The fifty-nine factors of 5040

1	2	3	4	5	6	7	8	9	10
12	14	15	16	18	20	21	24	28	30
35	36	40	42	45	48	56	60	63	70
72	80	84	90	105	112	120	126	140	144
168	180	210	240	252	280	315	336	360	420
504	560	630	720	840	1008	1260	1680	2520	5040

To find political *harmonia* in the quest for the principle by which a number is convenient, one must begin with Plato's initial observations about the number 5040. If one tests Plato's description of its features, one finds that it is, indeed, divisible consecutively by the first ten integers and that it, indeed, has fifty-nine factors, not counting "one" ("one" is listed for convenience in Table 50; perhaps one would prefer not to count 5040, instead).

One notes first the remarkable divisibility of 5040 by the first ten integers. This is convenient not only because of the happy divisions of men it allows but because of the relations it allows among those divisions. The first ten numbers contain, *inter alia*, all of the basic musical ratios significant to ancient Greek music: the double: 1:2, 2:4, 3:6, 4:8, 5:10; the triple: 1:3, 2:6, 3:9; the disdiapason: 1:4, 2:8; the fifth: 3/2, 6/4; the fourth: 4/3, 8/6; and the whole tone: 9/8.[6] The topic is political harmony and the universality of the harmonic law of numbers; so, it is good to be able to divide men in such a way that the ratios among them can mimic the cosmic *harmonia*.

One can select twelve numbers among the fifty-nine factors of 5040 that are special on the basis of harmonic principles, in the same way. They are highlighted in Table 51, below, in yellow, accompanied by certain calculations capturing the character of their particularity.

The twelve factors of 5040 highlighted, in yellow, in Table 51, are special because they are the only numbers in the factor set that allow the articulation of the basic musical ratios: 9/8, 4:3, 3:2, 2:1, 3:1. They are convenient numbers because they embed the musical ratios. If one divides men, using, them, one can achieve a *harmonia*. Each is, therefore, assigned uniquely to one of the twelve gods of the Greek pantheon and one of the twelve sections of the *polis*.

6 The other possible ratios allow the search for alternative musical systems dependent upon different basic relations—but that search strays from the purpose of this study and, so, will receive no further comment here.

The twelve special factors are not just convenient in embedding the musical ratios. They are also especially fertile in their capacities, collectively, to generate four unique sources of the cosmic *harmonia*. Key to this fertility is another feature that all of them have in common. Each is divisible by nine (each, as represented in our decimal system, also has digits, the sum of which is nine).

Note that the twelve special factors may be grouped into four distinct sets of doubles, parallel to the four quadrants of the zodiac and the triads of gods, thereof, as follows: {36, 72, 144}, {180, 360, 720}, {252, 504, 1008}, and {1260, 2520, 5040}. Each set gives rise to its own unique expression of the very same cosmic *harmonia*.

Consider the set {36, 72, 144}. None of these numbers, as it stands, belongs to a scale of any kind; neither does the first multiple of 144, namely 288 (144 × 2). The next multiple, however, 432 (144 × 3), another number divisible by nine (and having digits adding to nine), is the very first number, in series with the indicated triad, occurring in any of the Greek scales. It makes its earliest appearance as the second step in a *bona fide* descending Dorian or ascending Lydian diatonic disdiapason as follows.

> {384 T 432 T 486 S 512 T 576 T 648 T 729 S [768} T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536]

In its second appearance, it is the root of a *bona fide* ascending or descending Phrygian diatonic disdiapason as follows:

> {432 T 486 S 512 T 576 T 648 T 729 S 768 T [864} T 972 S 1024 T 1152 T 1296 T 1458 S 1536 T 1728]

Accordingly, the first set of diatonic scale numbers connected with the set {36, 72, 144} is identical to the *Timaeus* set from 384; but the scale properly originating from it is an ascending or descending Phrygian disdiapason. In the *Timaeus*, too, the d.Dorian/a.Lydian and d.Phrygian/a.Phrygian patterns compete for prominence.

Now consider the set {180, 360, 720}. None of these numbers, as it stands, leads to a scale of any kind; neither does the first multiple of 720, namely 1440 (720 × 2). The next multiple, however, 2160 (720 × 3), another number divisible by nine (and having digits adding to nine), is the very first number, in series with the indicated triad, occurring in any of the Greek scales. It makes its earliest appearance as the second step in a *bona fide* descending Dorian or ascending Lydian disdiapason, as indicated immediately following Table 51.

TABLE 51 The fifty-nine factors of 5040: numbers with special features

1	2	3	4	5
12	14	15	16	18
35	36 *36 = 9/8 × 32; *36 = 2 × 18; *36=3 × 12 *36=3/2 × 24 *36=4/3 × 27	40	42	45
72 *72 = 9/8 × 64; *72 = 2 × 36; *72=3 × 24 *72=3/2 × 48 *72=4/3 × 54	80	84	90	105
168	180 *180 = 9/8 × 160; *180 = 2 × 90; *180=3 × 60 *180=3/2 × 120 *180=4/3 × 135	210	240	252 *252 = 9/8 × 224; *252 = 2 × 126; *252=3 × 84 *252=3/2 × 168 *252=4/3 × 189
504 *504 = 9/8 × 448; *504 = 2 × 252; *504=3 × 168 *504=3/2 × 336 *504=4/3 × 378	560	630	720 *720 = 9/8 × 640; *720= 2 × 360; *720=3 × 240 *720=3/2 × 480 *720=4/3 × 540	840

6	7	8	9	10
20	21	24	28	30
48	56	60	63	70
112	120	126	140	144 * 144 = 9/8 × 128; * 144 = 2 × 72; * 144=3 × 48 * 144=3/2 × 96 * 144=4/3 × 108
280	315	336	360 * 360 = 9/8 × 320; * 360 = 2 × 180; * 360=3 × 120 * 360=3/2 × 240 * 360=4/3 × 270	420
1008 * 1008 = 9/8 × 896; * 1008= 2 × 504; * 1008=3 × 336 * 1008=3/2 × 672 * 1008=4/3 × 756	1260 * 1260 = 9/8 × 1120; * 1260= 2 × 630; * 1260=3 × 420 * 1260=3/2 × 840 * 1260=4/3 × 945	1680	2520 * 2520 = 9/8 × 2240; * 2520= 2 × 1260; * 2520=3 × 840 * 2520=3/2 × 1680 * 2520=4/3 × 1890	5040 * 5040 = 9/8 × 4480; * 5040= 2 × 2520; * 5040=3 × 1680 * 5040=3/2 × 3360 * 5040=4/3 × 3780

{1920 T 2160 T 2430 S 2560 T 2880 T 3240 T 3645 S [3840} T 4320 T 4860 S 5120 T 5760 T 6480 T 7290 S 7680]

Clearly, the above scale is bereft of *Timaeus* numbers; however, it bears an analogous relationship to the primary *Timaeus* scale. One would likely be able to trace a similar process of degeneration of octave periodicity to fifth periodicity upon a sufficient number of repetitions, if one set up an appropriate Crantor matrix, generative of the series. One would also be able to derive all of the musical facts discovered on the basis of the *Timaeus* set, using a number matrix generative of the above scale.

Like 432, which stood in continuity with the first triad, 2160, standing in continuity with the second, makes its second appearance as the root of a *bona fide* ascending or descending Phrygian disdiapason:

{2160 T 2430 S 2560 T 2880 T 3240 T 3645 S 3840 T [4320} T 4860 S 5120 T 5760 T 6480 T 7290 S 7680 T 8640]

Note that every element of each scale associated with the second triad is the fifth multiple of the corresponding element in the same scale type associated with first triad of numbers.

Now consider the set {252, 504, 1008}. None of these numbers, as it stands, leads to a scale of any kind; neither does the first multiple of 1008, namely 2016 (1008 × 2). The next multiple, however, 3024 (1008 × 3), a number divisible by nine (and having digits adding to nine), is the very first number in series with the indicated triad, occurring in any of the Greek scales. It makes its earliest appearance as the second step in a *bona fide* descending Dorian or ascending Lydian disdiapason as follows:

{2688 T 3024 T 3402 S 3584 T 4032 T 4536 T 5103 S [5376} T 6048 T 6804 S 7168 T 8064 T 9072 T 10206 S 10752]

This scale, associated with the third triad, like the d.Dorian/a.Lydian scale associated with the second triad, is also bereft of *Timaeus* numbers; however, it bears an analogous relationship to the primary *Timaeus* scale. One would likely be able to trace a similar process of degeneration of octave periodicity to fifth periodicity upon a sufficient number of repetitions, if one set up an appropriate Crantor matrix, generative of the series. One would also be able to derive all of the musical facts discovered on the basis of the *Timaeus* set, using a number matrix generative of the above scale.

Like 432, in continuity with the first triad, and 2160, in continuity with the second, 3024 makes its second appearance as the root of a *bona fide* ascending or descending Phrygian disdiapason:

{3024 T 3402 S 3584 T 4032 T 4536 T 5103 S 5376 T [6048} T 6804 S 7168 T 8064 T 9072 T 10206 S 10752 T 12096]

Note that every element of each scale associated with the third triad is the seventh multiple of the corresponding element in the same scale type associated with first triad of numbers.

Consider, finally, the set {1260, 2520, 5040}. None of these numbers, as it stands, leads to a scale of any kind; neither does the first multiple of 5040, namely 10080 (5040 × 2). The next multiple, however, 15120 (5040 × 3), a number divisible by nine (and having digits adding to nine), is the very first number in series with the indicated triad, occurring in any of the Greek scales. It makes its earliest appearance as the second step in a *bona fide* descending Dorian or ascending Lydian disdiapason as follows:

{13440 T 15120 T 17010 S 17920 T 20160 T 22680 T 25515 S [26880} T 30240 T 34020 S 35840 T 40320 T 45360 T 51030 S 53760]

This scale contains no *Timaeus* numbers; however, as in the cases of the like scale associated with the second and third triads, it bears an analogous relationship to the primary *Timaeus* scale. One would likely be able to trace a similar process of degeneration of octave periodicity to fifth periodicity upon a sufficient number of repetitions, if one set up an appropriate Crantor matrix, generative of the series. One would also be able to derive all of the musical facts discovered on the basis of the *Timaeus* set, using a number matrix generative of the above scale.

As in the other cases, 15120 makes its second appearance as the root of a *bona fide* ascending or descending Phrygian disdiapason:

{15120 T 17010 S 17920 T 20160 T 22680 T 25515 S 26880 T [30240} T 34020 S 35840 T 40320 T 45360 T 51030 S 53760 T 60480]

Note that every element of the scales associated with the fourth triad is the thirty-fifth (5 × 7) multiple of the corresponding element in the scales associated with the first triad, the seventh multiple of the corresponding element in the scales associated with the second triad, and the fifth multiple of the corresponding element in the scales associated with the third triad.

TABLE 52 The fifty-nine factors of 5040: additional annotations

1	2	3	4	5
12	14	15	16	18
35	36 *36 = 9/8 × 32; *36 = 2 × 18; *36=3 × 12 *36=3/2 × 24 *36=4/3 × 27	40	42	45
72 *72 = 9/8 × 64; *72 = 2 × 36; *72=3 × 24 *72=3/2 × 48 *72=4/3 × 54	80	84	90	105
168	180 *180 = 9/8 × 160; *180 = 2 × 90; *180=3 × 60 *180=3/2 × 120 *180=4/3 × 135	210	240	252 *252 = 9/8 × 224; *252 = 2 × 126; *252=3 × 84 *252=3/2 × 168 *252=4/3 × 189
504 *504 = 9/8 × 448; *504 = 2 × 252; *504=3 × 168 *504=3/2 × 336 *504=4/3 × 378	560	630	720 *720 = 9/8 × 640; *720= 2 × 360; *720=3 × 240 *720=3/2 × 480 *720=4/3 × 540	840

6	7	8	9	10
20	21	24	28	30
48	56	60	63	70
112	120	126	140	144 * 144 = 9/8 × 128; * 144 = 2 × 72; * 144=3 × 48 * 144=3/2 × 96 * 144=4/3 × 108
280	315	336	360 * 360 = 9/8 × 320; * 360 = 2 × 180; * 360=3 × 120 * 360=3/2 × 240 * 360=4/3 × 270	420
1008 * 1008 = 9/8 × 896; * 1008= 2 × 504; * 1008=3 × 336 * 1008=3/2 × 672 * 1008=4/3 × 756	1260 * 1260 = 9/8 × 1120; * 1260= 2 × 630; * 1260=3 × 420 * 1260=3/2 × 840 * 1260=4/3 × 945	1680	2520 * 2520 = 9/8 × 2240; * 2520= 2 × 1260; * 2520=3 × 840 * 2520=3/2 × 1680 * 2520=4/3 × 1890	5040 * 5040 = 9/8 × 4480; * 5040= 2 × 2520; * 5040=3 × 1680 * 5040=3/2 × 3360 * 5040=4/3 × 3780

Note that the special numbers constituting roots of the cosmic *harmonia*, as articulated above, are not the only numbers among the factors of 5040 divisible by nine (and having digits that add up to nine). See the additional eight numbers, highlighted in green, in the Table 52, above, also sharing that feature.

If one multiplies any one of the numbers highlighted in green, in Table 52, a sufficient number of times, one eventually arrives at one of the special factors highlighted in yellow. Indeed, any one of the factors in the table, multiplied a sufficient number of times, will eventually arrive at a special number divisible by nine (and having digits adding to nine), a number in other words, that can be further multiplied to arrive at one of the four unique, but related, articulations of the cosmic *harmonia*. No number indivisible by nine can be the nearest root of the *harmonia;* and some numbers divisible by nine (the twelve special factors) are closer to the *harmonia* than others by their immediate, internal witness to the formulas for the rise of the musical scale. Numbers are more or less fertile, and so, divisions of things by those numbers are more or less fertile, on the basis of their proximity to the cosmic *harmonia*.

Plato would have all divisions of things by number in the *polis* as close to harmony as possible as quickly as possible. The division of the community into 5040 household units is the smallest division generating twelve special factors attributable to the twelve deities by their relation to harmony. The next larger division generating such factors, unfortunately, also permits increased inconsonant relations. The division into 5040 units clearly allows the greatest concentration of possible divisions conforming to the cosmic *harmonia*—and it turns out that this is no coincidence. The diatonic scale allows exactly 5040 melodic permutations ($7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$).

The unexpected prominence of the number nine in relation to the cosmic *harmonia* may suggest a relation to the *Ennead* in ancient creation myths of all kinds. Comments, at this juncture, on such a relationship are, at best, speculative; but some enterprising student of ancient mythological and Near Eastern religious lore could find the suggestion sufficiently intriguing for research. Let there be at least one such curious mind.

Concluding Remarks

The new analysis of *Timaeus* 35 A–36 D offered by this study is worthy of careful consideration because it appears adequately to account for all elements of Plato's text from 35 A–36 D. It also draws special attention to a pattern of generation, governed by the Decad, that emerges as normative over and over again in the *Timaeus*, namely: a move from (a) same to different (monad to duad) to (b) sameness within difference (triad) to (c) difference within sameness within difference (tetrad). The object generated at the level of difference within sameness within difference becomes a new, higher level monad; and the pattern repeats, leading to additional generation and greater complexity. Although the ancients interpreting Plato did not discover the solution to the *Timaeus* riddle offered by this analysis, the study stands well within the spirit of their endeavors. It confirms Speusippus' intuition that the exemplar hid by the *Timaeus*, as the pattern according to which the cosmos came into being, was, indeed, the Decad.[1] The presence of same structure at the heart of the musical sections of the *Laws* does not appear coincidental and, indeed, highlights its importance.

One can only marvel at the genius of the ancient thinkers who recognized that the universe was either too vast or two small to be measured in anything but relative terms, and so determined to measure it in harmonic units. Ordered relations characterize the cosmos, and they begin with the ordered relations determining the perceptual capacities of human beings. These, the *Timaeus* argues, are the harmonic relations implicit in the Decad. Harmonic relations must govern the whole, too, since man is part of the whole, and the same principle runs through all of it. The microcosm is, necessarily, a reflection of the macrocosm for Plato; and that is why the Decad is the "All Perfect Animal".

1 See John Dillon, "The *Timaeus* in the Old Academy," 82–83.

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Appendices

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APPENDIX 1

Verification of the *Diesis* Remaining after Insertion of Two Sesquioctave Intervals into a Sesquitertian Part for the Sample Sesquitertian Part, 2:8/3

First mode of verification	**First mode of verification**	**Names for the ratio 256/243**
(1) 162/64 × n = 8/3 n = 8/3 × 64/162 n = 512/486 Simplifying, one verifies 256/243	(2) 81/32 × n = 8/3 n = 8/3 × 32/81 n = 256/243	**512/486** and **256/243** *New numbers needed in the Crantor table to represent these fractions*: **243, 256, 486, 512**
Second mode of verification[1]	**Second mode of verification**	**Second mode of verification**
(1) 162/64 × **256/243** = 41472/15552.	(2) 81/32 × **256/243** = 20736/7776	*New numbers needed in the Crantor matrix to represent fractions arising from calculations* (1) **41472; 15552**
To get all of the names for this fraction, one must factor as far as one can go by 2's, then as far as one can go by 3's, then according to as many alternative ways of factoring by both 2's and 3's as possible.	To get all of the names for this fraction, one must factor as far as one can go by 2's, then as far as one can go by 3's, then according to as many alternative ways of factoring by both 2's and 3's as possible.	(2) **20736; 7776**
Factoring as far as one can by 2's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243. Division by 2 is no longer possible; so division by 3 continues: 216/81; 72/27; 24/9; **8/3.** √	Factoring as far as one can by 2's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243. Division by 2 is no longer possible; so division by 3 continues: 216/81; 72/27; 24/9; **8/3.** √	(1) In addition to above, 20736; 7776; 10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72. (2) In addition to above, 10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72. Thus, not repeating any numbers identified as new above in table or otherwise: **10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72.**

1 One factors by 2's and 3's, below, because division by doubles and triples is characteristic of Plato in the *Timaeus*.

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(*cont.*)

Second mode of verification	**Second mode of verification**	**Second mode of verification**
Factoring as far as one can by 3's yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 512/192. Division by 3 is no longer possible; so division by 2 continues: 256/96; 128/48; 64/24; 32/12; 16/6; **8/3.** √	Factoring as far as one can by 3's yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 256/96. Division by 3 is no longer possible; so division by 2 continues: 128/48; 64/24; 32/12; 16/6; **8/3.** √	(1) In addition to above, 13824; 4608; 1728; 1536; 576; 192. (2) 6912; 2304; 864; 768; 288. Thus, not repeating any numbers identified as new above in table or otherwise: **13824; 4608; 1728; 1536; 576; 192; 6912; 2304; 864; 768; 288.**
Factoring alternately first by 2's and then by 3's yields the following names for 41472/15552: 20736/7776; 6912/2592; 3456/1296; 1152/432; 576/216; 192/72; 96/36; 32/12; 16/6. Alternate division by 3 is no longer possible; so division by 2 continues: **8/3.** √	Factoring alternately first by 2's and then by 3's yields the following names for 20736/7776: 10368/3888; 3456/1296; 1728/648; 576/216; 288/108; 96/36; 48/18; 16/6; **8/3.** √	(1) 3456; 1152; 432 (2) 3456; 1728; 648 Thus, not repeating any numbers identified as new above in table or otherwise: **3456; 1152; 432; 1728; 648**
Factoring alternately first by 3's and then by 2's yields the following names for 41472/15552: 13824/5184; 6912/2592; 2304/864; 1152/432; 384/144; 192/72; 64/24; 32/12. Alternate division by 3 is no longer possible; so division by 2 continues: 16/6; **8/3.** √	Factoring alternately first by 3's and then by 2's yields the following names for 20736/7776: 6912/2592; 3456/1296; 1152/432; 576/216; 192/72; 96/36; 32/12; 16/6. Alternate division by 3 is no longer possible; so division by 2 continues: **8/3.** √	(1) No new numbers. (2) No new numbers.
Factoring alternately in some way first by 2's and then by 3's continues to yield some new fractional expressions and one new number.	Factoring alternately in some way first by 2's and then by 3's continues to yield some new fractional expressions and one new number.	
For example, factoring first by two 2's and then by 3, yields the following names for 41472/15552: 20736/7776; 10368/3888; 3456/1296; 1728/648; 864/324; 288/108; 144/54; 72/27; 24/9. Alternate division by 2 is no longer possible. Division by 3 continues: **8/3.** √	For example, factoring first by two 2's and then by 3, yields the following names for 20736/7776: 10368/3888; 5184/1944; 1728/648; 864/324; 432/162; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	(1) 324 (2) 324 Thus, not repeating any numbers identified as new above in table or otherwise: **324**

(*cont.*)

Second mode of verification	Second mode of verification	Second mode of verification
Factoring first by two 2's and then two three's yields the following names for 41472/15552: 20736/7776; 10368/3888; 3456/1296; 1152/432; 576/216; 288/108; 96/36; 32/12; 16/6; **8/3.** √	Factoring first by two 2's and then two three's yields the following names for 20736/7776: 10368/3888; 5184/1944; 1728/648; 576/216; 288/108; 144/54; 48/18; 16/6; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by three 2's and one three yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 1728/648; 864/324; 432/162; 216/81; 72/27. Alternate division by 2 is no longer possible; so division continues by 3: 24/9; **8/3.** √	Factoring by three 2's and one three yields the following names for 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 432/162; 216/81. Division by 2 is no longer possible. Division by 3 continues: 72/27; 24/9; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by three 2's and two 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 1728/648; 576/216; 288/108; 144/54; 72/27; 24/9; **8/3.** √	Factoring by three 2's and two 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by three 2's and three 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 1728/648; 576/216; 192/72; 96/36; 48/18; 24/9; **8/3.** √	Factoring by three 2's and three 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 96/36; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by four 2's and one three yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 432/162; 216/81. Division by 2 is no longer possible; so division continues by 3: 72/27; 24/9; **8/3.** √	Factoring by four 2's and one 3 yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 216/81. Division by 2 is no longer possible. Division by 3 continues: 72/27; 24/9; **8/3.** √	(1) No new numbers (2) No new numbers

(*cont.*)

Second mode of verification	Second mode of verification	Second mode of verification
Factoring by four 2's and two 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 144/54;72/27. Division by two is no longer possible. Division by 3 continues: 24/9; **8/3.** √	Factoring by four 2's and two 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by four 2's and three 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 96/36; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3.** √	Factoring by four 2's and three 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by four 2's and four 3's: yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 96/36; 32/12; 16/6; **8/3.** √	Factoring by four 2's and four 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 16/6; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by five 2's and one three yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 216/81. Division by 2 is no longer possible. Division by 3 continues: 72/27; 24/9; **8/3.** √	Factoring by five 2's and one 3 yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81. Division by 2 is no longer possible. Division by 3 continues: 72/27; 24/9; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by five 2's and two 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	Factoring by five 2's and two 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	(1) No new numbers (2) No new numbers

(*cont.*)

Second mode of verification	Second mode of verification	Second mode of verification
Factoring by five 2's and three 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3. √**	Factoring by five 2's and three 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; 72/27; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3. √**	(1) No new numbers (2) No new numbers
Factoring by five 2's and four three's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 16/6; **8/3. √**	Factoring by five 2's and four 3's yields the following names for 20736/7776: 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; 72/27; 24/9; **8/3. √**	(1) No new numbers (2) No new numbers
Factoring by five 2's and five threes yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 16/6. Division by three is not possible. Division by 2 continues: **8/3. √**	Factoring by combination of five two's in combination with 3's complete with above.	(1) No new numbers (2) Not applicable
Factoring by six 2's and one three yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; Division by two no longer possible. Division by 3 continues: 72/27; 24/9; **8/3. √**	Factoring by six 2's and one three is not possible. Factoring possibilities taking now 2's then 3's complete.	(1) No new numbers (2) Not applicable
Factoring by six 2's and two 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3. √**		(1) No new numbers (2) Not applicable

(*cont.*)

Second mode of verification	Second mode of verification	Second mode of verification
Factoring by six 2's and three 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; 72/27; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3. √**	Likewise, factoring alternately in some way first by 3's and then by 2's yields no new numbers. Factoring first by two 3's and one 2 yields the following names for 20736/7776: 6912/2592; 2304/864; 1152/432; 384/144; 128/48; 64/24. Division by 3 is no longer possible. Division by 2 continues as follows: 32/12; 16/6; **8/3. √**	(1) No new numbers (2) No new numbers
Factoring by six 2's and four 3's yields the following names for 41472/15552: 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243; 216/81; 72/27; 24/9; **8/3. √**	Factoring by two 3's and two 2's yields the following names for 20736/7776: 6912/2592; 2304/864; 1152/432; 576/216; 192/72; 64/24; 32/12; 16/6. Division by 3 is no longer possible; division by 2 continues: **8/3. √**	(1) No new numbers (2) No new numbers
Factoring by six 2's and five 3's: No such sequence. Factoring possibilities by combinations of six 2's and 3's of any number end with sequence above.	Factoring by three 3's and one 2 yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 384/144; 128/48. Division by 3 is no longer possible. Division by 2 continues as follows: 64/24; 32/12; 16/6; **8/3. √**	(1) Not applicable (2) No new numbers
Factoring by six 2's and six 3's. No such sequence. See note immediately above.	Factoring by three 3's and two 2's yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 384/144; 192/72; 64/24. Division by 3 is no longer possible. Division by 2 continues as follows: 32/12; 16/6; **8/3. √**	(1) Not applicable (2) No new numbers
Factoring by seven 2's. No such sequence. Factoring possibilities by combinations taking first 2 and then 3 end with six 2's and four 3's.	Factoring by three 3's and three 2's yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 384/144; 192/72; 96/36; 32/12. Division by 3 is no longer possible. Division by 2 continues: 16/6; **8/3. √**	(1) Not applicable (2) No new numbers

(*cont.*)

Second mode of verification	Second mode of verification	Second mode of verification
Likewise, factoring alternately in some way first by 3's and then by 2's yields no new numbers.	Factoring by four 3's and one 2 yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 256/96; 128/48. Division by 3 is no longer possible. Division by 2 continues: 64/24; 32/12; 16/6; **8/3.** √	(1) Not applicable (2) No new numbers
Factoring first by two 3's and one 2 yields the following names for 41472/15552: 13824/5184; 4608/1728; 2304/864; 768/288; 256/96; 128/48. Division by 3 is no longer possible. Division by 2 continues. 64/24; 32/12; 16/6; **8/3.** √	Factoring by four 3's and two 2's yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 256/96; 128/48; 64/24. Division by 3 is no longer possible. Division by 2 continues: 32/12; 16/6; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by two 3's and two 2's yields the following names for 41472/15552: 13824/5184; 4608/1728; 2304/864; 1152/432; 384/144; 128/48; 64/24; 32/12. Division by 3 is no longer possible. Division by 2 continues: 16/6; **8/3.** √	Factoring by four 3's and three 2's yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 256/96; 128/48; 64/24; 32/12. Division by 3 is no longer possible. Division by 2 continues. 16/6; **8/3.** √	(1) No new numbers (2) No new numbers
Factoring by three 3's and one 2 yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 768/288; 256/96. Division by 3 is no longer possible. Division by 2 continues: 128/48; 64/24; 32/12; 16/6; **8/3.** √	Factoring by four 3's and four 2's yields the following names for 20736/7776: 6912/2592; 2304/864; 768/288; 256/96; 128/48; 64/24; 32/12; 16/6. Division by 3 is no longer possible. Division by 2 continues: 8/3.	(1) No new numbers (2) No new numbers
Factoring by three 3's and two 2's yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 768/288; 384/144; 128/48. Division by 3 is no longer possible. Division by 2 continues: 64/24; 32/12; 16/6; **8/3.** √	Factoring by five 3's is and then by 2 is not possible. Factoring possibilities now by 3 and then by 2 are exhausted with the above.	(1) No new numbers (2) Not applicable

(*cont.*)

Second mode of verification	Second mode of verification	Second mode of verification
Factoring by three 3's and three 2's yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 768/288; 384/144; 192/72; 64/24. Division by 3 is no longer possible. Division by 2 continues: 32/12; 16/6; **8/3.** √		(1) No new numbers (2) Not applicable
Factoring by four 3's and one 2 yields the following names for 41472/15552: yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 512/192; 256/96. Division by 3 is no longer possible. Division by 2 continues: 128/48; 64/24; 32/12; 16/6; **8/3.** √		(1) No new numbers (2) Not applicable
Factoring by four 3's and two 2's yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 512/192; 256/96; 128/48. Division by 3 is no longer possible. Division by 2 continues. 64/24; 32/12; 16/6; **8/3.** √		(1) No new numbers (2) Not applicable
Factoring by four 3's and three 2's yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 512/192; 256/96; 128/48; 64/24. Division by 3 is no longer possible. Division by 2 continues: 32/12; 16/6; **8/3.** √		(1) No new numbers (2) Not applicable
Factoring by four 3's and four 2's yields the following names for 41472/15552: 13824/5184; 4608/1728; 1536/576; 512/192; 256/96; 128/48; 64/24; 32/12. Division by 3 is no longer possible. Division by 2 continues: 16/6; **8/3.** √		(1) No new numbers (2) Not applicable

(*cont.*)

Second mode of verification	**Second mode of verification**	**Second mode of verification**
Options for factoring now by 3's in some combination and then by two's are exhausted with the possibility immediately above.		(1) Not applicable (2) Not applicable. *Total new numbers needed in the Crantor matrix to represent fractions for the first and second modes of verifying 256/243*: **243; 256; 486; 512; 41472; 15552; 20736; 7776; 10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72; 13824; 4608; 1728; 1536; 576; 192; 6912; 2304; 864; 768; 288; 3456; 1152; 432; 1728; 648; 324.**
Variant of second mode of verification	**Variant of second mode of verification**	**Variant of second mode of verification**
(1A) 162/64 × **512/486** = 82944/31104	Variant does not exist in the case of 81/32 since only value yielded for the remainder over in calculations based on this fraction is 256/243.	*New numbers needed in the Crantor matrix to represent fractions arising from calculations*
Factoring as far as one can by 2's yields the following names for 82944/31104; 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243. Division by 2 is no longer possible; so division by 3 continues: 216/81; 72/27; 24/9; **8/3. √**		(1A) 82944; 31104 (2) Not applicable Thus, not repeating any numbers identified as new above in table or otherwise: **82944; 31104**
Factoring as far as one can by 3's yields the following names for 82944/31104: 27648/10368; 9216/3456; 3072/1152; 1024/384. Division by 3 is no longer possible. Division by 2 continues: 512/192; 256/96; 128/48; 64/24; 32/12; 16/6; **8/3. √**		(1A) 27648; 9216; 3072; 1024 (2) Not applicable Thus, not repeating any numbers identified as new above in table or otherwise: **27648, 9216, 3072, 1024**

(*cont.*)

Variant of second mode of verification	**Variant of second mode of verification**	**Variant of second mode of verification**
Factoring alternately first by 2's and then by 3's yields the following names for 82944/31104: 41472/15552; 13824/5184; 6912/2592; 2304/864; 1152/432; 384/144; 192/72; 64/24; 32/12. Alternate division by 3 is no longer possible; so division by 2 continues: 16/6; **8/3.** √		(1A) No new numbers (2) Not applicable
Factoring alternately first by 3's and then by 2's yields the following names for 82944/31104: 27648/10368; 13824/5184; 4608/1728; 2304/864; 768/288; 384/144; 128/48; 64/24. Division by 3 is no longer possible. Division by 2 continues: 32/12; 16/6; **8/3.** √		(1A) No new numbers (2) Not applicable.
Factoring alternately in some way first by 2's and then by 3's continues to yield some new fractional expressions but no new numbers. For example, factoring first by two 2's and then by 3, yields the following names for 82944/31104: 41472/15552; 20736/7776; 6912/2592; 3456/1296; 1728/648; 576/216; 288/108; 144/54; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues **8/3.** √	For (1A) 162/64 × **512/486** = 82944/31104 Factoring alternately in some way first by 3's and then by 2's continues to yield some new fractional expressions but no new numbers. For example, factoring first by two 3's and then by 2, yields the following names for 82944/31104: 27648/10368; 9216/3456; 4608/1728; 1536/576; 512/192; 256/96. Division by 3 is no longer possible. Division by 2 continues: 128/48; 64/24; 32/12; 16/6; **8/3.** √	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable

(*cont.*)

Variant of second mode of verification	**Variant of second mode of verification**	**Variant of second mode of verification**
Factoring by two 2's and two 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 6912/2592; 2304/864; 1152/432; 576/216; 192/72; 64/24; 32/12; 16/6. Division by 3 is no longer possible. Division by 2 continues. **8/3.** √	Factoring by two 3's and two 2's yields the following names for 82944/31104: 27648/10368; 9216/3456; 4608/1728; 2304/864; 768/288; 256/96; 128/48; 64/24. Division by 3 is no longer possible. Division by 2 continues: 32/12; 16/6; **8/3.** √	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable
Factoring by three 2's and one 3 yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 3456/1296; 1728/648; 864/324; 432/162; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	Factoring by three 3's and one 2 yields the following names for 82944/31104; 27648/10368; 9216/3456; 3072/1152; 1536/576; 512/192. Division by 3 is no longer possible. Division by 2 continues: 256/96; 128/48; 64/24; 32/12; 16/6; **8/3.** √	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable
Factoring by three 2's and two 3's yields the following names for 82944/31104: 41472/15552; 20736/7776;10368/3888; 3456/1296; 1152/432; 576/216; 288/108; 144/54; 48/18; 16/6; **8/3.** √	Factoring by three 3's and two 2's yields the following names for 82944/31104; 27648/10368; 9216/3456; 3072/1152; 1536/576; 768/288; 256/96. Division by 3 is no longer possible. Division by 2 continues: 128/48; 64/24; 32/12; 16/6; **8/3.** √	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable
Factoring by three 2's and three 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 3456/1296; 1152/432; 384/144; 192/72; 96/36; 48/18; 16/6. Division by 3 is no longer possible. Division by 2 continues: **8/3.** √	Factoring by three 3's and three 2's yields the following names for 82944/31104; 27648/10368; 9216/3456; 3072/1152; 1536/576; 768/288; 384/144; 128/48. Division by 3 is no longer possible. Division by 2 continues: 64/24; 32/12; 16/6; **8/3.** √	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable

(*cont.*)

Variant of second mode of verification	**Variant of second mode of verification**	**Variant of second mode of verification**
Factoring by four 2's and one three yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 1728/648; 864/324; 432/162; 216/81. Division by 2 is no longer possible. Division by 3 continues. 72/27; 24/9; **8/3.** √	Factoring by four 3's and one 2 yields the following names for 82944/31104; 27648/10368; 9216/3456; 3072/1152; 1024/384; 512/192. Division by 3 is no longer possible. Division by 2 continues: 256/96; 128/48; 64/24; 32/12; 16/6; **8/3.** √	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable
Factoring by four 2's and two 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 1728/648; 576/216; 288/108; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √	Factoring by four 3's and two 2's yields the following names for 82944/31104; 27648/10368; 9216/3456; 3072/1152; 1024/384; 512/192; 256/96. Factoring must occur by 2 after this point: 128/48; 64/24; 32/12; 16/6; **8/3.** √ The factoring possibilities proceeding from four 3's and a combination of 2's end here.	(1A-col.1) No new numbers (1A-col.2) No new numbers (Case 2) Not applicable
Factoring by four 2's and three 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 1728/648; 576/216; 192/72; 96/36; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3.** √	Factoring by five 3's and one 2 is not possible. The factoring sequence now by 3's and then by 2's ends here.	(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by four 2's and four 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 1728/648; 576/216; 192/72; 64/24; 32/12; 16/6; **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable

(*cont.*)

Variant of second mode of verification	**Variant of second mode of verification**	**Variant of second mode of verification**
Factoring by five 2's and one 3 yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 432/162; 216/81. Division by 2 is no longer possible. Division by 3 continues: 72/27; 24/9; **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by five 2's and two 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by five 2's and three 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 96/36; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by five 2's and four 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 864/324; 288/108; 96/36; 32/12; 16/6; **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by five 2's and five 3's is not possible.		(1A-col.1) Not applicable (1A-col.2) Not applicable (Case 2) Not applicable

(*cont.*)

Variant of second mode of verification	Variant of second mode of verification	Variant of second mode of verification
Factoring by six 2's and one 3 yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 216/81. Division by 2 is no longer possible. Division by 3 continues: 72/27; 24/9; **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by six 2's and two 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 72/27. Division by 2 is no longer possible. Division by 3 continues: 24/9; **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by six 2's and three 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 24/9. Division by 2 is no longer possible. Division by 3 continues: **8/3.** √		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by six 2's and four 3's yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 432/162; 144/54; 48/18; 16/6; **8/3.**√		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by six 2's and five 3's is not possible.		(1A-col.1) Not applicable (1A-col.2) Not applicable (Case 2) Not applicable

(*cont.*)

Variant of second mode of verification	Variant of second mode of verification	Variant of second mode of verification
Factoring by six 2's and six 3's is not possible.		(1A-col.1) Not applicable (1A-col.2) Not applicable (Case 2) Not applicable
Factoring by seven 2's and one 3 yields the following names for 82944/31104: 41472/15552; 20736/7776; 10368/3888; 5184/1944; 2592/972; 1296/486; 648/243. Factoring must occur by 3 hereafter: 216/81; 72/27; 24/9; 8/3. √ This repeats the very first factoring sequence for 82944/31104. It exhausts the factoring possibilities by 2's and 3's. This process exhausts all calculations pertaining to all modes of verifying 256/243 with their relevant variants.		(1A-col.1) No new numbers (1A-col.2) Not applicable (Case 2) Not applicable
		*Collective set of **new** numbers to be added to the Crantor matrix from first mode, second mode, and variant of second mode in first column*: **243; 256; 486; 512; 41472; 15552; 20736; 7776; 10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72; 13824; 4608; 1728; 1536; 576; 192; 6912; 2304; 864; 768; 288; 3456; 1152; 432; 648; 324; 82944; 31104; 27648; 9216; 3072; 1024.** *Additional numbers to be inserted deriving from the names of sesquioctave intervals:* **81, 64, 162.**

Points:

1. Clearly, in some calculations relevant to determining the sesquioctave parts of the sesquitertian intervals and verifying the 256/243 remainder over, one will not have to perform the mathematics utilizing fractions which are simpler names for larger fractions involving parallel calculations. The factoring of the larger fraction will yield all names yielded by the calculations with the smaller fractions. However, some products, arising in the calculations of sesquioctave intervals and twofold verification of the remainder over, can be alternatively factored by either 2's or 3's, or, alternatively, factored now by 2 and then by 3 in a variety of patterns, as the calculations, above, for the "second mode of verification" show. Performing all of the relevant calculations with all of the smaller fractional names for a larger equivalent fraction involved in the same calculations is important in the cases of some sesquitertian intervals because it virtually guarantees that one will not overlook possible number names in a failure to notice factoring possibilities.
2. As this appendix indicates, the operations performed above for the sesquitertian interval 2:8/3 to verify the size of the remainder over, when one fills it with sesquioctave intervals, produce new numbers for the Crantor matrix, namely: **81; 64; 162; 243; 256; 486; 512; 41472; 15552; 20736; 7776; 10368; 3888; 5184; 1944; 2592; 972; 1296; 648; 216; 72; 13824; 4608; 1536; 576; 192; 6912; 2304; 864; 768; 288; 3456; 1152; 432; 1728; 648; 324; 82944; 31104; 27648; 9216; 3072; 1024.** A proliferation of numbers also results when parallel operations are performed for each of the thirteen remaining cases of sesquitertian parts.

APPENDIX 2

The Archytan Alternative in the Pythagorean School

As mentioned in note thirteen (13) to Chapter 4, Archytas did not follow the *Timaean* scheme, constituting the object of this study, to reach his ratios for the diatonic, chromatic, and enharmonic fourths but constructed a genuine Pythagorean alternative approach.[1] He worked within the same Philolaic harmonic framework as did Plato, recognizing the basic concords as the fourth (4/3), fifth (3/2), and tone (9/8); but he altered it in the endeavor to find purely superparticular ratios to describe the internal structures of the fourth and fifth, on the theory that such superparticularity was related to consonance. He apparently believed such ratios to be the means of rationalizing harmonic science with actual musical practice.[2] Clearly, the major divisions of the octave (fourth, fifth and tone), inherited from Philolaus and Pythagorean tradition, were all governed by epimoric ratios. The 256/243 remainder in the fourth and fifth, after internal divisions by two and three tones, however, was plainly unsatisfactory in this respect.

Archytas' divisions of the fourth for the three genera were as follows.

Diatonic: $9/8 \times 8/7 \times 28/27$
Enharmonic: $5/4 \times 36/35 \times 28/27$
Chromatic: $32/27 \times 243/224 \times 28/27$[3]

Clearly, he failed to respect the epimoric principle in his articulation of the chromatic fourth. Andrew Barker has noted the strangeness of Archytas' chromatic division given his epimoric preoccupations.[4]

It is not difficult to understand Archytas' scheme in comparison with the scheme presented by the *Timaeus*. The 8/7 ratio of Archytas' diatonic fourth, for example, is the harmonic mean between 1 and 4/3 $[(2 \times 1 \times 4/3)/(1 + 4/3) = 8/7]$. The *Timaeus* does

1 See Hagel, *Ancient Greek Science*, 448 for the characterization of Archytas' work and Plato's *Timaeus* as two Pythagorean approaches that parted ways.

2 See Andrew Barker, *Science of Harmonics*, 264, 290–291 for Philolaus' articulation of the major concords and the later quest for epimoric, also called superparticular, ratios in the articulation of musical intervals for their supposed reflection of the intelligible principle of musical commensurability.

3 Ibid., 301; Mathiesen, "Ancient Greek Music" 117.

4 Barker, *Science of Harmonics*, 298–299.

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not allow this division. The recipe for the world soul at 36 A allows the discovery of harmonic means (and arithmetic means) only once, specifically between the doubles and triples of Plato's original seven numbers. The 4/3 ratio is the harmonic means between 1 and 2 [$(2 \times 1 \times 2)/(1 + 2) = 4/3$]. Archytas clearly carried the means derivation process farther to reach an epimoric ratio that, in combination with 9/8, assures, mathematically, that the remainder over will be epimoric: $9/8 \times 8/7 \times 28/27 = 4/3$.

Note that Archytas' diatonic formula is not the only one possible for expressing the diatonic fourth epimorically. As Andrew Barker has noted, one could use the formula $8/7 \times 8/7 \times 49/48$. He has observed, however, that this possibility was too remote from actual musical usage to be a serious candidate for the diatonic fourth for Archytas' purposes.[5] Archytas' diatonic fourth, therefore, has two tones of slightly different size and a remainder over somewhat smaller ($1.0\overline{37}$) than the standard Philolaic (or, if one prefers, Platonic) remainder over of 256/243 (1.053497942). Whether this fourth was actually more consonant than the traditional fourth is a matter for musicological investigation. To reach the fifth, one simply adds a tone of the size 9/8.

The enharmonic fourth typically consisted in an undivided ditone and two smaller intervals.[6] If one experiments with the enharmonic, working with the superparticular hypothesis, the closest one may get, at first, is 7/6 (1.166666667) × 15/14 (1.071428571) × 112/105 (1.066666667). This is an odd structure, however, for an enharmonic fourth, because none of these intervals approaches the size of a ditone ($9/8 \times 9/8 = 1.265625$). Conveniently $7/6 \times 15/14$ = the epimoric 5/4 (1.25) which does make a respectable approximation to a ditone; and 112/105 can be expressed as $28/27 \times 36/35$, two mathematically happy epimoric candidates for the small intervals of the enharmonic. While it is uncertain that Archytas made exactly the foregoing observations, it is certainly quite possible, since it results in his very calculations, namely $5/4 \times 36/35 \times 28/27$. One achieves the enharmonic fifth in his system, necessarily, by adding a tone of the size 9/8. Archytas must have been very happy that the same interval, 28/27, that had surfaced in his analysis of the diatonic also emerged in the analysis of the enharmonic. One might further observe that the 5/4 ratio constituting his approximation to the undivided ditone is the arithmetic mean between 1 and 3/2, a division not permitted at *Timaeus* 36 A.

The chromatic fourth was, typically, an undivided tone and one half and two smaller intervals.[7] There are, at least, three ways to produce completely epimoric candidates for the chromatic fourth: 7/6 (1.16666667) × 10/9 (1.11111111) × 36/35 (1.028571429),

5 Ibid., 295 (and n. 22).

6 Reese, *Music*, 32 (relying on Ptolemy); Hagel, *Ancient Greek Music*, 112–113 (regarding continuity of basic Pythagorean framework [5th century B.C] through Ptolemy [2nd century A.D.]); D'ooge, "Greek Mathematics," 71 (for dating of Ptolemy).

7 Reese, *Music*, 31.

or, better, in terms of the closer equality of the smaller intervals, as Ptolemy suggested, much later than Archytas' time, 7/6 (1.16666667) × 12/11 (1.090909091) × 22/21 (1.047619048).[8] Barker and Hagel note a third possibility: 6/5 (1.2) × 15/14 (1.071428571) × 28/27 (1.037037037).[9] It would be difficult, mathematically, to choose between the second and third possibilities, since each offers a TS allowing two small tones roughly approaching similar sizes.

Archytas took none of the routes suggested in the previous paragraph, even though the first preserved a ratio, 36/35, that had been useful for articulating his enharmonic fourth. He was surely sufficiently sophisticated to have discovered all of mathematical possibilities suggested. One might speculate that his reasons had to do with the 6/5 and 7/6 intervals. Perhaps they did not sound consonant to him, although the 7/6 ratio, the arithmetic means between 1 and 4/3, is especially interesting since, taken together with the 8/7, another epimoric ratio, it formed a sesquitertian part.

Archytas' own chromatic formula, 32/27 (1.185185185) × 243/224 (1.084821429) × 28/27 (1.03737037), is not as strange as it first appears. The ratio 32/27 can be expressed, alternatively, either as (9/8 × 256/243) or (8/7 × 28/27). The virtue of the 32/27 ratio, therefore, was that it preserved the size of the standard TS (1.185185185) interval in the chromatic scale and could at least be expressed as the product of two epimoric ratios. It must have been musically important to retain the traditional size of the TS interval.

The 28/27 (1.037037037) is not too surprising as an interval in Archytas' chromatic fourth. It was the small interval relevant to his diatonic; and one would expect to feature the semitone of the diatonic as one of the semitones relevant to the chromatic scale. It also had the virtue of being epimoric.

The remaining interval 243/224 (1.084821429) was just the simplest form of the resulting remainder over following from the mathematics. It is not possible to express its value epimorically. However, note that the sum of the digits of 243 is 9 and the sum of the digits of 224 is 8. The sums are in epimoric relation, though the ancient Greeks would not have appreciated this point, since they did not use our system of digits. The proximity of the two smaller intervals in Archytas' chromatic was not as great as that between the 256/243 (1.053497942) and 2187/2048 (1.067871094) intervals associated with the chromatic fourth in the *Timaean* tradition relevant to this study, but it probably had the advantage of being less unwieldy for instrument tuning purposes.

Archytas' chromatic is interesting, perhaps, from the standpoint of being an embarrassment for the theory that epimoric intervals necessarily produced consonances superior to those that were not epimoric. Again, musicological investigation would be required to determine whether the three epimoric possibilities for the chromatic fourth suggested in this note are inferior, as to their consonance, to Archytas' choice for his chromatic fourth.

8 Hagel, "Ancient Greek Music," 202.

9 Ibid.; and Barker, *Science of Harmonics*, 299.

APPENDIX 3

Greater and Lesser Perfect Systems and Associated Questions

1 Greater Perfect System

1.1 *Overview*

According to the weight of scholarly opinion, the specific sequence (TTS [TTS T) (TTS] TTS T) represented the Greater Perfect System in ancient Greek music. The term "Greater Perfect System" ("GPS") earmarks, most generally, the significance of the two-octave sequence as the basis for Greek tonal structure.[1] According to Reese, "at least in the earlier centuries in which they (Greeks) discuss series of tones arranged in order of pitch," GPS assumes descending pitch values (so that the numbers used to represent intervals indicate string lengths that proceed from shorter lengths and higher pitch to longer lengths and lower pitch, rather than pitch indices proceeding from slower vibrations and lower pitch to faster vibrations and higher—as that would be ascending) and comprises a system of four fourths with an independent whole tone appearing in the middle of the system and an independent whole tone appearing at the end.[2]

The specific sequence indicated above, as the most popular scholarly postulate for GPS, based upon the existing evidence, includes a descending Dorian diatonic scale at the heart of a descending Hypodorian diatonic disdiapason. TTS T TTS, the descending Dorian diatonic sequence is marked, above, by the bold brackets. The descending Hypodorian diatonic disdiapason, (TTS TTST) (TTS TTST), is emphasized, above, as indicated, by the parenthetical markings. The descending Hypodorian diatonic sequence comprised a fourth followed by a fifth. The standard presentation of GPS in the Dorian *tonos* (with the descending Dorian diatonic scale at the heart) occurred in the two-octave interval (disdiapason) extending from a-A-a in its standard presentation in ancient texts.[3]

Scholars cannot be absolutely sure that GPS always assumes descending pitch values; so one must register the possibility that the ancients could present ascending pitch

1 Reese, *Music in the Middle Ages*, 21, 28; see, also, Isobel Henderson, "Ancient Greek Music," in *Ancient and Oriental Music*, ed. Egon Wellesz, vol. 1, *The New Oxford History of Music* (London: Oxford University Press, 1957; reprint, London: Oxford University Press, 1969), 346.

2 Reese, *Music in the Middle Ages*, 21–22.

3 Ibid., 21, 30; Mathiesen, "Greek Music Theory," 121–122.

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values within the GPS structure.[4] In that case, numbers in sequential order would signify a move from slower string vibrations (fewer impacts on air) and lower pitch (smaller numbers) to faster string vibrations (more impacts on air) and higher pitch (bigger numbers), rather than a move from shorter string lengths and higher pitch (smaller numbers) to longer string lengths and lower pitch (bigger numbers). If (TTS [TTS T) (TTS] TTS T) is ascending, rather than descending, then it presents an ascending *Hypophrygian* disdiapason with an ascending *Lydian* diapason at the heart of the system.[5] Numbers representing intervals in Greek texts are neutral as to the ascending or descending character of an intervallic sequence without contextual reference making clear whether they represent impacts or string lengths. The current study remains open to either possibility.

One might speculate that scholars assign GPS to the a-A-a two octave range specified above because they know something about pitches made at certain string lengths, on some common instrument, assuming some discernible unit of measure, and also customs and habits among the Greeks regarding pitch levels at which they were used to hearing particular musical sequences in relation to one another. It would seem, however, that one would have to have access to a large repository of ancient Greek musical texts to have any confidence concerning a set assignment of GPS to an a-A-a range. The repository of texts at today's scholarly disposal is not, in fact, large; and what exists is largely fragmentary.

Sometimes ancient texts (including medieval commentary upon them) themselves assign a note value to a number; in that case, scholars cannot be sure of a full correspondence between the ancient assignment and modern note values; nor can they be sure that such note values were absolute, rather than merely exemplary. Surviving texts do not, in fact, all support the same tonality assignments to the various octave species.[6]

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4 See Reese, *Music in the Middle Ages*, 21, indicating that, in earlier centuries, the descending pitch order was usual.

5 Ibid., 30 (for use as an aid in identifying the tone sequence).

6 Gustave Reese drew the following rough equation between our more familiar keys and the ancient Greek *tonoi*, noting that we must keep in mind regarding the comparison that the ancient Greek *tonoi* were bearers of GPS and not of our major and minor modes:

Mixolydian: D
Lydian: C-sharp
Phrygian: B
Dorian: A
Hypolydian: G-sharp
Hypophrygian: F-sharp
Hypodorian: E

Reese, *Music in the Middle Ages*, 30. It appears, however, that several ancient texts would allow variance from such an assignment. See, e.g., ibid., 36–45 and Mathiesen's presentation of

As this study shows, particularly in Chapter 6, many double intervals in the *Timaeus* number set, can be variously interpreted to express strings of different octave species. Indeed, the fairest interpretation, in many cases, is that multiple overlapping octave types manifest within the very same double interval. These overlapping octave strings contain some notes in common with one another and some differing notes. They are different flavors, so to speak, within the very same double interval. Accordingly, it may be erroneous to assign particular *tonoi* associated with the different octave species to particular modern keys or particular pitch ranges. Perhaps a *tonos* had more to do with the flavor of a pattern than it did a pitch range.

In our modern key system, a particular frequency, represented by a number, and the various multiples thereof are assigned a particular note flavor, so to speak, that we designate as E F G A B C D or the sharps and flats thereof. Modern western major keys share exactly the same scale structures and the "do" within each key has the same "flavor" in relation to the other steps of the scale, regardless of the key. For example, the diatonic octave for each modern western key has the ascending structure TTS T TTS (descending order is STT T STT), a structure roughly corresponding to the ascending Lydian diatonic octave of ancient Greek music. The comparison is only rough because the sizes of the tone and semitone steps are not the same as between the ancient Greek system and the modern western system. Likewise, the different minor keys of the modern western system also share the same internal ascending structure different from the internal ascending structure of a major key, namely, TSTTSTT (descending order is TTS T TST). The latter structure roughly corresponds to the structure of the ascending Hypodorian diatonic octave of ancient Greek music. Again, the "la" within each minor key has the same "flavor" in relation to other steps of the scale, regardless of the key. Various modes and modulations within modern major and minor keys are also uniform across keys.

The different Greek *tonoi*, in contrast, did not exhibit the same internal scale structures among themselves. They were associated with different and distinct octave species. They did not, therefore, bear the same relation to one another as modern major keys bear to each other. The diatonic scale structure of each particular *tonos* was unique to that *tonos*. The chromatic scale structure of each particular *tonos* was unique to that *tonos*. Each enharmonic scale structure of each particular *tonos* was unique to that tonos. The *tonoi* differed *from each other* in a manner analogous to the way in which a modern major western key differed from its relative minor.

Meanwhile, the chromatic and enharmonic varieties within each *tonos* were variations of the basic idea underlying the diatonic pattern of that *tonos* directed to a

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Aristoxenus, Ptolemy, and Aristides Quintilianus. Mathiesen, "Ancient Greek Music," 125, 127, and 128.

particular end, namely: the production, in the chromatic varieties, of two undivided TS sequences, a single tone, and four, instead of two, semitones in one diapasonic sequence and the production, in the enharmonic varieties either of two undivided double tone sequences, a single tone, and four quarter tone sequences in the same diapasonic sequence, or else, two undivided double tone sequences, a single tone, and two semitone sequences. Each *tonos* was distinct from every other in its placement of characteristic elements within the chromatic and enharmonic varieties; and it is likely that the result was intended.

For example, the descending Dorian diatonic scale pattern is TTS T TTS, displaying symmetrical structure around an independent whole tone. The Dorian chromatic pattern [TS]SS T [TS]SS retains the symmetry in identical patterns around an independent whole tone, but transposes the structure of the fourth by putting the TS ending each disjunct fourth in the diatonic pattern first, transforming it into an undivided TS interval, and dividing the remaining T into two semitones. The result is a single diapasonic structure containing two undivided TS intervals, a type of interval occurring in neither the diatonic nor the enharmonic diapason, and four, rather than two semitones.

The enharmonic pattern (assuming, just for example, a quarter tone enharmonic), [TT]QQ T [TT]QQ displays identical patterns around an independent whole tone, retaining the basic order of the Dorian diatonic scale but playing with the structure of the fourth using quarter tones and undivided [TT] sequences. The result is a diapasonic structure containing two undivided TT sequences, a type of interval occurring in neither the diatonic nor the enharmonic scales and four quarter tone sequences (or two semitone sequences). The Dorian flavor of the chromatic and enharmonic variations is defined by the derivative relationship of the chromatic, and enharmonic varieties to the diatonic variety of the *Dorian tonos*. One might define the overarching flavor of the Dorian as identical structures of the fourth on either side of an independent whole tone.

The chromatic and enharmonic varieties of the Lydian descending octave scale are achieved in a manner distinct from the derivation in the case of the Dorian scale. The Lydian descending diatonic scale pattern, STT T STT, like the Dorian, displays symmetrical structure around an independent whole tone. The chromatic and enharmonic variations of the Lydian scale, however, dispense with symmetry around the independent whole tone. The Lydian chromatic transforms the diatonic into two conjunct structures, both of which are the same size but dissimilarly composed (the first structure contains one free semitone, and the second structure contains three). The descending Lydian chromatic pattern is achieved by dividing the central independent whole tone and the last whole tone of the diatonic sequence into semitones and assigning the parts to create undivided TS sequences of two of the remaining T's proximate to a new semitone after the division. Only one independent tone remains, located after the first original semitone of the diatonic sequence. The result, by a means distinct

from that employed in the case of the Dorian or any other *tonos*, is the production of four semitones, rather than two in a diapasonic sequence, two undivided TS intervals, and a single independent tone.

The descending Lydian enharmonic pattern splits the initial semitone of the chromatic pattern, keeps one of the resulting parts at the beginning and moves one to the end of the sequence; merges the first of the middle semitones with the undivided TS sequence of the chromatic preceding it to arrive at a ditone interval; splits the second of the middle semitones into two quarter tones; and merges the S ending the chromatic diapason with the undivided TS sequence preceding it to arrive at another ditone interval. The result, by a means distinct from that employed in the case of the Dorian or any other *tonos*, is a diapason with no semitones, two undivided TT sequences, and four quarter tones. Because of their derivation from the Lydian diatonic, the Lydian chromatic and enharmonic preserve a "Lydian" flavor.

The motivation behind the chromatic and enharmonic variations within the different *tonoi* may have been the pathless quest to find a semitone that would represent the perfect split of the Greek whole tone; and so, it was important to attempt to generate semitones or putative parts thereof in as many different ways as possible. As the present study shows, the quest was impossible. Indeed, the quest for the size of the semitone seems to have driven the entire ancient Greek musical system.

Western modern music has nothing really equivalent to the chromatic and enharmonic varieties of the Greek diatonic *tonoi;* the only reasonable analogue would be distinct modulations within each modern key; but as noted, the same modulations would be possible within all of the major keys and would not be unique to any major key; nor would the modulations be motivated by the quest to find an interval. Clearly, the Greek *tonoi* are unique and so, inexplicable, with reference to the modern western key system.

The point is that the Greek system of music was a system so distinctly different in its musical preoccupations and emphases from the modern western system that the *tonoi* really cannot be compared to modern western keys and cannot be explained with reference to the modern western key system. Furthermore, when modern and even ancient and medieval scholars, assign note values, such as A and F#, that they recognize as pitch values, to sequences of *Timaeus* set numbers, an analyst must abstain from any temptation to compare the assignments to the articulation of something like a modern key structure. Such assignments are really just more or less knowledgeable, educated guesses concerning the location of the Greek voice or a particular instrument relative to the pitch systems of the persons making the assignments. One certainly cannot really confine *tonoi* to particular pitch ranges in any absolute sense.

1.2 *The* Timaeus *and GPS Possibilities*

One might expect GPS, as defined in the overview above, to begin among the *Timaeus* numbers at 288, since 384 is the beginning of the first completely articulated descending Dorian diatonic octave among the *Timaeus* numbers. Such an assignment would set the GPS as a descending *Hypodorian* two-octave sequence (disdiapason). GPS, in its standard presentation, does not begin among the *Timaeus* numbers at 288, however, as the requisite mathematics does not produce whole numbers fitting the pattern.

The first completely articulable GPS sequence among the *Timaeus* numbers, in the standard presentation known to scholars, begins at 576 and runs to 2304 as follows:

> 576 T 648 T 729 S [**768 T 864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536**] T 1728 T 1944 S 2048 T 2304

The above sequence is a descending two-octave *Hypodorian* sequence (disdiapason), having starting tone number 576 and ending tone number 2304 with a descending *Dorian* sequence in the middle of the system running from 768 to 1536. Alternatively, it is an ascending two-octave *Hypophrygian* sequence, with the same bounds, having an ascending *Lydian* diapason in the middle of the system running from 768 to 1536. Note that a fourth precedes the middle octave and that a fifth follows it. No attempt is made here to assign a key designation to the number sequence for the reasons set forth above.

Observe that GPS, as articulated just above, is merely the standard presentation of GPS in the *Dorian tonos* (or Lydian, if ascending), i.e., with the Dorian (or Lydian) *tonos* at the heart of the system. With sensitivity to the circumstance that scholars have no evidence for GPS in any other presentation, one might observe that GPS, theoretically, should be capable of alternative expressions in other *tonoi*. As noted above, 384 is the beginning of the first completely articulable octave in the *Timaeus* set; it is also, then, the beginning of the first completely articulable disdiapason. If one thought that the first articulable disdiapason of the set should have some special significance because it is the first, one might experiment with redefining GPS, as follows:

> TTS [T TTS TTS] T TTS

The first subset of numbers in the *Timaeus* number set expresses precisely such a descending sequence:

> 384 T 432 T 486 S [**512 T 576 T 648 T 729 S 768 T 864 T 972 S 1024**] T 1152 T 1296 T 1458 S 1536

The sequence immediately above marks a descending diatonic *Dorian* disdiapason or an ascending diatonic *Lydian* disdiapason, running from 384 to 1536. The octave sequence in the middle is a descending diatonic *Mixolydian* sequence or an ascending diatonic *Hypolydian* sequence running from 512 to 1024. Note that this alternative suggestion for GPS, like the structure representing the standard presentation, has a fourth preceding the middle octave and a fifth following it. The argument for any such alternative GPS is its first appearance among the *Timaeus* numbers, as well as its emphasis on Dorian and Lydian-type scales.

Note that one might experiment with GPS, further, shifting the middle by a tone to reverse the order of the fourth and fifth around the middle octave. Using the sequence immediately above as the basis for such experimentation, one arrives at the following articulation:

TTS T [TTS TTS T] TTS

384 T 432 T 486 S 512 T [**576 T 648 T 729 S 768 T 864 T 972 S 1024 T 1152**] T 1296 T 1458 S 1536

Clearly, the fifth now precedes the middle octave, and the fourth follows it. On the terms of such a redefinition, the octave in the middle would begin with 576 and end with 1152; but this tone sequence, assuming a descending scale, is Hypodorian, not Mixolydian, and, if ascending, it is Hypophrygian, not Hypolydian. The disdiapason is still Dorian, if descending, and Lydian, if ascending. Assuming that such redefinition is legitimate, then, GPS can be expressed in two different *tonoi* (based on the octave in the middle) within any given disdiapasonic diatonic sequence, depending upon the location of the fourth and fifth around the central octave. Further, as has been noted all along, two different overarching interpretations of GPS are possible, depending upon whether one regards sequences as descending or ascending.

One might attempt a similar shift on the standard presentation of GPS, to achieve a transposition of the fourth and fifth around the middle octave as follows:

576 T 648 T 729 S 768 T [**864 T 972 S 1024 T 1152 T 1296 T 1458 S 1536 T 1728**] T 1944 S 2048 T 2304

The above sequence is a descending two-octave Hypodorian sequence (disdiapason), having starting tone number 576 and ending tone number 2304 with a descending Phrygian sequence in the middle of the system running from 864 to 1728. Alternatively, it is an ascending two-octave Hypophrygian sequence, with the same bounds, having an ascending Phrygian diapason in the middle of the system running from 768 to 1536. Note that a fourth precedes the middle octave and that a fifth follows it.

Observe, as well, that the Phrygian diatonic tonos is its own reciprocal, a feature that would probably have fascinated the Greeks. This feature is an argument for the primacy of the Phrygian over the other Greek *tonoi*, in competition with the Dorian, and so an argument for the primacy of the particular version of GPS presented immediately above.[7]

Working with the idea that the Phrygian diatonic *tonos* was especially remarkable because of its character as its own reciprocal, one might wonder what GPS would look like if one used the Phrygian disdiapason as the overarching structure. The first such possible disdiapason begins with 432 and ends with 1728. The scale at the center, provided that one positions it after a fourth and before a fifth, so that it begins with 576 and ends with 1152, is Hypodorian, rather than Dorian, if descending and Hypophrygian if ascending. The structure is TST [TTS **T**TS T] TTS **T**, as follows.

> 432 T 486 S 512 T [**576 T 648 T 729 S 768 T 864 T 972 S 1024 T 1152**] T 1296 T 1458 S 1536 T 1728

Note that the above sequence is a good candidate for an understanding of GPS alternative to the scholarly favorite because, like the standard presentation, it allows an independent whole tone both in the middle and at the end. It is, indeed, very close to the standard model in containing a central descending diapason that is, in some way, related to the Dorian scale.

If one transposes the positions of the fourth and fifth around the central octave for a Phrygian GPS alternative, then one achieves the following structure within the overarching Phrygian descending or ascending diatonic disdiapason: TST T [TS T**T**S T T]TS **T**. Note that this structure also allows an independent whole tone in the middle of the central octave and an independent whole tone at the end, making it another good candidate as a rival to the standard presentation of GPS best known to scholars. Filling out the pattern on the basis of the same numbers presented just above one obtains:

> 432 T 486 S 512 T 576 T [**648 T 729 S 768 T 864 T 972 S 1024 T 1152 T 1296**] T 1458 S 1536 T 1728

The central octave is a descending Hypophrygian sequence or an ascending Hypodorian sequence, in other words directly opposite the sequences of the first Phrygian alternative as to the descending and ascending possibilities.

The chief problem with the alternative articulations of GPS suggested in this appendix is the lack of surviving evidence in support of them. Scholars are reasonably cer-

7 See Plato *Republic* 398 E–399 D, containing Socrates' opinion of the various *tonoi*.

tain that the structure of GPS was TTS [TTS T TTS] TTS T because the surviving evidence supports it; but the path that led ancient thinkers to that particular structure is obscure at best.

If the Dorian scale had primacy in ancient Greek music, then the GPS alternatives centered on the Dorian disdiapason may shed some light on the reasons that ancient musicians vacillated on the identity of the model octave. Some set it at 384 to 768 and others set it as 768 to 1536. The argument for the former choice is that the 384 to 768 sequence is the first diatonic scale, indeed the first octave scale of any kind, which arises among the *Timaeus* numbers. It happens to be a descending Dorian scale or an ascending Lydian scale. One could use the descending Dorian disdiapason beginning from 384 as the overarching structure for GPS but, then, the central octave would not be Dorian. The argument for the 768 to 1536 sequence as the model octave is that it is the first diatonic scale arising as the central octave within the standard presentation of GPS among the *Timaeus* numbers. It, too, is a Dorian sequence.

One wonders, aside from theoretical objections already noted above, why a descending Dorian sequence from 384 to 1536 was not chosen as the standard presentation of a GPS with a descending Mixolydian sequence, 512 to 1024, at the center as suggested above. A simple explanation is, possibly, that the range of the system would then have been set too high for the male voice. The centrality of the Mixolydian octave, running from 512 to 1024, to a GPS system defined by a descending Dorian disdiapason beginning with 384 (shortest string length), a theoretically pure starting point since 384 was truly the beginning of the first articulable diatonic octave in the *Timaeus* set, was better suited to the female voice, a voice not valued in ancient Greek music. A GPS defined on the basis of the descending Hypodorian disdiapason running from 576 to 2304 sets the central octave as the Dorian octave beginning at 768, an entire fifth below the putative center, 512, for the female voice. The suggestion, is, of course, speculative, but it is sufficiently intriguing to warrant further investigation in relation to a possible deliberate suppression of the female in ancient Greek culture.

One might suggest that Greek males suppressed the theoretical preference that rightly belonged to the female voice by redefining an original GPS that had indeed run, descending, from 384 to 1536 and that had a 512 to 1024 descending Mixolydian center (or a 576 to 1152 descending Hypodorian center—though this is a less likely possibility, since this arrangement transposes the position of the fourth and fifth around the central octave in a manner that evidence has not yet presented). They replaced it with a GPS that put the Dorian diatonic octave (theoretically important because it is the first octave scale of any kind fully articulable on the basis of the *Timaeus* numbers) at the center of the system but subordinated the Dorian disdiapason to the Hypodorian disdiapason as the basis of GPS. The Dorian disdiapason as the basis for GPS was perhaps actually properly suited to female voices. The Dorian, long associated with masculinity and the male in ancient Greek culture may have been, correspondingly, a secondary

association that arose with the suppression of the importance of the female in Greek culture. The original association of the Dorian was, perhaps, with the female voice. Certainly, there is some evidence of the suppression of the female in ancient Greek music; so the idea is not out of bounds. It occurs in connection with the Lydian and mixed Lydian modes in their association with the female voice, however, in Plato *Republic* 398 E–399 A, where Socrates characterized these modes as dirgelike, lax, unfit for warriors and men, and useless even to women who are to make the best of themselves. The Dorian he praised as desirable within the *polis* because it fittingly imitates the sounds made by a brave man in war or other enforced business and, also, the Phrygian, because it is best suited to works of persuasion in times of peace. See Plato *Republic* 399A. It must not be forgotten that, in the diatonic, the Lydian and Dorian are exact reciprocals.

2 Lesser Perfect System

The sequence [TTS {TTS T] TST} was known as the Lesser Perfect System ("LPS") in ancient Greek music. Modulation was possible between GPS, in its standard presentation, and LPS because the two systems shared eight steps of their scales in both directions (whether ascending or descending). The sequence in bold blue brackets in the foregoing sentence is a descending Hypodorian or ascending Hypophrygian diatonic octave sequence. The sequence in bold lavender braces is also a descending Hypodorian or ascending Hypophrygian diatonic octave sequence. The first instance of the LPS tone sequence among the *Timaeus* numbers begins with 1728 and ends with 4608, the tone a fourth below the octave ending with 3456 (or above, if one is working in ascending order), as follows:

1728 T 1944 T **2187** S 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608

Note that the complete articulation of LPS, unlike the complete articulation of GPS, depends upon the emergence of chromatic numbers (here 2187), i.e., the numbers that arise as a result of distortions to the original descending Dorian or ascending Lydian diatonic octave of the *Timaeus* numbers that occur as the octave repeats.

Note further that, on the standard interpretation of GPS, above, any direct continuity between GPS and LPS seems dubious, although modulation between the two systems is certainly possible precisely from the point where 1728 appears in standard GPS (see above section). If one redefines GPS, alternatively, within the framework of a descending or ascending Phrygian disdiapason, however, direct continuity is possible.

A GPS, set from 432 to 1728 with pattern TST TTS T TST TTS T, comprising a descending or ascending Phrygian disdiapason—the first such pattern occurring in the *Timaeus* table—and having a descending Hypodorian or ascending Hypophrygian cen-

ter from 576 to 1152 (thus, TST [TTS T TST] TTS T—note the independent whole tone in the middle of the system and the independent whole tone at the end), would allow an LPS with the standard definition starting from 1728 and ending at 4608 to follow seamlessly upon GPS redefined in the *Timaeus* number set as follows:

> *Begin Redefined GPS* (432 T 486 S 512 T 576) [(T 648 T 729 S 768) T 864 (T 972 S 1024 T 1152)] (T 1296 T 1458 S 1536) T {1728} *End Redefined GPS, begin Standard LPS* T 1944 T **2187** S 2304 T 2592 T 2916 S 3072 T 3456 T 3888 S 4096 T 4608 *End Standard LPS*

Modulation between the redefined GPS and standard LPS is anticipated because of the contiguity; but such modulation, as belonging to a single UPS system, assumes a redefined relationship between GPS and LPS to which scholars might object. On the standard interpretation both GPS and LPS comprise descending Hypodorian or ascending Hypophrygian sequences. In the alternative, LPS is a descending Hypodorian or ascending Hypophrygian sequence, a characteristic it shares only with the central octave in the redefined GPS.

One might well justifiably argue that any genuine alternative to GPS would require a redefined LPS bearing the same relation to the alternative as the LPS of standard interpretation bears to GPS of standard interpretation.[8] In other words, the LPS of the redefined system should, *inter alia*, comprise a descending or ascending Phrygian sequence. Perhaps such a suggestion is well-placed, and perhaps it is not. Perhaps it goes too far in insisting upon a relation that may or may not have been real in the minds of the ancient Greeks.

The primary focus, above, on the descending Hypodorian or ascending Hypophrygian sequence in LPS does harmonize quite nicely, after all, with the central position of a descending Hypodorian or ascending Hypophrygian diatonic octave sequence of a GPS consisting in an ascending or descending Phrygian disdiapason, and fits well with some speculation in the ancient world that the Phrygian disdiapason was the superior sequence in ancient Greek music. Observe, that since the Phrygian disdiapason is its own reciprocal, it has a "hermaphroditic" character neutral to appropriation by either gender.

The evidence is probably insufficient for scholars to determine whether the relation between GPS and LPS, as they typically understand it, was actually real for the Greeks, rather than just appealing to academics. To give the scholars the benefit of the doubt, this study will assume that the weight of their opinion is correct.

8 Mathiesen, "Greek Music Theory," 122.

The GPS alternative consisting in a Phrygian disdiapason anticipates interesting possibilities for modulation to the standard GPS system when the redefined system is directly followed by standard LPS. Observe below that what becomes a particular articulation of standard GPS is represented by the numbers in red, highlighted in various colors. The number highlighted in blue is the center of one articulation of standard GPS. The number highlighted in green is the place of modulation within the articulation of standard LPS necessary to complete the articulation of standard GPS begun within Redefined GPS.

> *Begin Redefined GPS* (3456 T 3888 S 4096 T 4608) [(T 5184 T 5832 S 6144) T 6912 (T 7776 S 8192 T 9216)] (T 10368 T 11664 S 12288) T {13824} *End Redefined GPS, begin standard LPS* T 15552 T 17496 S **18432** T **20736** T **23328** S **24576** T **27648** T **31104** S **32768** T **36864** *End Lesser Perfect System*

> 15552 S 16384 T 18432 shows the modulation to standard GPS through a standard LPS, not within the same UPS articulation as the standard GPS to which modulation is occurring, but the next one. The emboldened numbers are actually the numbers that the articulation of standard LPS, in question, has in common with the articulation of GPS to which it properly corresponds; and that articulation of GPS has no overlap with Redefined GPS. At 18432, the articulation of standard GPS, to which the LPS in question corresponds, departs, above, by a tone, instead of a semitone, with two conjunct TTS fourths immediately above the semitone.

There is, of course, no surviving evidence that an ancient Greek musician ever attempted modulation between two different, theoretically possible GPS systems. It is interesting, nonetheless, to contemplate the possibility.

APPENDIX 4

Alternative Perfect Systems

One might speculate whether one can find among the *Timaeus* numbers alternatives for GPS and LPS different from the one that modern scholars favor. The set of possibilities for descending GPS is the following:

(a) Mixolydian disdiapason with Dorian center following first fifth:

TTTS **TTSTTTS** TTS

(b) Phrygian disdiapason with Hypodorian center following first fourth:

TST **TTSTTST** TTST

(c) Phrygian disdiapason with Hypophrygian center following first fifth

TSTT **TSTTSTT** TST

(d) Lydian disdiapason with Hypophrygian center after the first fourth

STT **TSTTSTT** TSTT

(d) Lydian disdiapason with Hypolydian center after first fifth

STTT **STTSTTT** STT

(e) Dorian disdiapason with Mixolydian center after the first fourth

TTS **TTTSTTS** TTTS

(f) Dorian disdiapason with Hypodorian center following the first fifth

TTST **TTSTTST** TTS

(g) Hypolydian disdiapason with Lydian center following the first fourth

STT **STTTSTT** STTT

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(h) Hypophrygian disdiapason with Lydian center after the first fifth

TSTT **STTTSTT** STT

(i) Hypophrygian disdiapason with Phrygian center after the first fourth

TST **TSTTTST** TSTT

(j) Hypodorian disdiapason with Dorian center after first fourth: standard model

TTS **TTSTTTS** TTST

(k) Hypodorian disdiapason with Phrygian center following first fifth

TTST **TSTTTST** TST

The overlap with redefined LPS, in each of the above systems, begins after the first unit, whether it be a fourth or fifth, by analogy to the relationship between GPS and LPS in the standard model. Observe that one would not have to take such an approach to alternative LPS systems, but rather, simply look for the earliest opportunity to articulate it within the confines of alternative GPS. The latter approach is, in fact, one of the approaches to *Timaeus* UPS in the text of Chapter 5.

Two of the many different possibilities for alternative GPS and LPS systems are exemplified below, based, first, on a GPS consisting in a Mixolydian disdiapason with a Dorian center following the first fifth and, second on a GPS consisting in a Hypophrygian disdiapason with a Phrygian center after the first fourth. This appendix does not purport to have worked out the alternatives perfectly, but simply to point the way to new possibilities.

The template provided for the calibration of the system in each example is abstracted from the structure of the standard system as presented by Thomas Mathiesen.[1] Each example assumes different note spans for the alternative systems than the A-a-A standard system, relating them to the standard system for the note spans assigned. Observe that nothing dictates such an approach. In the text of Chapter 5, an A-a-A span has been assumed for *Timaeus* UPS. As Appendix 3 and Chapter 6 of this study indicate, almost every given disdiapason among the *Timaeus* numbers is subject to multiple interpretations as to *tonoi* represented.

1 See Mathiesen, "Ancient Greek Music," 118, 122.

1 Alternative Unmodulating Perfect System: Example 1

TABLE 53 Sample alternative UPS, example no. 1, calibrated on the basis of GPS redefined as a descending Mixolydian disdiapason [TTTS (TTS TTTS) TTS] with a descending Dorian center that is clearly framed by a preceding descending Mixolydian fifth and a subsequent descending Mixolydian fourth

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A full tone structure suggested by the *Timaeus* number set, for alternative GPS, thus redefined, adequate to account for all diatonic, chromatic, and enharmonic possibilities in such a system is the following:

[(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS][(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS]

Not all components internal to the structure of "T" will manifest as note values within the GPS system, as the chart below shows.

Diatonic LPS in the new system is redefined, in a fashion analogous to the relation between GPS and LPS in the standard system, as the following:

[TTTS TTS TTS]

(In other words, it consists in the descending Mixolydian diatonic octave with repetition of the last descending fourth, just as in the standard system LPS is the descending Hypodorian octave with a repetition of the last descending Hypodorian fourth.)

The full articulation of LPS suggested by the *Timaeus* number set as necessary to preserve the proper relation between GPS and LPS and to account for all diatonic, chromatic, and enharmonic possibilities in LPS is the following:

[(T= S′S)T(T=SS′)S] [(T= S′S)(T=SS′)S][(T= S′S)TS]

Again, not all components internal to the structure of "T" will manifest as note values within the LPS system, as the chart below shows. LPS begins calibration against GPS, in the redefined system, at the end of the interval of GPS marked in blue above. The relation of the LPS to GPS in the area of overlap is indicated below by the mapping of LPS onto GPS. The tone structure that LPS has in common with GPS is indicated in alternating lavender and yellow. The tone structure wherein it differs from GPS is indicated in alternating red and green.
GPS Overlaid with LPS

[(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS][(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS]

A few observations are key to elucidating why the GPS and LPS tone structures provided above allow for all chromatic and enharmonic, as well as diatonic possibilities for the redefined GPS and LPS.

Observe that the structure of a descending Mixolydian chromatic disdiapason (the chromatic GPS), embedding a central descending Dorian chromatic octave (see the sequence in yellow below) is the following:

T [TS] S′S [TS] S′S T [TS] S′S [TS] S′S

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

Note that the semitones in red are semitones of the 2187/2048, rather than 256/243 variety. A mapping of the chromatic sequence onto GPS manifests that the above semitone specifications are correct. The chromatic sequence maps onto the GPS tone structure articulated above as the print and brackets in red and alternate highlighting in green and blue below indicates:

[(T= S′S)(T= S′S)([T = SS′)S]] [(T= [S′S])TS][(T= [S′S])(T= S′S)([T = SS′)S]] [(T= [S′S])[TS]]

(T=[S′S])

Note that the S′S unit needed at the end of the chromatic sequence requires the beginning of a second articulation of alternative GPS. Knowing where the articulation of the chromatic GPS sequence begins in relation to diatonic GPS helps one to determine the exact positions of the chromatic notes in the alternative system.

Observe that the structure of a descending Mixolydian enharmonic disdiapason (the enharmonic GPS), embedding a central descending Dorian enharmonic octave (highlighted in yellow below) is the following:

T [TT] QQ [TT] QQ T [TT] QQ [TT] QQ

This sequence maps onto the GPS tone structure articulated above from the beginning of that sequence, as indicated by alternating yellow, blue, and lavender highlighting and red brackets and lettering, with the lavender being reserved to mark the *loci* of quarter tones.

[(T= S′S)([T= S′S)(T = SS′)] S] [([T= S′S)T]S][(T= S′S)([T= S′S)(T=SS′)]S] [([T= S′S)T]S]

Observe that the structures of chromatic Mixolydian LPS and enharmonic Mixolydian LPS must also be accommodated in the calibration of the alternative system. They are not exactly parallel to the chromatic Mixolydian GPS and enharmonic Mixolydian GPS.

Note that the chromatic structure of Mixolydian LPS is the following:

T [TS] S′S [TS] S′S [TS] S′S

Observe that one cannot imbed a Dorian chromatic octave within the above sequence without partially repeating the LPS chromatic sequence (underlined portion) as follows:

T [TS] S′S [TS] S′S [TS] S′S T [TS] S′S

Such repetition is unnecessary in the alternative system since the articulation of chromatic Mixolydian GPS gives the Dorian chromatic octave a central place.

Note that the S′ sequences above, indicate semitones of the 2187/2048, rather than the 256/243 variety. The overlay of the chromatic sequence on LPS, below, shows that the semitone identification is correct.

Chromatic Mixolydian LPS maps onto the diatonic LPS structure given above, as indicated by the alternating blue and yellow highlighting and red lettering and bracket. Full articulation requires a partial repetition of diatonic LPS, as the lavender highlighting below indicates:

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

[(T= S′S)T([T = SS′)S]][(T= S′S)([T = SS′)S]][(T= S′S)[TS]](T= S′S). . .

Observe that the repeated (T= S′S) sequence necessary for the complete calibration of chromatic Mixolydian LPS is exactly the same (T= S′S) sequence necessary for the complete calibration of chromatic Mixolydian GPS.

Note that the enharmonic structure of Mixolydian LPS is the following:

T [TT] QQ [TT] QQ [TT] QQ

Observe that one cannot imbed a Dorian enharmonic octave within the above sequence without repeating the LPS enharmonic sequence (underlining and red letter) as follows:

T [TT] QQ [TT] QQ [TT] QQ T [TT] QQ

Such repetition is unnecessary in the alternative system since the articulation of enharmonic Mixolydian GPS gives the Dorian enharmonic octave a central place.

In any case, the enharmonic structure of Mixolydian LPS maps onto the diatonic Mixolydian LPS structure suggested by the *Timaeus* set and articulated above, from its beginning, as the alternating yellow and blue highlighting and red lettering and brackets below indicate. Quarter tones must be calculated at the *loci* specified in red.

[(T= S′S)[T(T = SS′)]S] [([T= S′S)(T = SS′)]S][([T= S′S)T]S]

The above sequence relates to the diatonic and enharmonic Mixolydian GPS structures suggested by the *Timaeus* set, for the area of overlap between GPS and LPS suggested by the *Timaeus* set, as follows, vis-à-vis the *loci* for the calculation of quarter tones. Note that the top sequence is diatonic Mixolydian LPS marked for enharmonic Mixolydian LPS. The sequences in red are the *loci* of semitones that must be split to achieve enharmonic Mixolydian LPS. The second sequence compares diatonic Mixolydian GPS marked for enharmonic Mixolydian GPS with the first sequence vis-à-vis the semitones that must be split to achieve the respective LPS and GPS Mixolydian enharmonic structures. Note that the enharmonic structures of the two systems are not parallel. The LPS structure requires the splitting of three semitones, rather than four. One of the semitone splittings required for enharmonic Mixolydian LPS is unique to LPS. It shares the other two in common with enharmonic Mixolydian GPS. The third sequence below shows, in red, the only semitone splitting unique to enharmonic Mixolydian LPS within the context of diatonic Mixolydian LPS marked for enharmonic Mixolydian LPS. It also shows, in lavender, within that context, the *loci* of the semitones that must be split to achieve enharmonic Mixolydian GPS over the area of overlap between LPS and GPS. The fourth sequence below shows, within the context of diatonic Mixolydian GPS, marked for enharmonic Mixolydian GPS, all semitones that must be split in the GPS structure to allow for calibration of both enharmonic Mixolydian GPS and enharmonic Mixolydian LPS. The markings in lavender, in the fourth sequence, indicate the splittings in common between enharmonic Mixolydian GPS and enharmonic Mixolydian LPS. The markings in green indicate the splittings unique to enharmonic Mixolydian GPS. The markings in red indicate the splitting unique to enharmonic Mixolydian LPS.

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

[(T= S′S)[T(T = SS′)]S] [([T= S′S)(T = SS′)]S][([T= S′S)T]S]

[(T= S′S)([T= S′S)(T = SS′)] S] [([T= S′S)T]S][(T= S′S)([T= S′S)(T=SS′)]S][([T= S′S)T]S]

[(T= S′S)[T(T = SS′)]S] [([T= S′S)(T = SS′)]S][([T= S′S)T]S]

[(T= S′S)([T= S′S)(T = SS′)] S] [([T= S′S)T]S][(T= S′S)([T= S′S)(T=SS′)]S][([T= S′S)T]S]

The only additional quarter tone calculation required for enharmonic LPS, not already accommodated by the calculations needed for enharmonic GPS occur at the S marked immediately above. It is clear that enharmonic LPS and GPS have only two QT calculations in common, that LPS has one QT calculation unique to it, and that GPS has two QT calculations unique to it.

When all is said and done, it appears that the alternative UPS accommodating the new GPS and LPS considered in this example, must be calibrated as follows, including an extra S′S sequence beyond the disdiapason (indicated in green) to accommodate all chromatic possibilities in LPS and GPS. The *loci* for the calibration of QT's is indicated in yellow (unique to GPS), red (unique to LPS) and lavender (common to GPS and LPS). The S in red print is the last interval included within GPS only. GPS and LPS overlap for the remainder of the sequence.

[(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS][(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS](T= S′S).

If one were to compare the alternative UPS considered in this example to the standard model, one would find that the *nete hyperbolaion* of the alternative model correlates to the *mese* or [a^1] of the standard one. The two systems cannot be precisely mapped onto one another. *Nete hyperbolaion* [a^2] of the alternative system is an entire octave lower than the *nete hyperbolaion* of the standard system; and the centering of the Dorian diapason as between the two systems differs by a tone. The displacement of the center down by a tone in the alternative system is, in fact, the reason that an extra repeated (T= S′S) is needed within it at the low end of the scale. Names beyond *nete hyperbolaion*, *mese*, and *proslambanomenos* are unattempted; and note values, are mere educated guesses, as the alternative model analyzed in this chart is a theoretical construct.

Diatonic notes of GPS are highlighted in yellow. Chromatic notes of GPS are highlighted in blue. Enharmonic notes of GPS are highlighted in green. When a note belongs to more than one scale, then it is highlighted in the applicable number of colors. Diatonic notes of LPS are marked with a yellow asterisk *. Chromatic notes of LPS are marked with a blue asterisk *. Enharmonic notes of LPS are marked with a green asterisk *. When a note belongs to more than one scale, it is marked with the applicable number of colored asterisks. Observe that only elements in blue, green, and yellow correspond to actual note values within GPS and LPS. The numbers highlighted in red are notes implicit within the structure of "T" but never independently manifest, given the formulas for the diatonic, chromatic, and enharmonic Mixolydian scales in ancient Greek music. The unused elements in red point to theoretical possibilities for additional scales that were probably never realized by the ancient Greeks.

[a^2] *nete hyperbolaion*	GPS only	49152

S′ = 2187/2048

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

[g#*2]	Does not manifest as note value in GPS or LPS	52488	
S = 256/243			
[g2]	GPS only Calculation of chromatic GPS begins from this point.	55296	
S′ = 2187/2048			
[f#*2]	Does not manifest as note value within GPS or LPS	59049	
S = 256/243			
[f2]	GPS only	62208	
S= 256/243			
[e*2]	Does not manifest as note value in GPS or LPS	65536	
S′ = 2187/2048			
[e♭2]	GPS only	69984	
S= 256/243 End initial fifth of first Mixolydian diapason of two-octave system			
	Enharmonic QT belongs to GPS only	But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572	Calculations do not yield numbers reasonably reducible to whole numbers.

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

			(b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237 (c) (69984+ 73728)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 71856/69984 ≈ 1.026748971 and 73728/71856 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration.	**71856**
[d^2]	First shared note of LPS and GPS	73728 * *	GPS only	71856
S′ = 2187/2048				
[$c\#^2$]	GPS only	78732		
S = 256/243	Calculation of chromatic LPS begins from this point.			
[c]	GPS and LPS	82944 * * *		

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

T = 9/8				
[b♭²]	GPS and LPS	93312 * *		
S= 256/243 Ends first descending Mixolydian diapason in the two-octave GPS system	Enharmonic QT in GPS only		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by ancients) ≈ 1.029302237 (c) (93312 + 98304)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 95808/93312 ≈ 1.026748971 and 98304/95808 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are not the same as in the former case. However, they are not altogether to be unexpected. They appear to be close to a quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485).	Calculations do not yield numbers reasonably reducible to whole numbers. **95808**
[a¹] *Mese*	GPS only	98304	GPS only	95808

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

S′ = 2187/2048 Beginning of second Mixolydian diapason in two-octave system				
[g#^{1}] *Mese* in chromatic GPS and in diatonic and enharmonic LPS	GPS and LPS	104976 * *		
S=256/243	Enharmonic QT in LPS only		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237 (c) (104976 + 110592)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 107784/104976 ≈ 1.026748971 and 110592/107784 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size.	Calculations do not yield numbers reasonably reducible to whole numbers. 107784
[g^1]	GPS and LPS	110592* * *	LPS only	107784 *
S′ = 2187/2048				
[f#^{1}]	LPS only	118098 *		
S=256/243				
[f^1]	GPS and LPS	124416 * *		

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

S = 256/243				
[e^{1}]	Does not manifest as note value in GPS or LPS	131072		
S′ = 2187/2048				
[e♭^{1}]	GPS and LPS	139968* * *		
S=256/243 End of initial fifth of the second Mixolydian diapason in the two-octave system	Enharmonic QT in both LPS and GPS		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by ancients) ≈ 1.029302237 (c) (139968 + 147456)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 143712/139968 ≈ 1.026748971 and 147456/143712 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size.	Calculations do not yield numbers reasonably reducible to whole numbers. **143712**
[d^{1}]	GPS and LPS	147456* * *	GPS and LPS	143712 *
S′ = 2187/2048				
[c#^{1}]	GPS and LPS	157464 *		
S= 256/243				
[c^{1}]	GPS and LPS	165888 * *		
T=9/8				

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

[b♭¹]	GPS and LPS	186624 * *		
S=256/243		Enharmonic QT in both GPS and LPS	But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by ancients) ≈ 1.029302237 (c) (186624 + 196608)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 191616/186624 ≈ 1.026748971 and 196608/191616 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size.	Calculations do not yield numbers reasonably reducible to whole numbers. **191616**
[a] *Proslambanomenos* in both GPS and LPS; *Nete Hyperbolaion* for a new repetition	GPS and LPS	196608* * *	GPS and LPS	191616 *
S′ = 2187/2048				
	GPS and LPS	209952 *		
S = 256/243				
	GPS and LPS	221184 *		

Thus ends this sample alternative system comprised by *Timaeus* numbers and arriving at quarter tones needed for enharmonic scales solely by the ancient method of splitting semitones.

TABLE 53 Sample alternative UPS, example no. 1 (*cont.*)

Note: The placement of S′ is dictated by two considerations: (1) preservation of the *ordo* (TTTS TTS TTTS TTS) of the descending Mixolydian disdiapason needed for an alternative GPS in the Dorian *tonos* (S × S′ = 9/8); and (2) the manifest placements of S′ among the *Timaeus* numbers fitting the indicated pattern.

A visual mapping of LPS onto GPS of the above nonstandard perfect system, rendering the relationships between the two systems more readily apparent than they are from an initial inspection of the table above is as follows.

1.1 *Visual Mapping of LPS onto GPS on the Alternative Perfect System*

TABLE 54 Visual mapping of nonstandard LPS onto nonstandard UPS for example no. 1

Nonstandard LPS	Nonstandard GPS
Descending Mixolydian octave with repetition of the final fourth, [TTTS TTS TTS], structured thus: [(T= S′S)T(T=SS′)S] [(T= S′S)(T=SS′)S] [(T= S′S)TS] The intervals highlighted in green and red indicate the semitones that combine to make particular tones of the LPS sequence.	Descending Mixolydian disdiapason, [TTTS TTS TTTS TTS] structured thus: [(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS] [(T= S′S)(T= S′S)(T=SS′)S] [(T= S′S)TS] Calibration of LPS begins from the endpoint of the interval highlighted above in yellow. The intervals highlighted in lavender and blue indicate the semitones that combine to make particular tones of the GPS sequence.
S′ S	S′ S S′ S S S′ S S′ S

TABLE 54 Visual mapping of nonstandard LPS onto nonstandard UPS (*cont.*)

T	T
S	S
S′	S′
S	S
S′	S′
S	S
S	S
S′	S′
S	S
S′	S′
S	S
T	T
S	S
	S′ Begins GPS again; extra note value needed for the complete articulation of chromatic GPS and LPS. S extra note value needed for the complete articulation of chromatic GPS and LPS.

2 Alternative Unmodulating Perfect System: Example 2

TABLE 55 Sample alternative UPS, example no. 2, calibrated on the basis of GPS redefined as a descending Hypophrygian disdiapason [TST (TSTT TST) TSTT] with a descending Phrygian center that is clearly framed by a preceding descending Hypophrygian fourth and a subsequent descending Hypophrygian fifth

A full tone structure suggested by the *Timaeus* number set, for alternative GPS, thus redefined, adequate to account for all diatonic, chromatic, and enharmonic possibilities in such a system is the following:

[(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)] [(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)]

Not all components internal to the structure of "T" will manifest as note values within the GPS system, as the chart below shows.

Diatonic LPS in the new system is redefined, in fashion analogous to the relation between GPS and LPS in the standard system, as the following:

[TST TSTT STT]

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

(In other words, it consists in the descending Hypophrygian diatonic octave with repetition of the last descending fourth, just as in the standard system LPS is the descending Hypodorian octave with a repetition of the last descending Hypodorian fourth.)

The full articulation of LPS suggested by the *Timaeus* number set as necessary to preserve the proper relation between GPS and LPS and to account for all diatonic, chromatic, and enharmonic possibilities in LPS is the following:

[(T= SS′)ST] [(T= SS′)S(T=S′S)(T= SS′)] [S(T= S′S)T]

Again, not all components internal to the structure of "T" will manifest as note values within the LPS system, as the chart below shows. LPS begins calibration against GPS, in the redefined system, at the end of the interval of GPS marked in blue above. The relation of the LPS to GPS in the area of overlap is indicated below by the mapping of LPS onto GPS. The tone structure that LPS has in common with GPS is indicated in alternating lavender and yellow. The tone structure wherein it differs from GPS is indicated in alternating red and green. The blue marking is the last interval in GPS before the overlap with LPS begins.

GPS overlaid with LPS

[(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)] [(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)]

Note that there is no other means of overlaying GPS with LPS in the redefined system under consideration than the means illustrated above.

A few observations are key to elucidating why the GPS and LPS tone structures provided above allow for all chromatic and enharmonic, as well as diatonic possibilities for the redefined GPS and LPS.

Observe that the structure of a descending Hypophrygian chromatic disdiapason (the chromatic GPS), embedding a central descending Phrygian chromatic octave (see the sequence in yellow) is the following:

S′S [TS] S′S T [TS] S′S [TS] S′S T [TS]

Note that the semitones in red, above, are semitones of the 2187/2048, rather than 256/243 variety. A mapping of the chromatic sequence onto GPS manifests that the above semitone specifications are correct. The chromatic sequence maps onto the GPS tone structure articulated above as the alternating highlighting indicates, with the alternating blue and yellow indicating [TS] sequences, the alternating red and green indicating independent semitones and the lavender indicating independent tones, as follows:

[*(T= S*S′)S(T= SS′)] [(T=SS′)ST(T= SS′)] [(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)] *(T=S*S′)

Note that the S unit needed at the end of the chromatic sequence requires the beginning of a second articulation of alternative GPS. Knowing where the articulation of the chromatic GPS sequence begins in relation to diatonic GPS helps one to determine the exact positions of the chromatic notes in the system.

Observe that the structure of a descending Hypophrygian enharmonic disdiapason (the enharmonic GPS), embedding a central descending Phrygian enharmonic octave (highlighted in yellow below) is the following:

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

QQ [TT] QQ T [TT] QQ [TT] QQ T [TT]

This sequence maps onto the GPS tone structure articulated above from the beginning of that sequence, as indicated by alternating yellow, blue, and lavender highlighting, with the lavender indicating the semitones that must be split to achieve the [QQ] units, the yellow indicating the [TT] sequences and the blue indicating the intervals that combine to make the independent tones.

[(T= SS′)S(T= SS′)] [(T = SS′)ST(T= SS′)] [(T= SS′)S(T= SS′)] [(T = SS′)ST(T= SS′)]

Observe that the structures of chromatic Hypophrygian LPS and enharmonic Hypophrygian LPS must also be accommodated in the calibration of the alternative system. They are not exactly parallel with the chromatic Hypophrygian GPS and enharmonic Hypophrygian GPS.

Note that the chromatic structure of Hypophrygian LPS is the following:

S [TS] S′S (T= S′S) [TS] (T= S′S) [TS]

The internal structure of tones is indicated only as necessary to illuminate the relationship of the chromatic Hypophrygian LPS to an embedded Phrygian octave. Observe that one can completely imbed a Phrygian chromatic octave within the above sequence as indicated by the portion of the sequence below highlighted in yellow:

S [TS] S′S (T= S′S) [TS] (T= S′S) [TS]

Note that the articulation of chromatic Hypophrygian GPS already gives the Phrygian chromatic octave a central place.

Note that the S′ sequences noted above for chromatic Hypophrygian LPS, indicate semitones of the 2187/2048, rather than the 256/243 variety. The overlay of chromatic Hypophrygian LPS on diatonic Hypophrygian LPS, below, shows that the semitone identification is correct.

Chromatic Hypophrygian LPS maps onto the diatonic LPS structure given above, as indicated below, with the yellow and blue highlighting indicating independent semitones, the green highlighting indicating independent tones, and the lavender highlighting indicating TS units. Full articulation requires a partial repetition of diatonic Hypophrygian LPS, as the underlining, italicizing, and asterisk marking at the end of the sequence below indicate.

[*(T= SS′)*ST][(T= SS′)S(T=S′S)(T= SS′)][S(T= S′S) T]*[(*T* = SS′)*…]

Observe that the repeated S sequence after (T =) necessary for the complete calibration of chromatic Hypophrygian LPS is still *within the GPS articulation*, since LPS ends a whole tone (T=SS′) short of GPS.

Note that the enharmonic structure of descending Hypophrygian LPS is the following:

QQ [TT] QQ (T= S′S) [TT] [QQ splitting S] [TT]

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

The internal structure of the independent tone, above, is indicated only as necessary to illuminate how one achieves the extra fourth at the end of the enharmonic sequence, as would be consistent with an analogy that took the relationship between GPS and LPS in the standard system as the norm.

Observe that one can completely embed a Phrygian enharmonic octave within the above sequence as the yellow highlighting below indicates.

QQ [TT] QQ (T= S′S) [TT] [QQ splitting S] [TT]

Note that the articulation of enharmonic Hypophrygian GPS already gives the Phrygian enharmonic octave a central place in the system.

Note that enharmonic Hypophrygian LPS maps onto diatonic Hypophrygian LPS, as follows, where the red highlighting indicates semitones that are split to achieve [QQ] units, green highlighting indicates [TT] units, and blue highlighting indicates the independent tone:

[(T= SS′)ST] [(T= SS′)S(T=S′S)(T= SS′)] [S(T= S′S)T]

Enharmonic Hypophrygian LPS relates to the diatonic and enharmonic Hypophrygian GPS structures suggested by the *Timaeus* set, for the area of overlap between GPS and LPS suggested by the *Timaeus* set, as follows, vis-à-vis the *loci* for the calculation of quarter tones. Note that the top sequence is diatonic Hypophrygian LPS marked for enharmonic Hypophrygian LPS. The sequences in red are the *loci* of semitones that must be split to achieve enharmonic Hypophrygian LPS. The second sequence compares diatonic Hypophrygian GPS marked for enharmonic Hypophrygian GPS with the first sequence vis-à-vis the semitones that must be split to achieve the respective LPS and GPS Hypophrygian enharmonic structures. Note that the enharmonic structures of the two systems are not parallel. The LPS structure requires the splitting of three semitones, rather than four. One of the semitone splittings required for enharmonic Hypophrygian LPS is unique to LPS. It shares the other two in common with enharmonic Hypophrygian GPS. The third sequence below shows, in red, the only semitone splitting unique to enharmonic Hypophrygian LPS within the context of diatonic Hypophrygian LPS marked for enharmonic Mixolydian LPS. It also shows, in lavender, within that context, the *loci* of the semitones that must be split to achieve enharmonic Hypophrygian GPS over the area of overlap between LPS and GPS. The fourth sequence below shows, within the context of diatonic Hypophrygian GPS, marked for enharmonic Hypophrygian GPS, all semitones that must be split in the GPS structure to allow for calibration of both enharmonic Hypophrygian GPS and enharmonic Hypophrygian LPS. The markings in lavender, in the fourth sequence, indicate the splittings in common between enharmonic Hypophrygian GPS and enharmonic Hypophrygian LPS. The markings in green indicate the splittings unique to enharmonic Hypophrygian GPS. The markings in red indicate the splitting unique to enharmonic Hypophrygian LPS.

[(T= SS′)ST][(T= SS′)S(T=S′S)(T= SS′)][S(T= S′S)T]

[(T= SS′)S(T= SS′)][(T = SS′)ST(T= SS′)][(T= SS′)S(T= SS′)][(T = SS′)ST(T= SS′)]

[(T= SS′)ST][(T= SS′)S(T=S′S)(T= SS′)][S(T= S′S)T]

[(T= SS′)S(T= SS′)][(T = SS′)ST(T= SS′)][(T= SS′)S(T= SS′)][(T = SS′)ST(T= SS′)]

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

The only additional quarter tone calculation required for enharmonic LPS, not already accommodated by the calculations needed for enharmonic GPS occur at the S marked immediately above. It is clear that enharmonic LPS and GPS have only two QT calculations in common, that LPS has one QT calculation unique to it, and that GPs has two QT calculations unique to it.

When all is said and done, it appears that the alternative system accommodating the new GPS and LPS considered in this example, must be calibrated as follows, including an extra S sequence beyond the disdiapason (indicated in green) to accommodate all chromatic possibilities in GPS. The *loci* for the calibration of QT's is indicated in green (unique to GPS), red (unique to LPS) and lavender (common to GPS and LPS). The sequences highlighted in blue indicate the intervals included within GPS only. GPS and LPS overlap for the remainder of the sequence.

[(T= SS′)S(T= SS′)][(T=SS′)ST(T= SS′)][(T= SS′)S(T= SS′)][(T=SS′)ST(T= SS′)]

If one were to compare the alternative perfect system considered in this example to the standard perfect system, one would find that the *nete hyperbolaion* of the alternative system begins a TS sequence higher than in the standard system within a roughly comparable number range, correlating to the *parhypate hypaton* or the [c] of the octave above [a^2] of the standard system. The two systems clearly cannot be precisely mapped onto one another, as *nete hyperbolaion* is redefined, in the alternative system as [c] of the octave above the initial [a^2] of the standard calibration, i.e., [c^3] in relation to the standard system and as the new system has a Phrygian rather than Dorian center. Names beyond *nete hyperbolaion*, *mese*, and *proslambanomenos* are unattempted, and note values are mere educated guesses, as the alternative perfect system analyzed in this chart is a theoretical construct.

Diatonic notes of GPS are highlighted in yellow. Chromatic notes of GPS are highlighted in blue. Enharmonic notes of GPS are highlighted in green. When a note belongs to more than one scale, then it is highlighted in the applicable number of colors. Diatonic notes of LPS are marked with a yellow asterisk *. Chromatic notes of LPS are marked with a blue asterisk *. Enharmonic notes of LPS are marked with a green asterisk *. When a note belongs to more than one scale, it is marked with the applicable number of colored asterisks. Observe that only elements in blue, green, and yellow correspond to actual note values within GPS and LPS. The numbers highlighted in red are notes implicit within the structure of "T," but they never independently manifest, given the formulas for the diatonic, chromatic, and enharmonic Hypophrygian scales in ancient Greek music. The "unused" elements point to theoretical possibilities for additional scales that were probably never realized by the ancient Greeks.

[c^2] *nete hyperbolaion*	52488		
S=256/243		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by ancients) ≈ 1.029302237	Calculations do not yield numerical values reasonably reducible to whole numbers.

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

		(c) (52488 + 55296)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 54594/53892 ≈ 1.026748971 and 55296/53892 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration, making the system somewhat less rich than the standard system.	53892
	55296 (note from which chromatic GPS is calculated)		53892
S′ = 2187/2048			
	59049		
S = 256/243			
	62208		
S = 256/243			
	65536		
S′ = 2187/2048			
	69984 * *		

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

S = 256/243	First shared note as between LPS and GPS	But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237 (c) (69984 + 73728)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 71856/69984 ≈ 1.026748971 and 73728/71856 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration, making the system somewhat less rich than the standard system	Calculations do not yield numerical values reasonably reducible to whole numbers. 71856
	73728 *		71856 *
S′ = 2187/2048			
	78732 * * (note from which chromatic LPS is calculated)		
S = 256/243			
	82944 * *		
T=9/8			

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

	93312 * *		
S=256/243		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237 (c) (93312 + 98304)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 95808/93312 ≈ 1.026748971 and 98304/95808 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration, making the system somewhat less rich than the standard system.	Calculations do not yield numerical values reasonably reducible to whole numbers. 95808
	98304 * *		95808 *
S′ = 2187/2048			
	104976 * *		
S=256/243		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237	Calculations do not yield numerical values reasonably reducible to whole numbers.

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

		(c) (104976+ 110592)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 107784/104976 ≈ 1.026748971 and 110592/107784 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration, making the system somewhat less rich than the standard system.	107784
	110592 * * *		107784
S′ = 2187/2048			
	118098		
S = 256/243			
	124416 * *		
S = 256/243			
	131072		
S′ = 2187/2048			
	139968 * *		
S = 256/243		But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572	Calculations do not yield numerical values reasonably reducible to whole numbers.

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

		(b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237 (c) (139968+ 147456)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 143712/139968 ≈ 1.026748971 and 147456/143712 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration, making the system somewhat less rich than the standard system.	143712
	147456 * * *		143712 *
S′ = 2187/2048			
	157464		
S = 256/243			
	165888 * *		
T = 9/8	Last note value in common as between GPS and LPS		
	186624 * *		
S = 256/243			
	196608 *		

TABLE 55 Sample alternative UPS, example no. 2 (*cont.*)

S′ = 2187/2048			
	209952		
S = 256/243	Extra element beyond the dis-diapason needed for the complete articulation of chromatic Hypophrygian GPS	But if QT = (a) $(9/8)^{1/4}$ (ancient's true QT, incalculable by them) ≈ 1.029883572 (b) $[(2)^{1/6}]^{1/4}$ (true QT incalculable by the ancients) ≈ 1.029302237 (c) (209952 + 221184)/2 i.e., simple interval split of segment bounded by even numbers, probable ancient method of finding the QT, results in approx. equality of 215568/209952 ≈ 1.026748971 and 221184/215568 ≈ 1.026052104 i.e., method yields two quarter tones of slightly different size. The values of these unequal quarter tones are uniform throughout this table. Note that the quarter measure based on splits of semitones of the 2187/2048 variety (the square root of 1.067871094 ≈ 1.033378485) do not occur in this calibration, making the system somewhat less rich than the standard system.	Calculations do not yield numerical values reasonably reducible to whole numbers. 215568
	221184		215568

Thus ends this sample alternative perfect system comprised by *Timaeus* numbers and arriving at quarter tones needed for enharmonic scales solely by the ancient method of splitting semitones.

Note: The placement of S′ is dictated by two considerations: (1) preservation of the *ordo* (TST TSTT TST TSTT) of the descending Hypophrygian disdiapason needed for an alternative GPS in the Phrygian *tonos* (S × S′ = 9/8); and (2) the manifest placements of S′ among the *Timaeus* numbers fitting the indicated pattern.

A visual mapping of LPS onto GPS in the above alternative system, rendering the relationships between the two systems more readily apparent than they are from an initial inspection of the table above is as follows:

2.1 *Visual mapping of LPS onto GPS in the alternative model*

TABLE 56 Visual mapping of alternative LPS onto alternative GPS for example no. 2

Nonstandard LPS	Nonstandard GPS
Descending Hypophrygian octave with repetition of the final fourth, [TST TSTT STT], structured thus: [(T= SS′)ST] [(T= SS′)S(T=S′S)(T= SS′)] [S(T= S′S)T] The intervals highlighted in green and red indicate the semitones that combine to make particular tones of the LPS sequence.	Descending Hypophrygian disdiapason, [TST TSTT TST TSTT] structured thus: [(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)] [(T= SS′)S(T= SS′)] [(T=SS′)ST(T= SS′)] Calibration of LPS begins from the endpoint of the interval highlighted above in yellow. The intervals highlighted in lavender and blue indicate the semitones that combine to make particular tones of the GPS sequence.
S S′ S T S S′ S S′ S S	S S′ S S S′ S S′ S T S S′ S S′ S S

TABLE 56 Visual mapping of alternative LPS onto alternative GPS for example no. 2 (*cont.*)

S′	S′
S	S
S′	S′
S	S
T	T
	S
	S′
	S (begins GPS anew; needed to accommodate chromatic GPS)

APPENDIX 5

Two Overlapping Sequences of Doubles, Including Coincident Diatonic Octaves within Each, Bounded Entirely by Chromatic Factors of 1719926784

1 Key

(a) "Coincident diatonic octaves" are octaves of different species sharing the same octave span.
(b) Numbers in bold indicate chromatic nonfactors of 1719926784.
(c) Numbers in plain type, belong to the subset of chromatic factors of 1719926784; they comprise the endpoints defining the continuously overlapping doubles in this appendix.
(d) Italicized numbers also comprise scalar elements within the original octave periodicity, i.e., the original model scale numbers and their multiples, prior to the complete degeneration of the primary *Timaeus* scale.
(e) Plain numbers in brackets are both chromatic factors of 1719926784 and nonscalar elements in relation to the original octave periodicity, i.e., that of the primary d.Dorian/a.Lydian scale of the *Timaeus*, as they arise after that periodicity has completely degenerated. See Table 26, "Fully Annotated Table of Timaeus Numbers Arranged in Numerical Order from Least to Greatest with Annotations."
(f) Note that the two series of overlapping doubles represented in Appendix 6 and this Appendix 5, originating, respectively, from the chromatic nonfactors and chromatic factors of 1719926784, are distinct series vis-à-vis each other. Each is characterized by a unique set of overlapping octave chains, each of which, in itself, includes overlapping coincident octaves; both sets overlap each other, too, and also interfere with the original periodicity.

2 Preliminary Notes

Preliminary note one: Observe how the subsets of octaves articulated in this appendix reflect the mixed character of the world soul in the numbers composing them. As the reader will recall, the world soul is a mixture of same, different, and the mixture of same and different. On the level of the kinds of numbers composing the octaves below, "same" corresponds to the model scale numbers of the original octave periodicity and their multiples, all of which evenly divide 1719926784. "Different" corresponds

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to the chromatic nonfactors of 1719926784. Mixed corresponds to the chromatic factors of 1719926784. The chromatic factors correspond to a "mixture of same and different" because, while they are not model scale numbers or their multiples, they share the feature, in common with the model scale numbers and their multiples, of being even divisors of 1719926784. The chromatic nonfactors are neither model scale numbers, their multiples, nor even divisors of 1719926784.

Preliminary note two: Appendix 5 evidences two chains of contiguous octaves. The two chains continuously overlap each other. The first chain is comprised by the octaves labeled (1), (2), (4), (6), (8), (10), (12), (14), (16), (18), (20), (22), (24), (26), (28), (30), (31), and (32). The second is comprised by the octaves labeled (3), (5), (7), (9), (11), (13), (15), (17), (19), (21), (23), (25), (27), and (29). The first chain actually gives rise to the second. The beginning tone of the second chain, 6561, arises as the tone a fifth removed from 4374, the starting tone of the second octave in the first string. The two overlapping octave chains of Appendix 5 are, then, removed from one another by the sesquialter interval.

The second chain of Appendix 5, in turn, mediates a relationship to the doubles of Appendix 6, based upon chromatic *nonfactors* of 1719926784. The beginning tone of the first chain of doubles based upon chromatic nonfactors, 19683, arises as the tone a fifth removed from 13122, the starting tone of the second octave of the second chain of doubles based on chromatic factors. The two chains are, then, removed from one another by the sesquialter interval.

3 Octave Sequences Originating from Chromatic Factors of 1719926784

(1) 2187 *2916* 4374

4374:2187 = 2:1= octave ratio
2916:2187 = 4:3 = sesquitertian ratio
4374:2916 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

2187 *2304 2592 2916* · *3072 3456 3888* 4374
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2

Note one: The scale specified is a d.Hypolydian/a.Mixolydian scale. One might wonder if a person could rearrange the order of *leimma* and whole tone intervals in the parts

of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 2187 as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

(2) 4374 6561 8748

8748:4374 = 2:1 = the octave ratio
6561:4374 = 3:2 = the sesquialter ratio
8748:6561 = 4:3 = the sesquitertian ratio
$3/2 \times 4/3 = 2$

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

4374 *4608 5184 5832* 6561 · *6912 7776* 8748
$[256/243 \times 9/8 \times 9/8 \times 9/8 = 3/2] \times [256/243 \times 9/8 \times 9/8 = 4/3] = 2$
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 4374 and 8748, based on the *Timaeus* numbers, which assumes 4374 as the foundation for an initial fourth, rather than a fifth. There are two such alternatives, constituting, in fact, coincident octaves, vis-à-vis each other and the octave formulation above, as follows:

(a) 4374 *4608 5184 5832* · *6144 6912 7776* 8748
$[256/243 \times 9/8 \times 9/8 = 4/3] \times [256/243 \times 9/8 \times 9/8 \times 9/8 = 3/2] = 2$
d.Hypolydian/a.Mixolydian scale

(b) 4374 *4608 5184 5832* · 6561 *6912 7776* 8748
$[256/243 \times 9/8 \times 9/8 = 4/3] \times [9/8 \times 256/243 \times 9/8 \times 9/8 = 3/2] = 2$
d.Lydian/a.Dorian scale

Note three: Observe that the option in (b) is identical to the original formulation, despite the rearrangement of principal constituent parts it assumes. The reproduction of the original sequence, in such rearrangement, side-by-side with a unique option, similarly produced, represents the process whereby an original monad becomes a duad. The duad becomes a possibility when one can legitimately interpret the original sequence in more than one way, e.g., as beginning with either a fourth or fifth. The emergence of the duad from only one of those perspectives shows that the choice of interpretation is not a matter of indifference. The choice of interpretation bears particular generative consequences.

Note four: Although one can analyze the chromatically grounded diatonic octave indicated, as illustrated above, starting with either the fourth, marked by *5832* or the fifth, marked by 6561, it is more fitting to begin with the fifth, as the primary articulation. As an intermediate, chromatic node, 6561 is similar to the endpoints in its relation to the original periodicity of the chain; further, like each of the endpoints, 6561 is a chromatic factor of 1719926784.

(3) 6561 8748 13122

13122:6561 = 2:1 = the octave ratio
8748:6561 = 4:3 = the sesquitertian ratio
13122:8748 = 3:2 = the sesquialter ratio
$4/3 \times 3/2 = 2$

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

6561 *6912 7776* 8748 · *9216 10368 11664* 13122
$[256/243 \times 9/8 \times 9/8 = 4/3] \times [256/243 \times 9/8 \times 9/8 \times 9/8 = 3/2] = 2$
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 6561 as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a whole number.

Note three: One unaccustomed arrangement of the octave is possible not allowing any standard division into fourths and fifths, namely, STSTTTT, represented as follows:

6561 *6912 7776 8192 9216 10368 11664* 13122
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(4) 8748 1312 17496

17496:8748 = 2:1 = the octave ratio
13122:8748 = 3:2 = sesquialter ratio
17496:13122 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

8748 *9216 10368 11664* 13122 · *13824 15552* 17496
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian scale

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 8748 and 17496, based on the *Timaeus* numbers, which assumes 8748 as the foundation for an initial fourth, rather than a fifth. There are indeed two such alternatives, constituting, in fact, coincident octaves, vis-à-vis each other and the octave formulation above, as follows:

(a) 8748 *9216 10368 11664* · *12288 13824 15552* 17496
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian scale.

(b) 8748 *9216 10368 11664* · 13122 *13824 15552* 17496
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian scale.

Note three: Observe that the option in (b) is identical to the original formulation, despite the rearrangement of principal constituent parts it assumes. The reproduction of the original sequence, in such rearrangement, side-by-side with a unique option, similarly produced, represents the process whereby an original monad becomes a duad. The duad becomes a possibility when one can legitimately interpret the original sequence

of the chain in more than one way, e.g., as beginning with either a fourth or pentachord. The emergence of the duad from only one of those perspectives shows that the choice of interpretation is not a matter of indifference. The choice of interpretation bears particular generative consequences.

(5) 13122 17496 26244

26244:13122 = 2:1 = octave ratio
17496:13122 = 4:3 = sesquitertian ratio
26244:17496 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 13122 *13824 15552* 17496 · *18432 20736 23328* 26244
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 13122 *13824 15552* 17496 · **19683** *20736 23328* 26244
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

Note one: Observe that the second full articulation of the octave, constituting a coincident octave with the first, arises as a result of rearranging the order of *leimma* and whole tone intervals in the second primary part. The rearrangement allows an original monad to become a pair, a duad.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 13122 as the beginning of a fifth rather than a fourth. There is, indeed such an alternative formulation, constituting a coincident octave with those already indicated, as follows:

13122 *13824 15552* 17496 **19683** · *20736 23328* 26244
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2

Inspection shows that it is identical to option (b) above.

Note three: The same unaccustomed octave sequence, STSTTTT, possible for octave (3), not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (5), as follows:

13122 *13824 15552 16384 18432 20736 23328* 26244
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(6) 17496 26244 34992

34992:17496 = 2:1 = the octave ratio
26244:17496 = 3:2 = sesquialter ratio
34992:26244 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 17496 *18432 20736 23328* 26244 · *27648 31104* 34992
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 17496 **19683** *20736 23328* 26244 · *27648 31104* 34992
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the second full articulation of the octave arises as a result of rearranging the order of *leimma* and whole tone intervals in its first primary part. The process of rearrangement allows the emergence of a duad from an original monad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 17496 and 34992, based on the *Timaeus* numbers, which assumes 17496 as the foundation for an initial fourth, rather than a fifth. There are four such alternatives, constituting coincident octaves, vis-à-vis each other and the octave formulations above, as follows:

(a) 17496 *18432 20736 23328* · *24576 27648 31104* 34992
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 17496 **19683** *20736 23328* · *24576 27648 31104* 34992
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed sequence: TST STTT

(c) 17496 *18432 20736 23328* · 26244 *27648 31104* 34992
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(d) 17496 **19683** *20736 23328* · 26244 *27648 31104* 34992
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that options (c) and (d) are identical to the original formulations, assuming a different order of primary principal parts. The reproduction among the secondary formulations of the original formulations plus two unique sequences is well-suited to representing the movement of the monad to the duad in each case. Since one might accurately assess that a primary movement of the monad to the duad occurs at the level of the original formulations, one might also view the secondary level as a progression from a duad to a duad. A tetrad, relating to the original monad, arises when the duad it generates moves to a duad. The sixth octave articulation, then, shows a complete progression of a monad to four, in harmony with the importance the *Timaeus* text, in general gives to that progression.

(7) 26244 34992 52488

52488:26244 = 2:1 = the octave ratio
34992:26244 = 4:3 = the sesquitertian ratio
52488:34992 = 3:2 = the sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 26244 *27648 31104* 34992 · *36864 41472 46656* 52488
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 26244 *27648 31104* 34992 · **39366** *41472 46656* 52488
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

Note one: Observe that the second full articulation of the octave arises as a result of rearranging the order of *leimma* and whole tone intervals in its second primary part. The process of rearrangement allows the emergence of a duad from an original monad.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 26244 as the beginning of a fifth rather than a fourth. There is one such alternative constituting a coincident octave with those already articulated as follows:

26244 *27648 31104* 34992 **39366** · *41472 46656* 52488
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note three: Observe that the above option is identical to option (b) among the original formulations, despite the rearrangement of principal constituent parts it assumes.

Note four: The same unaccustomed octave sequence, STSTTTT, possible for octaves (3) and (5), not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (7), as follows:

26244 *27648 31104 32768 36864 41472 46656* 52488
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(8) 34992 52488 69984

69984:34992 = 2:1 = the octave ratio
52488:34992 = 3:2 = the sesquialter ratio
69984:52488 = 4:3 = the sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 34992 *36864 41472 46656* 52488 · *55296 62208* 69984
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 34992 *36864 41472 46656* 52488 · **59049** *62208* 69984
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed sequence: STTT TST

(c) 34992 **39366** *41472 46656* 52488 · *55296 62208* 69984
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(d) 34992 **39366** *41472 46656* 52488 · **59049** *62208* 69984
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. With the eighth octave, possibilities of rearrangement expand from the first primary constituent part of the octave to the second, as well. The latter phenomenon is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 34992 and 69984, based on the *Timaeus* numbers, which assumes 34992 as the foundation for an initial fourth, rather than a fifth. There are, indeed, such alternatives, constituting coincident octaves vis-à-vis each other and the original formulations, as follows:

(a) 34992 *36864 41472 46656* · *49152 55296 62208* 69984
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 34992 *36864 41472 46656* · 52488 *55296 62208* 69984
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 34992 *36864 41472 46656* · 52488 **59049** *62208* 69984
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed sequence: STT TTST

(d) 34992 **39366** *41472 46656* · *49152 55296 62208* 69984
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed sequence: TST STTT

(e) 34992 **39366** *41472 46656* · 52488 *55296 62208* 69984
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(f) 34992 **39366** *41472 46656* · 52488 **59049** *62208* 69984
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Six secondary possibilities arise with octave set (8) because of yet a new movement from monad to duad. Previously, there had been only one way to rearrange whole tones and *leimmata* in the secondary primary component of the octave in the secondary set. Now there are two ways. Observe that four of the above sequences, (b),

(c), (e), and (f) are identical to options among the original formulations, namely, (a), (b), (c), and (d), respectively. Only two sequences, (a) and (d), among the secondary set, are unique.

(9) 52488 69984 104976

104976:52488 = 2:1 = the octave ratio
69984:52488 = 4:3 = the sesquitertian ratio
104976:69984= 3:2 = the sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 52488 *55296 62208* 69984 · *73728 82944 93312* 104976
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 52488 *55296 62208* 69984 · **78732** *82944 93312* 104976
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 52488 **59049** *62208* 69984 · *73728 82944 93312* 104976
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed sequence: TST STTT

(d) 52488 **59049** *62208* 69984 · **78732** *82944 93312* 104976
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not one could produce alternative formulations of the given octaves by treating 52488 as the beginning of a fifth, rather than a fourth. There are, indeed two such alternative formulations, constituting coincident octaves, as follows:

(a) 52488 *55296 62208* 69984 **78732** · *82944 93312* 104976
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 52488 **59049** *62208* 69984 **78732** · *82944 93312* 104976
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternative formulations, (a) and (b) are identical, respectively, with options (b) and (d) among the original formulations.

Note four: The same unaccustomed octave sequence, STSTTTT, possible for octaves (3), (5), and (7), not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (9), as follows:

52488 *55296 62208 65536 73728 82944 93312* 104976
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

Note five: Yet another unaccustomed octave sequence, TSSTTTT arises in octave (9) not allowing for any standard division of the octave into fourths and fifths, as follows:

52488 **59049** *62208 65536 73728 82944 93312* 104976
[9/8 × 256/243 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(10) 69984 104976 139968

139968:69984 = 2:1 = the octave ratio
104976:69984 = 3:2 = the sesquialter ratio
139968:104976 = 4:3 = the sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 69984 *73728 82944 93312* 104976 · *110592 124416* 139968
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 69984 *73728 82944 93312* 104976 · **118098** *124416* 139968
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed sequence: STTT TST

(c) 69984 **78732** *82944 93312* 104976 · *110592 124416* 139968
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(d) 69984 **78732** *82944 93312* 104976 · **118098** *124416* 139968
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 69984 and 139968, based on the *Timaeus* numbers, which assumes 69984 as the foundation for an initial fourth, rather than a fifth. There are indeed such alternatives, constituting coincident octaves vis-à-vis each other and those articulated above, as follows:

(a) 69984 *73728 82944 93312* · *98304 110592 124416* 139968
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 69984 *73728 82944 93312* · 104976 *110592 124416* 139968
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 69984 *73728 82944 93312* · 104976 **118098** *124416* 139968
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed sequence: STT TTST

(d) 69984 **78732** *82944 93312* · *98304 110592 124416* 139968
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed sequence: TST STTT

(e) 69984 **78732** *82944 93312* · 104976 *110592 124416* 139968
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(f) 69984 **78732** *82944 93312* · 104976 **118098** *124416* 139968
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Secondary possibilities arise because of a particular movement from monad to duad. At one point, there had been only one way to rearrange whole tones and *leimmata* in the secondary primary component of the octave in the secondary set. Now there are two ways. Observe that four of the above sequences, (b), (c), (e), and (f) are identical to options among the original formulations, namely, (a), (b), (c), and (d), respectively. Only two sequences, (a) and (d), among the secondary set, are unique.

(11) 104976 139968 209952

209952:104976 = 2:1 = the octave ratio
139968:104976 = 4:3 = sesquitertian ratio
209952:139968 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers, as follows:

(a) 104976 *110592 124416* 139968 · *147456 165888 186624* 209952
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 104976 *110592 124416* 139968 · **157464** *165888 186624* 209952
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 104976 **118098** *124416* 139968 · *147456 165888 186624* 209952
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed sequence: TST STTT

(d) 104976 **118098** *124416* 139968 · **157464** *165888 186624* 209952
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent

parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 104976 as the beginning of a fifth rather than a fourth. There are, indeed, two such alternative formulations as follows:

(a) 104976 *110592 124416* 139968 **157464** · *165888 186624* 209952
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 104976 **118098** *124416* 139968 **157464** · *165888 186624* 209952
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternative formulations, (a) and (b) are identical, respectively, with options (b) and (d) among the original formulations.

Note four: The same unaccustomed octave sequence, STSTTTT, possible for octaves (3), (5), (7) and (9) not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (11), as follows:

104976 *110592 124416 131072 147456 165888 186624* 209952
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

Note five: The unaccustomed octave sequence, TSSTTTT arising in octave (9), not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (11), as follows:

104976 **118098** *124416 131072 147456 165888 186624* 209952
[9/8 × 256/243 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(12) 139968 209952 279936

279936:139968 = 2:1 = the octave ratio
209952:139936 = 3:2 = the sesquialter ratio
279936:209952 = 4:3 = the sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves, from the set of *Timaeus* numbers as follows:

(a) 139968 *147456 165888 186624* 209952 · *221184 248832* 279936
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
a.Lydian/d.Dorian

(b) 139968 *147456 165888 186624* 209952 · **236196** *248832* 279936
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed arrangement: STTT TST

(c) 139968 **157464** *165888 186624* 209952 · *221184 248832* 279936
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a. Hypodorian

(d) 139968 **157464** *165888 186624* 209952 · **236196** *248832* 279936
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 139968 and 279936, based on the *Timaeus* numbers, which assumes 139968 as the foundation for an initial fourth, rather than a fifth. There are, indeed, such alternatives, constituting coincident octaves as follows:

(a) 139968 *147456 165888 186624* · *196608 221184 248832* 279936
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 139968 *147456 165888 186624* · 209952 *221184 248832* 279936
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 139968 *147456 165888 186624* · 209952 **236196** *248832* 279936
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed: STT TTST

(d) 139968 **157464** *165888 186624* · *196608 221184 248832* 279936
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed: TST STTT

(e) 139968 **157464** *165888 186624* · 209952 *221184 248832* 279936
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(f) 139968 **157464** *165888 186624* · 209952 **236196** *248832* 279936
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Secondary possibilities arise because of a particular movement from monad to duad. At one point, there had been only one way to rearrange whole tones and *leimmata* in the secondary primary component of the octave in the secondary set. Now there are two ways. Observe that four of the above sequences, (b), (c), (e), and (f) are identical to options among the original formulations, namely, (a), (b), (c), and (d), respectively. Only two sequences, (a) and (d), among the secondary set, are unique.

(13) 209952 279936 419904

419904:209952 = 2:1 = the octave ratio
279936:209952 = 4:3 = sesquitertian ratio
419904:279936 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves, from the set of *Timaeus* numbers as follows:

(a) 209952 *221184 248832* 279936 · *294912 331776 373248* 419904
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 209952 *221184 248832* 279936 · **314928** *331776 373248* 419904
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 209952 **236196** *248832* 279936 · *294912 331776 373248* 419904
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed arrangement: TST STTT

(d) 209952 **236196** *248832* 279936 · **314928** *331776 373248* 419904
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 209952 as the beginning of a fifth rather than a fourth. There are, indeed two such alternative formulations, constituting coincident octaves vis-à-vis each other and those above as follows:

(a) 209952 *221184 248832* 279936 **314928** · *331776 373248* 419904
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 209952 **236196** *248832* 279936 **314928** · *331776 373248* 419904
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternative formulations, (a) and (b) are identical, respectively, with options (b) and (d) among the original formulations.

Note four: The same unaccustomed octave sequence, STSTTTT, possible for octaves (3), (5), (7), (9), and (11), not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (13), as follows:

209952 *221184 248832 262144 294912 331776 373248* 419904
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

Note five: Yet another unaccustomed octave sequence, TSSTTTT, arising in octaves (9) and (11), not allowing for any standard division of the octave into fourths and fifths, is also possible for octave (13) as follows:

209952 **236196** *248832 262144 294912 331776 373248* 419904
9/8 × 256/243 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8

(14) 279936 419904 559872

559872:279936 = 2:1 = the octave ratio
419904:279936 = 3:2 = the sesquialter ratio
559872:419904 = 4:3 = the sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves, from the set of *Timaeus* numbers as follows:

(a) 279936 *294912 331776 373248* 419904 · *442368 497664* 559872
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 279936 *294912 331776 373248* 419904 · **472392** *497664* 559872
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed sequence: STTT TST

(c) 279936 **314928** *331776 373248* 419904 · *442368 497664* 559872
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(d) 279936 **314928** *331776 373248* 419904 · **472392** *497664* 559872
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 279936 and 559872, based on the *Timaeus* numbers, which assumes 279936 as the foundation for an initial fourth, rather than a fifth. There are, indeed, such alternatives, constituting coincident octaves as follows:

(a) 279936 *294912 331776 373248* · *393216 442368 497664* 559872
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 279936 *294912 331776 373248* · 419904 *442368 497664* 559872
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 279936 *294912 331776 373248* · 419904 **472392** *497664* 559872
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed: STT TTST

(d) 279936 **314928** *331776 373248* · *393216 442368 497664* 559872
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed: TST STTT

(e) 279936 **314928** *331776 373248* · 419904 *442368 497664* 559872
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(f) 279936 **314928** *331776 373248* · 419904 **472392** *497664* 559872
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Six secondary possibilities arise because of a particular movement from monad to duad. At one point, there had been only one way to rearrange whole tones and *leimmata* in the secondary primary component of the octave in the secondary set. Now there are two ways. Observe that four of the above sequences, (b), (c), (e), and (f) are identical to options among the original formulations, namely, (a), (b), (c), and (d), respectively. Only two sequences, (a) and (d), among the secondary set, are unique.

(15) 419904 559872 839808

839808:419904 = 2:1 = the octave ratio
559872:419904 = 4:3 = sesquitertian ratio
839808:559872 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 419904 *442368 497664* 559872 · *589824 663552 746496* 839808
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 419904 *442368 497664* 559872 · **629856** *663552 746496* 839808
[256/243 × 9/8 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 419904 **472392** *497664* 559872 · *589824 663552 746496* 839808
[9/8 × 256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed: TST STTT

(d) 419904 **472392** *497664* 559872 · **629856** *663552 746496* 839808
[9/8 × 256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 419904 as the beginning of a fifth rather than a fourth. There are two such alternative formulations, constituting coincident octaves, vis-à-vis each other and those above, as follows:

(a) 419904 *442368 497664* 559872 **629856** · *663552 746496* 839808
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 419904 **472392** *497664* 559872 **629856** · *663552 746496* 839808
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternative formulations, (a) and (b) are identical, respectively, with options (b) and (d) among the original formulations.

(16) 559872 839808 1119744

1119744:559872 = 2:1 = octave ratio
839808:559872 = 3:2 = sesquialter ratio
1119744:839808 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves, from the set of *Timaeus* numbers as follows:

(a) 559872 *589824 663552 746496* 839808 · *884736 995328* 1119744
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 559872 *589824 663552 746496* 839808 · **944784** *995328* 1119744
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed: STTT TST

(c) 559872 **629856** *663552 746496* 839808 · *884736 995328* 1119744
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(d) 559872 **629856** *663552 746496* 839808 · **944784** *995328* 1119744
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 559872 and 1119744, based on the *Timaeus* numbers, which assumes 559872 as the foundation for an initial fourth, rather than a fifth. There are indeed such alternatives, constituting coincident octaves both with each other and those above, as follows:

(a) 559872 *589824 663552 746496* · *786432 884736 995328* 1119744
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 559872 *589824 663552 746496* · 839808 *884736 995328* 1119744
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 559872 *589824 663552 746496* · 839808 **944784** *995328* 1119744
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed: STT TTST

(d) 559872 **629856** *663552 746496* · *786432 884736 995328* 1119744
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed: TST STTT

(e) 559872 **629856** *663552 746496* · 839808 *884736 995328* 1119744
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(f) 559872 **629856** *663552 746496* · 839808 **944784** *995328* 1119744
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Six secondary possibilities arise because of a particular movement from monad to duad. At one point, there had been only one way to rearrange whole tones and *leimmata* in the secondary primary component of the octave in the secondary set. Now there are two ways. Observe that four of the above sequences, (b), (c), (e), and (f) are identical to options among the original formulations, namely, (a), (b), (c), and (d), respectively. Only two sequences, (a) and (d), among the secondary set, are unique.

(17) 839808 1119744 1679616

1679616:839808 = 2:1 = octave ratio
1119744:839808 = 4:3 = sesquitertian ratio
1679616:1119744 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves, from the set of *Timaeus* numbers as follows:

(a) 839808 *884736 995328* 1119744 · *1179648 1327104 1492992* 1679616
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 839808 *884736 995328* 1119744 · **1259712** *1327104 1492992* 1679616
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(c) 839808 **944784** *995328* 1119744 · *1179648 1327104 1492992* 1679616
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed: TST STTT

(d) 839808 **944784** *995328* 1119744 · **1259712** *1327104 1492992* 1679616
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 839808 as the beginning of a fifth rather than a fourth. There are, indeed, two such alternative formulations, constituting coincident octaves both with each other and those above, as follows:

(a) 839808 *884736 995328* 1119744 **1259712** · *1327104 1492992* 1679616
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 839808 **944784** *995328* 1119744 **1259712** · *1327104 1492992* 1679616
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternative formulations, (a) and (b) are identical, respectively, with options (b) and (d) among the original formulations.

(18) 1119744 1679616 2239488

2239488:1119744 = 2:1 = octave ratio
1679616:1119744 = 3:2 = sesquialter ratio
2239488:1679616 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave, indeed four coincident octaves, from the set of *Timaeus* numbers as follows:

(a) 1119744 *1179648 1327104 1492992* 1679616 · *1769472 1990656* 2239488
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 1119744 *1179648 1327104 1492992* 1679616 · **1889568** *1990656* 2239488
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed: STTT TST

(c) 1119744 **1259712** *1327104 1492992* 1679616 · *1769472 1990656* 2239488
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(d) 1119744 **1259712** *1327104 1492992* 1679616 · **1889568** *1990656* 2239488
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 1119744 and 2239488, based upon the *Timaeus* numbers, which assumes 1119744 as the foundation for an initial fourth, rather than a fifth. There are, indeed, such alternative formulations, constituting coincident octaves both with one another and those above, as follows:

(a) 1119744 *1179648 1327104 1492992* · 1679616 *1769472 1990656* 2239488
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(b) 1119744 *1179648 1327104 1492992* · 1679616 **1889568** *1990656* 2239488
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed: STT TTST

(c) 1119744 **1259712** *1327104 1492992* · 1679616 *1769472 1990656* 2239488
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(d) 1119744 **1259712** *1327104 1492992* · 1679616 **1889568** *1990656* 2239488
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Observe that the usual pattern of six alternatives has been broken. The interval 256/243 can no longer be made with the value marking the top of the primary fourth. Observe that all existing alternative formulations are now identical to the original ones.

(19) 1679616 2239488 3359232

3359232:1679616 = 2:1 = octave ratio
2239488:1679616 = 4:3 = sesquitertian ratio
335232:2239488 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves from the set of *Timaeus* numbers as follows:

(a) 1679616 *1769472 1990656* 2239488 · *2359296 2654208 2985984* 3359232
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) 1679616 *1769472 1990656* 2239488 · **2519424** *2654208 2985984* 3359232
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 =3/2] = 2
d.Lydian/a.Dorian

(c) 1679616 **1889568** *1990656* 2239488 · *2359296 2654208 2985984* 3359232
[9/8 × 256/243 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
Unaccustomed: TST STTT

(d) 1679616 **1889568** *1990656* 2239488 · **2519424** *2654208 2985984* 3359232
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 =3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two`: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 1679616 as the beginning of a fifth rather than a fourth. There are, indeed, two such alternative formulations, constituting coincident octaves both with one another and those above, as follows:

(a) 1679616 *1769472 1990656* 2239488 **2519424** · *2654208 2985984* 3359232
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 1679616 **1889568** *1990656* 2239488 **2519424** · *2654208 2985984* 3359232
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternative formulations, (a) and (b) are identical, respectively, with options (b) and (d) among the original formulations.

(20) 2239488 3359232 4478976

4478976:2239488 = 2:1 = octave ratio
3359232:2239488 = 3:2 = sesquialter ratio
4478976:3359232 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octaves, from the set of *Timaeus* numbers as follows:

(a) 2239488 *2359296 2654208 2985984* 3359232 · *3538944 3981312* 4478976
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 2239488 *2359296 2654208 2985984* 3359232 · **3779136** *3981312* 4478976
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
Unaccustomed: STTT TST

(c) 2239488 **2519424** *2654208 2985984* 3359232 · *3538944 3981312* 4478976
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(d) 2239488 **2519424** *2654208 2985984* 3359232 · **3779136** *3981312* 4478976
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe that the production of four different options on the primary level, representing the movement of the monad to the tetrad on that plane, results from the possibility of rearranging whole tones and *leimmata* in both of the primary constituent parts of the octave. The expansion of the possibilities of rearrangement from the first primary constituent part of the octave to the second is itself a movement of monad to duad.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 2239488 and 4478976, based upon the *Timaeus* numbers,

which assumes 2239488 as the foundation for an initial fourth, rather than a fifth. There are, in fact, four such alternative formulations, constituting coincident octaves both with one another and those above, as follows:

(a) 2239488 *2359296 2654208 2985984* · 3359232 *3538944 3981312* 4478976
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(b) 2239488 *2359296 2654208 2985984* · 3359232 **3779136** *3981312* 4478976
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
Unaccustomed: STT TTST

(c) 2239488 **2519424** *2654208 2985984* · 3359232 *3538944 3981312* 4478976
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(d) 2239488 **2519424** *2654208 2985984* · 3359232 **3779136** *3981312* 4478976
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Observe that the usual pattern of six alternatives remains broken. The interval 256/243 can no longer be made with the value marking the top of the primary fourth. Observe that all existing alternative formulations are identical to the original ones.

(21) 3359232 4478976 6718464

6718464:3359232 = 2:1 = octave ratio
4478976:3359232 = 4:3 = sesquitertian ratio
6718464:4478976 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify two full coincident octaves from the set of *Timaeus* numbers as follows:

(a) 3359232 *3538944 3981312* 4478976 · **5038848** *5308416 5971968* 6718464
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(b) 3359232 **3779136** *3981312* 4478976 · **5038848** *5308416 5971968* 6718464
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the usual pattern for the odd numbered sequences of four alternatives has been broken. The interval 256/243 can no longer be made with the value marking the top of the primary fourth to begin the second primary part of the octave.

Note two: Observe that the elements in connection with each of the two series of overlapping octaves originating from the chromatic factors of 1719926784, that are chromatic with respect to any given octave in the series, are the elements of the octaves overlapping it that are not also properly included in the octave in question, together with the elements of the original octave periodicity that are not properly included in the octave in question.

Note three: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 3359232 as the beginning of a fifth, rather than a fourth. There are, indeed, two such alternative formulations corresponding to two coincident octaves as follows:

(a) 3359232 *3538944 3981312* 4478976 **5038848** · *5308416 5971968* 6718464
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 3359232 **3779136** *3981312* 4478976 **5038848** · *5308416 5971968* 6718464
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note four: Observe that the two alternative formulations are now identical to the original sequences above.

(22) 4478976 6718464 8957952

897952:4478976 = 2:1 = octave ratio
6718464:4478976 = 3:2 = sesquialter ratio
8957952:6718464 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the following two full coincident octaves from the set of *Timaeus* numbers as follows:

(a) 4478976 **5038848** *5308416 5971968* 6718464 · *7077888 7962624* 8957952
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

(b) 4478976 5038848 *5308416 5971968* 6718464 · 7558272 *7962624* 8957952
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: Observe a further breaking down of the pattern displayed by the even numbered sequences in that no formulation depending upon the articulation of the *leimma* at the bottom of the first primary part of the octave is now possible.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 4478976 and 8957952, based on the *Timaeus* numbers, which assumes 4478976 as the foundation for an initial fourth, rather than a fifth. There are, in fact, two distinct alternative formulations, constituting coincident octaves both with one another and those above:

(a) 4478976 5038848 *5308416 5971968* · 6718464 *7077888 7962624* 8957952
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

(b) 4478976 5038848 *5308416 5971968* · 6718464 7558272 *7962624* 8957952
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Observe that the alternative formulations are identical to the original ones.

(23) 6718464 8957952 13436928

13436928:6718464 =2:1 = octave ratio
8957952:6718464 = 4:3 = sesquitertian ratio
13436928:8957952 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify elements of the octave from the set of *Timaeus* numbers as follows:

(a) 6718464 *7077888 7962624* 8957952 · 10077696 *10616832 11943936* 13436928
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(b) 6718464 7558272 *7962624* 8957952 · 10077696 *10616832 11943936* 13436928
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the usual pattern for the odd numbered sequences of four alternatives remains broken. The interval 256/243 can no longer be made with the value marking the top of the primary fourth to begin the second primary part of the octave.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 6718464 as the beginning of a fifth rather than a fourth. There are, indeed, two alternative formulations constituting coincident octaves both with each other and those above:

(a) 6718464 *7077888 7962624* 8957952 **10077696** · *10616832 11943936* 13436928
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

(b) 6718464 **7558272** *7962624* 8957952 **10077696** · *10616832 11943936* 13436928
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the alternatives are identical to the original formulations.

(24) 8957952 13436928 17915904

17915904:8957952 = 2:1 = octave ratio
13436928:8957952 = 3:2 = sesquialter ratio
17915904:13436928 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can a full octave from the set of *Timaeus* numbers as follows:

(a) 8957952 **10077696** *10616832 11943936* 13436928 · **15116544** *15925248* 17915904
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not there is an alternative formulation of the octave interval between 8957952 and 17915904, based on the *Timaeus* numbers, which assumes 8957952 as the foundation for an initial fourth, rather than a fifth. There is indeed one alternative formulation as follows:

(a) 8957952 **10077696** *10616832 11943936* · 13436928 **15116544** *15925248* 17915904
[9/8 × 243/256 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

Note three: Observe that the alternative is identical to the original.

(25) 13436928 17915904 26873856

26873856:17915904 = 2:1 = octave ratio
17915904:13436928 = 4:3 = sesquitertian ratio
26873856:17915904 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify one full octave from the set of *Timaeus* numbers as follows:

(a) 13436928 **15116544** *15925248* 17915904 · **20155392** *21233664 23887872* 26873856
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Hypophrygian/a.Hypodorian

Note one: Observe that the usual pattern for the odd numbered sequences of four alternatives has further broken down. The interval 256/243 can no longer be made at the top of either primary part of the octave.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 13436928 as the beginning of a fifth rather than a fourth. There is, indeed, an alternative formulation, as follows:

(a) 13436928 **15116544** *15925248* 17915904 **20155392** · *21233664 23887872* 26873856
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Hypophrygian/a.Hypodorian

Note three: Observe that the usual pattern for the odd numbered sequences of four alternatives has further broken down. The interval 256/243 can no longer be made at the top of the first primary part of the octave.

(26) 17915904 26873856 35831808

35831808:17915904 = 2:1 = octave ratio
26873856:17915904 = 3:2 = sesquialter ratio
35831808:26873856 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify elements of the octave from the set of *Timaeus* numbers as follows:

(a) 17915904 **20155392** *21233664 23887872* 26873856 · **30233088** *31850496* 35831808
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
d.Phrygian/a.Phrygian

Note: One might wonder whether or not there is an alternative formulation of the octave interval between 17915904 and 35831808, based upon the *Timaeus* numbers, which assumes 17915904 as the foundation for an initial fourth, rather than a fifth. There is an alternative, containing elements of a coincident octave as follows:

(a) 17915904 **20155392** *21233664 23887872* · 26873856 **30233088** *31850496* 35831808
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
d.Phrygian/a.Phrygian

It is identical to the original formulation.

(27) 26873856 35831808 53747712

53747712:26873856 = 2:1 = octave ratio
35831808:26873856 = 4:3 = sesquitertian ratio
53747712:35831808 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, the most complete octave that one can identify is as follows:

(a) 26873856 **30233088** *31850496* 35831808 · **40310764** *47775744* 53747712
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × (256/243 × 9/8) × 9/8 = 3/2] = 2
(ST combined)
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × (9/8 × 256/243) × 9/8 = 3/2] = 2
(TS combined)

Note one: Clearly, octave periodicity has broken down in the second series of octaves marked by chromatic factors of 1719926784. As the reader can observe, the most complete, incomplete octave is ambiguous as between two formulas.

Note two: For the first time at the twenty-seventh octave, it is impossible to identify a full octave following any permissible pattern. The two octave series marked by the chromatic factors of 1719926784, thus reflect the overarching bound of the *Timaeus*

extension in their own manner. Note that they mark it differently than the two octave series defined by the chromatic nonfactors of 1719926784 that clearly emphasize the importance of the Decad, another overarching limiting concept of the *Timaeus*.

Note three: One might wonder whether or not one could produce an alternative formulation of the given, partial octave by treating 26873856 as the beginning of a fifth rather than a fourth. The most complete alternative formulation is as follows:

26873856 **30233088** *31850496* 35831808 **40310784** · *47775744* 53747712
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] · x · [(256/243 × 9/8) × 9/8 = 4/3] = 2
(ST combined)
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [(9/8 × 256/243) × 9/8 = 4/3] = 2
(TS combined)

Note: As the reader can observe, the incomplete alternative is ambiguous as between two possibilities.

(28) 35831808 53747712 71663616

71663616:35831808 = 2:1 = octave ratio
53747712:35831808 = 3:2 = sesquialter ratio
71663616:53747712 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, the most complete octave that one can identify from the set of *Timaeus* numbers is as follows:

35831808 **40310784** *47775744* 53747712 · **60466176** *63700992* 71663616
[9/8 × (256/243 × 9/8) × 9/8 = 3/2] × [9/8 × 256/243 × 9/8 = 4/3] = 2
(ST combined)

Note: The above sequence has an undivided TS sequence. One might wonder whether there is an alternative formulation of the octave interval between 35831808 and 71663616 based on the *Timaeus* numbers, which assumes 35831808 as the foundation for an initial fourth, rather than a fifth. There is not a perfect alternative to the incomplete octave indicated, but, rather, an incomplete sequence containing elements of a coincident octave as follows:

35831808 **40310784** *47775744* · 53747712 **60466176** *63700992* 71663616
[9/8 × (256/243 × 9/8) = 4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
(combined ST)

[9/8 × (9/8 × 256/243) =4/3] × [9/8 × 9/8 × 256/243 × 9/8 = 3/2] = 2
(combined TS)

Note: The alternative formulation is ambiguous as between the two octave formulas. Clearly, octave periodicity has broken down in the first series of octaves marked by chromatic factors of 1719926784.

(29) 53747712 71663616 107495424

107495424:53747712 = 2:1 = octave ratio
71663616:53747712 = 4:3 = sesquitertian ratio
107495424:71663616 = 3:2 = sesquialter ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, the most complete octave one can formulate is as follows:

53747712 **60466176** *63700992* 71663616 · **80621568** *95551488* 107495424
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × (256/243 × 9/8) × 9/8 = 3/2] = 2
(ST combined)
[9/8 × 256/243 × 9/8 = 4/3] × [9/8 × (9/8 × 256/243) × 9/8 =3/2] = 2
(TS combined)

Note one: The partial octave is ambiguous as between two different octave formulas.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 53747712 as the beginning of a fifth rather than a fourth. There is, indeed, a formulation alternative to the above incomplete octave reflecting an incomplete octave, ambiguous between two possibilities, and containing the following elements:

53747712 **60466176** *63700992* 71663616 **80621568** · *95551488* 107495424
[9/8 × 256/243 × 9/8 × 9/8 = 3/2] × [(256/243 × 9/8) × 9/8 = 4/3] = 2
(ST combined)
[9/8 xx 256/243 × 9/8 × 9/8 = 3/2] × [(9/8 × 256/243) × 9/8 = 4/3] = 2
(TS combined)

(30) 71663616 107495424 143327232

143327232:71663616 = 2:1 = octave ratio
107495424:71663616 = 3:2 = sesquialter ratio

143327232:107495424 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, the most complete octave one can formulate is as follows:

71663616 80**621568** *95551488* 107495424 · **120932352** 143327232
[9/8 × (256/243 × 9/8) × 9/8 = 3/2] × [9/8 × (256/243 × 9/8) = 4/3] = 2
(ST combined) (ST combined)
[9/8 × (9/8 × 256/243) × 9/8 = 3/2] × [9/8 × (9/8 × 256/243) = 4/3] = 2
(TS combined) (TS combined)
[9/8 × (256/243 × 9/8) × 9/8 = 3/2] × [9/8 × (9/8 × 256/243) = 4/3] = 2
(ST combined) (TS combined)
[9/8 × (9/8 × 256/243) × 9/8 = 3/2] × [9/8 × (256/243 × 9/8) = 4/3] = 2
(TS combined) (ST combined)

The partial octave is ambiguous among four different octave formulas.

Note one: One might wonder whether there is an alternative formulation of the octave interval between 71663616 and 143327232 based on the *Timaeus* numbers, which assumes 71663616 as the foundation for an initial fourth, rather than a fifth. The most complete alternative one can formulate is the following:

71663616 80**621568** *95551488* · 107495424 **120932352** 143327232
[9/8 × (256/243 × 9/8) = 4/3] × [9/8 × 9/8 × (256/243 × 9/8) = 3/2] = 2
(ST combined) (ST combined)
[9/8 × (9/8 × 256/243) = 4/3] × [9/8 × 9/8 × (256/243 × 9/8) = 3/2] = 2
(TS combined) (ST combined)
[9/8 × (256/243 × 9/8) = 4/3] × [9/8 × 9/8 × (9/8 × 256/243) = 3/2] = 2
(ST combined) (TS combined)
[9/8 × (9/8 × 256/243) = 4/3] × [9/8 × 9/8 × (9/8 × 256/243) = 3/2] = 2
(TS combined) (TS combined)

Note two: The partial alternative formulation is ambiguous as among four different octave formulas.

(31) 143327232 *191102976* [286654464]

[286654464]:143327232 = 2:1 = octave ratio
191102976:143327232 = 4:3 = sesquitertian ratio
[286654464]: 191102976 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, the most complete octave one can identify from the set of *Timaeus* numbers is as follows:

143327232 *191102976* · 214990848 [286654464]
(a) **[(256/243 × 9/8 × 9/8) = 4/3]** × [9/8 × **(256/243 × 9/8 × 9/8)** = 3/2] = 2
(b) **[(9/8 × 256/243 × 9/8) = 4/3]** × [9/8 × **(256/243 × 9/8 × 9/8)** = 3/2] = 2
(c) **[(256/243 × 9/8 × 9/8) = 4/3]** × [9/8 × **(9/8 × 256/243 × 9/8)** = 3/2] = 2
(d) **[(256/243 × 9/8 × 9/8) = 4/3]** × [9/8 × **(9/8 × 9/8 × 256/243)** = 3/2] = 2
(e) **[(9/8 × 256/243 × 9/8) = 4/3]** × [9/8 × **(9/8 × 256/243 × 9/8)** = 3/2] = 2
(f) **[(9/8 × 256/243 × 9/8) = 4/3]** × [9/8 × **(9/8 × 9/8 × 256/243)** = 3/2] = 2
(g) **[(9/8 × 9/8 × 256/243) = 4/3]** × [9/8 × **(256/243 × 9/8 × 9/8)** = 3/2] = 2
(h) **[(9/8 × 9/8 × 256/243) = 4/3]** × [9/8 × **(256/243 × 9/8 × 9/8)** = 3/2] = 2
(i) **[(9/8 × 9/8 × 256/243) = 4/3]** × [9/8 × **(9/8 × 256/243 × 9/8)** = 3/2] = 2
(j) **[(9/8 × 9/8 × 256/243) = 4/3]** × [9/8 × **(9/8 × 9/8 × 256/243)** = 3/2] = 2
(k) **[(9/8 × 9/8 × 256/243) = 4/3]** × [9/8 × **(9/8 × 256/243 × 9/8)** = 3/2] = 2
(l) **[(9/8 × 9/8 × 256/243) = 4/3]** × [9/8 × **(9/8 × 9/8 × 256/243)** = 3/2] = 2
All sequences emboldened and in red indicate combined intervals.

The partial octave is ambiguous among twelve different octave formulas.

Note one: The thirty-first sequence is the first sequence that loses a chromatic factor of 1719926784 as the marker of the endpoint the octave. Another sort of *Timaeus* number that cannot be said to be chromatic with respect to the original scale, instead, enters into the octave periodicity, with the originating chromatic factor, because the original scale has completely degenerated with the advent of the bracketed number. Because the original scale has disappeared by the time the bracketed number arises, it cannot be said to be chromatic with respect to the original scale. It remains part of a scale in which one of the elements that is chromatic with respect to the original scale and one of the elements that is part of the original periodicity are also elements; but the former scale radically departs from the original scale. After the thirty-first sequence, there is no trace of the original periodicity.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating 143327232 as the beginning of a fifth rather than a fourth. There is, indeed, a formulation alternative to the above incomplete octave reflecting another incomplete octave and containing the following elements:

143327232 *191102976 214990848* · [286654464]
(a) **[(256/243 × 9/8 × 9/8)** × 9/8 = 3/2] × **[(256/243 × 9/8 × 9/8) = 4/3]** = 2
(b) **[(9/8 × 256/243 × 9/8)** × 9/8 = 3/2] × **[(256/243 × 9/8 × 9/8) = 4/3]** = 2
(c) **[(9/8 × 9/8 × 256/243)** × 9/8 = 3/2] × **[(256/243 × 9/8 × 9/8) = 4/3]** = 2
(d) **[(256/243 × 9/8 × 9/8)** × 9/8 = 3/2] × **[(9/8 × 256/243 × 9/8) = 4/3]** = 2

(e) [(9/8 × 256/243 × 9/8) × 9/8 = 3/2] × [(9/8 × 256/243 × 9/8) = 4/3] = 2
(f) [(9/8 × 9/8 × 256/243) × 9/8 = 3/2] × [(9/8 × 256/243 × 9/8) = 4/3] = 2
(g) [(256/243 × 9/8 × 9/8) × 9/8 = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2
(h) [(9/8 × 256/243 × 9/8) × 9/8 = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2
(j) [(9/8 × 9/8 × 256/243) × 9/8 = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2

The partial octave is ambiguous as among nine different octave formulations.

(32) [286654464] [429981696] [573308928]

[573308928]:[286654464] = 2:1 = octave periodicity
[429981696]:[286654464] = 3:2 = sesquialter ratio
[573308928]:429981696] = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify elements of the octave from the set of *Timaeus* numbers as follows:

[286654464] [429981696] · [573308928]
(a) [(256/243 × 9/8 × 9/8 × 9/8) = 3/2] × [(256/243 × 9/8 × 9/8) = 4/3] = 2
(b) [(9/8 × 256/243 × 9/8 × 9/8) = 3/2] × [(256/243 × 9/8 × 9/8) = 4/3] = 2
(c) [(9/8 × 9/8 × 256/243 × 9/8) = 3/2] × [(256/243 × 9/8 × 9/8) = 4/3] = 2
(d) [(9/8 × 9/8 × 9/8 × 256/243) = 3/2] × [(256/243 × 9/8 × 9/8) = 4/3] = 2
(e) [(256/243 × 9/8 × 9/8 × 9/8) = 3/2] × [(9/8 × 256/243 × 9/8) = 4/3] = 2
(f) [(9/8 × 256/243 × 9/8 × 9/8) = 3/2] × [(9/8 × 256/243 × 9/8) = 4/3] = 2
(g) [(9/8 × 9/8 × 256/243 × 9/8) = 3/2] × [(9/8 × 256/243 × 9/8) = 4/3] = 2
(h) [(9/8 × 9/8 × 9/8 × 256/243) = 3/2] × [(9/8 × 256/243 × 9/8) = 4/3] = 2
(i) [(256/243 × 9/8 × 9/8 × 9/8) = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2
(j) [(9/8 × 256/243 × 9/8 × 9/8) = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2
(k) [(9/8 × 9/8 × 256/243 × 9/8) = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2
(l) [(9/8 × 9/8 × 9/8 × 256/243) = 3/2] × [(9/8 × 9/8 × 256/243) = 4/3] = 2
Sequences emboldened and in red are combined sequences.

Note one: The partial octave is the first to contain no original model scale numbers and no chromatic factors of 1719926784. It is ambiguous among twelve different alternative octave formulas.

Note two: One might wonder whether there is an alternative formulation of the octave interval between [286654464] and [573308928] based on the *Timaeus* numbers, which assumes [286654464] as the foundation for an initial fourth, rather than a fifth. There is no such alternative marked by *Timaeus* numbers:

(33) [5733o8928]

There is no element among the *Timaeus* numbers that forms an octave ratio with [5733o8928] that is beyond that number. Instead, [573308928] forms a fifth with [859963392] and the latter number comprises an octave with [1719926784], the limit of the *Timaeus* expansion. Otherwise stated, [573308928] forms a triple ratio with [1719926784]. When one observes that a triple is composed of an octave and a fifth, one recalls that the octave periodicity of the original sequence eventually broke down to fifth periodicity and gradually degenerated to nothing. It is not surprising therefore, that the two series of overlapping and coincident octaves dependent for their definition on chromatic factors of 1719926784 break down to the fifth as well.

APPENDIX 6

Two Overlapping Sequences of Doubles, Including Coincident Diatonic Octaves within Each, Bounded Entirely by Chromatic Nonfactors of 1719926784

1 Key

(a) Numbers in bold indicate chromatic numbers not constituting factors of 1719926784; they comprise the endpoints defining the continuously overlapping doubles in this appendix.
(b) Numbers in plain type belong to the subset of chromatic factors of 1719926784.
(c) Italicized numbers also comprise scalar elements within the original octave periodicity, i.e., the original model scale numbers and their multiples, prior to the complete degeneration of the primary *Timaeus* scale.
(d) Plain numbers in brackets are both chromatic factors of 1719926784 and nonscalar elements in relation to the original octave periodicity, i.e., that of the primary d.Dorian/a.Lydian scale of the *Timaeus*, as they arise after that periodicity has completely degenerated. See Table 26, "Fully Annotated Table of Timaeus Numbers Arranged in Numerical Order from Least to Greatest with Annotations."

2 Preliminary Notes

Preliminary note one: The set of octaves articulated in this Appendix 6 reflects the mixed character of the world soul on the level of the numbers composing it. As the reader will recall, the world soul is a mixture of same, different, and the mixture of same and different. With respect to the kinds of numbers composing the octaves below, "same" corresponds to the model scale numbers of the original octave periodicity and their multiples, all of which evenly divide 1719926784. "Different" corresponds to the chromatic nonfactors of 1719926784. Mixed corresponds to the chromatic factors of 1719926784. The chromatic factors correspond to a "mixture of same and different" because, while they are not model scale numbers or their multiples, they share the feature, in common with the model scale numbers and their multiples, of being even divisors of 1719926784. The chromatic nonfactors are neither model scale numbers nor even divisors of 1719926784.

Preliminary note two: Appendix 6 evidences two chains of contiguous octaves. The two chains continuously overlap each other. The first chain is comprised by the octaves

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labeled (1), (2), (4), (6), (8), (10), (12), (14), (16), (18), (20), and (22). The second chain consists in the octaves labeled (3), (5), (7), (9), (11), (13), (15), (17), (19), (21), (23), and (24). The second chain actually arises from the first. The starting tone of the second chain, 59049, arises as the fifth above 39366, the starting tone of the second octave of the first chain. The two chains are, then, separated by the sesquialter interval.

3 Octaves Originating from Chromatic Nonfactors of 1719926784

(1) **19683** 26244 **39366**

36366:19683 = 2:1 = octave ratio
26244:**19683** =4:3 = sesquitertian ratio
39366: 26244= 3:2 = sesquialter ratio
4/3 × 3/2 = 2

As one might suspect, it is possible to fill in the sesquitertian and sesquialter parts of this octave interfering with both the original octave periodicity and the other octave chain of this appendix, among others, entirely from numbers within the *Timaeus* set. The intermediate numbers within the primary parts of the interfering octave belong to the original scale, as well, although the end markers both of the interfering octave and its primary constituent parts are, respectively, chromatic nonfactors and chromatic factors of 1719926784. Recalling that a sesquitertian part can be filled in the pattern of (9/8 × 9/8 × 256/243) or (9/8 × 256/243 × 9/8) or (9/8 × 9/8 × 256/243), one can identify the full octave as follows:

19683 *20736 23328* 26244 · *27648 31104* 34992 **39366**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not a person could produce an alternative formulation of the given octave by treating **19683** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

19683 *20736 23328 24576 27648 31104* 34992 **39366**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(2) **39366 59049 78732**

78732:39366 = 2:1 = octave ratio
59049:39366 = 3:2 = sesquialter ratio
78732:59049 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Note one: The new octave periodicities indicated by the first two chromatically grounded diatonic octaves above are contiguous not overlapping. They appear, therefore, to be part of the same order of octave periodicity, but they really are not: the orders of fifth and fourth are reversed for each with respect to the other. This difference in the order becomes a basis for overlap and interference among the octaves based on the chromatic nonfactors of 1719926784 as those octaves progress. Two distinct octave series within the latter set of octaves actually originate from the two different arrangements. The octave periodicities that they, respectively, mark clearly interfere with the original order of octave periodicity of the *Timaeus*, as well as with each other, among others.

Note two: Observe that all of the octaves in the appendix originating from chromatic nonfactors of 1719926784, from the second octave going forward, have articulations containing both endpoints and intermediate primary nodes (nodes marking the internal bound of a constituent fourth or fifth) entirely within the subset of chromatic nonfactors of 1719926784, with the exception of the very last octave and selected coincident octaves first discussed below in Note Four. The coincident octaves follow the example of the first octave in having a chromatic factor of 1719926784 mark the internal bound of the primary constituent parts.

Recalling that a sesquitertian part can be filled in the pattern of (9/8 × 9/8 × 256/243) or (9/8 × 256/243 × 9/8) or (9/8 × 9/8 × 256/243) and that a sesquialter part can be filled in the pattern of (4/3 × 9/8) or (9/8 × 4/3) or (9/8 × 9/8 × 256/243 × 9/8 × 9/8) or (9/8 × 256/243 × 9/8 × 9/8) or (256/243 × 9/8 × 9/8 × 9/8) or (9/8 × 9/8 × 9/8 × 256/243) or (9/8 × 9/8 × 256/243 × 9/8) one can identify the full octave from the set of *Timaeus* numbers as follows:

39366 *41472 46656* 52488 **59049** · *62208* 69984 **78732**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 =4/3] = 2
d.Lydian/a.Dorian

Note three: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note four: There are two alternative formulations of the octave interval between **39366** and **78732**, based on the *Timaeus* numbers, which assume **39366** as the foundation for an initial fourth, rather than a fifth. These alternatives constitute coincident octaves vis-à-vis both each other and the octave articulated above. Because they stand opposite to the octave articulated above in the order of their internal arrangement, they constitute a new source of interference within the set of octaves based on the chromatic nonfactors of 1719926784.

39366 52488 **78732**

(a) **39366** *41472 46656* 52488 · *55296 62208* 69984 **78732**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **39366** *41472 46656* 52488 · **59049** *62208* 69984 **78732**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8]
d.Lydian/a.Dorian

Note five: Observe that the alternatively formulated octave in (b) is actually identical in sequence to the original octave, even though the order of the primary constituent parts is reversed. The fifth in the "coincident" version might, then, actually best be regarded either as a disjunct sesquioctave interval plus a fourth having the *leimma* at the beginning of the chord or a fourth having the *leimma* in the second position plus a disjunct sesquioctave interval at the end of the chord.

Observe, too, that the identity of one of the alternatively formulated octaves with the original is well suited to expressing the idea that what started as single thing results in a pair. The relationship between the first articulation of the octave and the formulation of the pair is actually then analogous to the movement of the monad to the duad. The same relationship holds for all other octave sequences in this appendix producing a pair of alternative articulations based upon a reversal of the order of the primary parts of the octave.

Note six: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

39366 *41472 46656 49152 55296 62208* 69984 **78732**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(3) **59049 78732 118098**

118098:59049 = 2:1 = octave ratio
78732:59049 = 4:3 = sesquitertian ratio
118098:78732 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Note one: The third chromatically grounded diatonic octave marked by nonfactors of 1719926784 overlaps not only with the original octave periodicity of the *Timaeus*, but also, among others, with the octave periodicities indicated by the chromatically grounded diatonic octave no. 2 of this appendix (with its coincidents).

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

59049 *62208* 69984 **78732** · *82944 93312* 104976 **118098**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note two: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note three: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **59049** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note four: There are two possible unaccustomed arrangements of the octave not allowing any standard division into fourths and fifths. The first is STSTTTT, as follows:

(a) **59049** *62208* 69984 *73728 82944 93312* 104976 **118098**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

The second is SSTTTTT, articulated below.

(b) **59049** *62208 65536 73728 82944 93312* 104976 **118098**
[256/243 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 9/8]

(4) **78732 118098 157464**

157464:78732 = 2:1 = octave ratio
118098:78732 = 3:2 = sesquialter ratio
157464:118098 = 4:3 = sesquitertian ratio
3:2 × 4:3 = 2

Note one: The fourth chromatically grounded diatonic octave marked by nonfactors of 1719926784 overlaps not only the original octave periodicity of the *Timaeus*, but also the octave periodicity comprised by the third chromatically grounded diatonic octave. The third chromatically grounded diatonic octave, then, interferes with two different sets of chromatically grounded diatonic octave periodicities overlapping it and with the original order of periodicity of the *Timaeus*, among others. In relation to the previous three, the fourth octave completes a distinct tetrad or unit of interfering octave periodicities. All chromatically grounded diatonic octaves marked by nonfactors of 1719926784 beginning with the third chromatically grounded diatonic octave and excluding the very last one are similarly characterizable in terms of their interference.

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

78732 *82944 93312* 104976 **118098** · *124416* 139968 **157464**
[256/243 × 9/8 × 9/8 × 9/8= 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note two: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note three: There are two alternative formulations of the octave interval between **78732** and **157464**, based on the *Timaeus* numbers, which assume **78732** as the foundation for an initial fourth, rather than a fifth. They constitute coincident octaves vis-à-vis both each other and the octave articulated above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **78732** *82944 93312* 104976 · *110592 124416* 139968 **157464**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **78732** *82944 93312* 104976 · **118098** *124416* 139968 **157464**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

Note four: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

78732 *82944 93312 98304 110592 124416* 139968 **157464**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(5) **118098 157464 236196**

236196:118098 = 2:1 = octave ratio
157464:118098 = 4:3 = sesquitertian ratio
236196:157464 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

118098 *124416* 139968 **157464** · *165888 186624* 209952 **236196**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] =2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **118098** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note three: There are two possible unaccustomed arrangements of the octave not allowing any standard division into fourths and fifths. The first is STSTTTT, as follows:

(a) **118098** *124416* 139968 *147456 165888 186624* 209952 **236196**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

The second is SSTTTTT, articulated below.

(b) **118098** *124416 131072 147456 165888 186624* 209952 **236196**
[256/243 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 9/8]

(6) **157464 236196 314928**

314928:157464 = 2:1 = octave ratio
236196:157464 = 3:2 = sesquialter ratio
314928:236196 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

157464 *165888 186624* 209952 **236196** · *248832* 279936 **314928**
[256/243 × 9/8 × 9/8 × 9/8= 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There are two alternative formulations of the octave interval between **157464** and **314928**, based on the *Timaeus* numbers, which assume **157464** as the foundation for an initial fourth, rather than a fifth. They constitute coincident octaves vis-à-vis both each other and the octave above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **157464** *165888 186624* 209952 · *221184 248832* 279936 **314928**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **157464** *165888 186624* 209952 · **236196** *248832* 279936 **314928**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT not allowing any standard division into fourths and fifths, as follows:

157464 *165888 186624 196608 221184 248832* 279936 **314928**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(7) **236196 314928 472392**

472393:236196 = 2:1 = octave ratio
314928:236196 = 4:3 = sesquitertian ratio
472392:314928 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

236196 *248832* 279936 **314928** · *331776 373248* 419904 **472392**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2]
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **236196** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note three: There are two possible unaccustomed arrangements of the octave not allowing any standard division into fourths and fifths. The first is STSTTTT, as follows:

(a) **236196** *248832* 279936 *294912 331776 373248* 419904 **472392**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

The second is SSTTTTT, articulated below.

(b) **236196** *248832 262144 294912 331776 373248* 419904 **472392**
[256/243 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 9/8]

(8) **314928 472392 629856**

629856:314928 = 2:1 = octave ratio
472392:314928 = 3:2 = sesquialter ratio
629856:472392 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

314928 *331776 373248* 419904 **472392** · *497664* 559872 **629856**
[256/243 × 9/8 × 9/8 × 9/8= 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There are two alternative formulations of the octave interval between **314928** and **629856**, based on the *Timaeus* numbers, which assume **314928** as the foundation for an initial fourth, rather than a fifth. They constitute coincident octaves vis-à-vis both each other and the octave above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **314928** *331776 373248* 419904 · *442368 497664* 559872 **629856**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **314928** *331776 373248* 419904 · 472392 *497664* 559872 **629856**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

314928 *331776 373248 393216 442368 497664* 559872 **629856**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(9) **472392 629856 944784**

944784:472392 = 2:1 = octave ratio
629856:472392 = 4:3 = sesquitertian ratio
944784:629856 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

472392 *497664* 559872 **629856** · *663552 746496* 839808 **944784**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **472392** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

472392 *497664* 559872 *589824 663552 746496* 839808 **944784**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(10) **629856 944784 1259712**

1259712:629856 = 2:1 = octave ratio
944784:629856 = 3:2 = sesquialter ratio
1259712:944784 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

629856 *663552 746496* 839808 **944784** · *995328* 1119744 **1259712**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the interfering octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There are two alternative formulations of the octave interval between **629856** and **1259712**, based on the *Timaeus* numbers, which assume **629856** as the foundation for an initial fourth, rather than a fifth. They constitute coincident octaves vis-à-vis both each other and the octave above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **629856** *663552 746496* 839808 · *884736 995328* 1119744 **1259712**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **629856** *663552 746496* 839808 · **944784** *995328* 1119744 **1259712**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

629856 *663552 746496 786432 884736 995328* 1119744 **1259712**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(11) **944784 1259712 1889568**

1889568:944784 = 2:1 = octave ratio
1259712:944784 = 4:3 = sesquitertian ratio
1889568:1259712 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

944784 *995328* 1119744 **1259712** · *1327104 1492992* 1679616 **1889568**
[256/243 × 9/8 × 9/8 =4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **944784** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

944784 *995328* 1119744 *1179648 1327104 1492992* 1679616 **1889568**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(12) **1259712 1889568 2519424**

2519424:1259712 = 2:1 = octave ratio
1889568:1259712 = 3:2 = sesquialter ratio
2519424:1889568 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

1259712 *1327104 1492992* 1679616 **1889568** · *1990656* 2239488 **2519424**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There are two alternative formulations of the octave interval between **1259712** and **2519424**, based on the *Timaeus* numbers, which assume **1259712** as the foundation for an initial fourth, rather than a fifth. They constitute coincident octaves vis-à-vis both each other and the octave above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **1259712** *1327104 1492992* 1679616 · *1769472 1990656* 2239488 **2519424**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **1259712** *1327104 1492992* 1679616 · **1889568** *1990656* 2239488 **2519424**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(13) **1889568 2519424 3779136**

3779136:1889568 = 2:1 = octave ratio
2519424:1889568 = 4:3 = sesquitertian ratio
3779136:2519424 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

1889568 *1990656* 2239488 **2519424** · *2654208 2985984* 3359232 **3779136**
[256/243 × 9/8 × 9/8 =4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the interfering octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **1889568** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

Note three: There is one possible unaccustomed arrangement of the octave, STSTTTT, not allowing any standard division into fourths and fifths, as follows:

1889568 *1990656* 2239488 *2359296 2654208 2985984* 3359232 **3779136**
[256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8]

(14) **2519424 3779136 5038848**

5038848:2519424 = 2:1 = octave ratio
3779136:2519424 = 3:2 = sesquialter ratio
5038848:3779136 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

2519424 *2654208 2985984* 3359232 **3779136** · *3981312* 4478976 **5038848**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There are two alternative formulations of the octave interval between **2519424** and **5038848**, based on the **Timaeus** numbers, which assume **2519424** as the

foundation for an initial fourth, rather than a fifth. They constitute coincident octaves both vis-à-vis each other and the octave above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **2519424** *2654208 2985984* 3359232 · *3538944 3981312* 4478976 **5038848**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] =2
d.Hypolydian/a.Mixolydian

(b) **2519424** *2654208 2985984* 3359232 · **3779136** *3981312* 4478976 **5038848**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(15) **3779136 5038848 7558272**

7558272:3779136 = 2:1 = octave ratio
5038848:3779136 = 4:3 = sesquitertian ratio
7558272:5038848 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

3779136 *3981312* 4478976 **5038848** · *5308416 5971968* 6718464 **7558272**
[256/243 × 9/8 × 9/8 =4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **3779136** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

(16) **5038848 7558272 10077696**

10077696:5038848 = 2:1 = octave ratio
7558272:5038848 = 3:2 = sesquialter ratio
10077696:7558272 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

5038848 *5308416 5971968* 6718464 **7558272** · *7962624* 8957952 **10077696**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There are two alternative formulations of the octave interval between **5038848** and **10077696**, based on the *Timaeus* numbers, which assume **5038848** as the foundation for an initial fourth, rather than a fifth. They constitute coincident octaves vis-à-vis both each other and the octave above. One, as previously noted for the first such example in this appendix, is identical to the original octave.

(a) **5038848** *5308416 5971968* 6718464 · *7077888 7962624* 8957952 **10077696**
[256/243 × 9/8 × 9/8 = 4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

(b) **5038848** *5308416 5971968* 6718464 · **7558272** *7962624* 8957952 **10077696**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(17) **7558272 10077696 15116544**

15116544:7558272 = 2:1 = octave ratio
10077696:7558272 = 4:3 = sesquitertian ratio
15116544:10077696 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

7558272 *7962624* 8957952 **10077696** · *10616832 11943936* 13436928 **15116544**
[256/243 × 9/8 × 9/8 =4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **7558272** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

(18) **10077696 15116544 20155392**

20155392:10077696 = 2:1 = octave ratio
15116544:10077696 = 3:2 = sesquialter ratio
20155392:1511544 = 4:3 = sesquitertian ratio
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

10077696 *10616832 11943936* 13436928 **15116544** · *15925248* 17915904 **20155392**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 =4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: Observe that there is now only one alternative formulation of the octave interval between **10077696** and **20155392**, based on the *Timaeus* numbers, which assumes **10077696** as the foundation for an initial fourth, rather than a fifth. It constitutes a coincident octaves vis-à-vis the octave above. It is, in fact, the identical alternative. The elimination of a second alternative, at this point in the repetition of octaves based on the chromatic nonfactors of 1719926784, indicates that the octave periodicities of that order are beginning to break down, although they are still largely intact. The movement of monad to duad is eliminated, at least on one level.

10077696 *10616832 11943936* 13436928 · **15116544** *15925248* 17915904 **20155392**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian

(19) **15116544 20155392 30233088**

30233089:15116544 = 2:1 = octave ratio
20155392:15116544 = 4:3 = sesquitertian ratio
30233089:20155392 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

15116544 *15925248* 17915904 **20155392** · *21233664 23887872* 26873856 **30233088**
[256/243 × 9/8 × 9/8 =4/3] × [256/243 × 9/8 × 9/8 × 9/8 = 3/2] = 2
d.Hypolydian/a.Mixolydian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **15116544** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

(20) **20155392 30233089 40310784**

40310784:20155392 = 2:1 = octave ratio
30233089:20155392 = 3:2 = sesquialter ratio
40310784:30233089 = 4:3 = sesquitertian ration
3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify the full octave from the set of *Timaeus* numbers as follows:

20155392 *21233664 23887872* 26873856 **30233088** · *31850496* 35831808 **40310784**
[256/243 × 9/8 × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
d.Lydian/a.Dorian

Note one: One might wonder whether or not a person could rearrange the order of *leimma* and whole tone intervals in the parts of the above octave to achieve an alternative full formulation of the octave from the *Timaeus* numbers. It is not possible. Attempts to do so result in values that are not *Timaeus* numbers.

Note two: There is, again, only one alternative formulation of the octave interval between **20155392** and **40310784**, based on the *Timaeus* numbers, which assumes **20155392** as the foundation for an initial fourth, rather than a fifth. It constitutes a coincident octave vis-à-vis the octave above, and it is, in fact, the identical alternative to that octave.

20155392 *21233664 23887872* 26873856 · **30233088** *31850496* 35831808 **40310784**
[256/243 × 9/8 × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
d.Lydian/a.Dorian Scale

Note three: As further inspection of the appendix will reveal that the coincident octave associated with the twentieth octave listed is the tenth and last fully articulated coincident octave.

(21) **30233088 40310784 60466176**

60466176:3023308 = 2:1 = octave ratio
40310784:30233088 = 4:3 = sesquitertian ratio
60466176:40310784 = 3:2 =sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify elements of the octave from the set of *Timaeus* numbers as follows:

30233088 *31850496* 35831808 **40310784** *47775744* 53747712 **60466176**
TS combined interval

The incomplete octave is somewhat ambiguous as between the following two patterns:

[256/243 × 9/8 × 9/8 =4/3] × [(256/243 × 9/8) × 9/8 × 9/8 = 3/2] = 2
TS combined interval
[256/243 × 9/8 × 9/8 = 4/3] × [(9/8 × 256/243) × 9/8 × 9/8 = 3/2] = 2
TS combined interval

Note one: The twenty-first octave of the series of overlapping doubles marked by chromatic nonfactors of 1719926784 is incomplete, indicating that the octave periodicities of the overlapping series based on that set of numbers manifestly begin to degenerate at this point. The unmistakable degeneration at the twenty-first octave reflects the pattern of degeneration of the original octave periodicity after the repetition of ten complete octaves. Mark that there are two kinds of octaves in the series charted by

this appendix. They clearly alternate in their capacity to produce coincident octaves or not, reflecting the contrast of same and different, so central to the *Timaeus*. There are exactly ten fully articulated examples of each type of octave prior to the twenty-first. After a complete decad of each, both subsets break down. The breakdown of octave periodicity after ten complete articulations in each of the two subsets mirrors the breakdown of the original periodicity after ten complete articulations. Such phenomena reinforce the centrality of the Decad as the pattern that the demiurge uses to construct the universe in the *Timaeus*.

Note two: Observe, for further purposes of comparison with phenomena accompanying the repetition and degeneration of the original periodicity, that for any given octave periodicity of whatever kind articulated in this appendix, foreign elements arising with the repetition of the subject periodicity, arise as elements belonging to overlapping octaves or to the original primary octave periodicity of the *Timaeus* that are not also properly included in the octave periodicity in question.

Note three: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **30233088** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

(22) **40310784 60466176 80621568**

> **80621568:40310784** = 2:1 = octave ratio
> **60466176:40310784** = 3:2 = sesquialter ratio
> **80621568:60466176** = 4:3 = sesquitertian ratio
> 3/2 × 4/3 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify elements of the octave from the set of *Timaeus* numbers as follows:

> **40310784** *47775744* 53747712 **60466176** *63700992* 71663616 **80621568**
> TS combined sequence

The partial octave is somewhat ambiguous as between the following two patterns:

> [(256/243 × 9/8) × 9/8 × 9/8 = 3/2] × [256/243 × 9/8 × 9/8 = 4/3] = 2
> TS combined sequence
> [(9/8 × 256/243) × 9/8 × 9/8 = 3/2] × [256/243 × 9/3 × 9/8 = 4/3] = 2
> TS combined sequence

Note: There is an alternative formulation of elements of the octave interval between **40310784** and **80621568**, based on the *Timaeus* numbers, which assumes 5374712 as the foundation for an initial fourth, rather than a fifth; but this time it is not complete. The incomplete formulation indicates a coincident octave but not one fully belonging to the *Timaeus* set. Note once more that the partial octave is identical to the original, although the order of primary constituent parts has been reversed.

40310784 *47775744* 53747712 **60466176** *63700992* 71663616 **80621568**
TS combined sequence

The partial octave is ambiguous as between the following two patterns:

[(256/243 × 9/8) × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2 or
TS combined sequence
[(9/8 × 256/243) × 9/8 = 4/3] × [9/8 × 256/243 × 9/8 × 9/8 = 3/2] = 2
TS combined sequence

(23) **60466176 80621568 120932352**

120932352:60466176 = 2:1 = octave ratio
80621568:60466176 = 4:3 = sesquitertian ratio
120932352:80621568 = 3:2 = sesquialter ratio
4/3 × 3/2 = 2

Noting the various ways of filling sesquitertian and sesquialter parts, one can identify a partial octave from the set of *Timaeus* numbers as follows:

60466176 *63700992* 71663616 **80621568** *95551488* 107495424 **120932352**
TS combined sequence

The incomplete octave is ambiguous as between the following two patterns.

[256/243 × 9/8 × 9/8 =4/3] × [(256/243 × 9/8) × 9/8 × 9/8 = 3/2] = 2 or
TS combined sequence
[256/243 × 9/8 × 9/8 = 4/3] × [(9/8 × 256/243) × 9/8 × 9/8 = 3/2] = 2
TS combined sequence

Note: One might wonder whether or not one could produce an alternative formulation of the given octave by treating **60466176** as the beginning of a fifth rather than a fourth. An alternative formulation is not possible because the number that would stand at the end is not a *Timaeus* number.

(24) **120932352**

Note one: The endpoints of the fourth and fifth that would normally be the primary constituent part of the octave are missing. Noting the various ways of filling sesquitertian and sesquialter parts, one can identify elements of a partial octave from the set of *Timaeus* numbers as follows. Note the strangeness of the formulation. Because of the missing intervals, a tone/semitone unit precedes a diatessaron that is followed by a lone tone. The diatessaron, in effect, splits a theoretical fifth. No Greek octave was structured in such a way.

TS combined sequence
120932352 143327232 *191102976* [214990848] _____
{x 9/8} Missing endpoint
TS combined DT combined lone tone and **missing end**
[256/243 × 9/8 × [9/8 × 9/8 × 256/243 = 4/3] × 9/8 × **9/8**] = 2 or
[256/243 × 9/8 × [256/243 × 9/8 × 9/8 = 4/3] × 9/8 × **9/8**] = 2 or
[256/243 × 9/8 × [9/8 × 256/243 × 9/8 = 4/3] × 9/8 × **9/8**] = 2 or
[9/8 × 256/243 × [9/8 × 9/8 × 256/243 = 4/3] × 9/8 × **9/8**] = 2 or
[9/8 × 256/243 × [256/243 × 9/8 × 9/8 = 4/3] × 9/8 × **9/8**] = 2 or
[9/8 × 256/243 × [9/8 × 256/243 × 9/8 = 4/3] × 9/8 × **9/8**] = 2 or

Appendix 6 demonstrates the richness of the overlapping doubles and coincident octaves based upon the chromatic nonfactors of 1719926784. If one similarly analyses the chromatic numbers that are factors of 1719926784, one finds that they define the entirely distinct set of overlapping doubles and coincident octaves set forth in Appendix 5.

APPENDIX 7

Continuously Overlapping and Contiguous Chains of Doubles, Including Coincident Diatonic Octaves within Each, Bounded Entirely by Model Scale Numbers and Their Multiples

1 Key

(a) Numbers in bold indicate chromatic numbers not constituting factors of [1719926784].
(b) Numbers in plain type, belong to the subset of chromatic factors of [1719926784].
(c) Italicized numbers comprise scalar elements within the original octave periodicity based on the model octave. These numbers are the starting and ending tones for the octaves in the chains belonging to this appendix.
(d) Underlined numbers are numbers preceding the rise of the original model scale.
(e) Plain numbers in brackets are both chromatic factors of 1719926784 and non-scalar elements in relation to the original octave periodicity, i.e., that of the primary d.Dorian/a.Lydian scale of the *Timaeus*, as they arise after that periodicity has completely degenerated. See Table 26, "Fully Annotated Table of Timaeus Numbers Arranged in Numerical Order from Least to Greatest with Annotations."
(f) The table begins with the first completely articulable double, *384:768*. Thirty incomplete doubles precede the model octave. They are the following: 1:2, 2:4, 3:6; 4:8; 6:12, 8:16; 9:18, 12:24, 16:32, 18:36, 24:48, 27:54, 32:64, 36:72, 48:96, 54:108, 64:128, 72:144, 81:162, 96:192, 108:216, 128:256, 144:288, 162:324, 192:*384*, 216:*432*, 243:*486*, 256:*512*, 288:*576*, 324:*648*

2 Preliminary Notes

Preliminary note one: This appendix sets forth all doubles in the *Timaeus* number set based upon original model scale numbers and their multiples. Sample analyses of several of the doubles appear below to illustrate particular points.

Preliminary note two: The symbol Δ marks the octaves of the chain beginning with the model octave scale, followed so closely in Table 26. In general, six other kinds of octave strings, taking their starting points from successive steps in the model octave scale, begin or continue between repetitions of the model octave. They are labeled α–ζ below.

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Preliminary note three: The seven bands of octaves are unequal to one another in length, taking into account both their accustomed patterns and all variations possible for them beyond their original patterns, due to the invasion of chromatic elements. Octaves counted include all completely articulable octaves, i.e., those articulable only in the primary pattern for any of the seven chains; the ambiguous ones articulable both in the primary pattern and one or more additional patterns made possible by the creep of chromatic elements into the chain; and, also, those, both ambiguous and not, that are articulable in some octave pattern enabled by chromatic invasion, after the primary sequence for the chain has disappeared. For example, the model octave string (Δ-string) is eleven octaves long, including such variations. There are eleven octaves, rather than ten, because a descending Mixolydian or ascending Hypolydian diatonic scale occurs after the tenth octave containing the last descending Dorian or ascending Lydian pattern. The α-string is fourteen octaves long. The β-string is sixteen octaves long. The γ-string is nine octaves long. The δ-string is twelve octaves long. The ε-string is fifteen octaves long. The ζ-string is fifteen octaves long.

Preliminary note four: Observe that the seven octave strings, in their primary articulations, set forth, below, in this appendix, respectively, as octaves numbered 1–7, together exemplify all seven of the ancient arrangements of the diatonic octave, i.e., Mixolydian, Lydian, Phrygian, Dorian, Hypolydian, Hypophrygian, and Hypodorian, in both ascending and descending varieties. The following strings exhibit perfect disdiapasons, i.e., two-octave sequences undisturbed by chromatic elements: Δ-string, α-string, β-string, and γ-string.

3 Octave Analysis for Strings Originating from Sequential Steps of the Model Octave Scale

Δ(1) *384:768*
There is one possible tone sequence for the octave. It is the following:

384 432 486 512 576 648 729 768
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Lydian (impacts on air) or descending Dorian (string lengths) scale.

α(2) *432:864*
There is one possible tone sequence for the octave. It is the following:

432 486 512 576 648 729 768 864
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTST. Accordingly, the scale reflected by the octave articulated is either an ascending Phrygian (impacts on air) or a descending Phrygian (string lengths) scale. The Phrygian scale is its own perfect reciprocal.

β(3) *486:972*
There is one possible tone sequence for the octave. It is the following:

486 512 576 648 729 768 864 972
[256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is STTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Dorian (impacts on air) or descending Lydian (string lengths) scale. Observe that this string moves in the exact opposite direction of Δ-string.

γ(4) *512:1024*
There is one possible tone sequence for the octave. It is the following:

512 576 648 729 768 864 972 1024
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTTS Accordingly, the scale reflected by the octave articulated is either an ascending Hypolydian (impacts on air) or descending Mixolydian (string lengths) scale.

δ(5) *576:1152*
There is one possible tone sequence for the octave. It is the following:

576 648 729 768 864 972 1024 1152
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is

TTSTTST. Accordingly, the scale reflected by the octave articulated is either an ascending Hypophrygian (impacts on air) or descending Hypodorian (string lengths) scale.

ε(6) *648:1296*
There is one possible tone sequence for the octave. It is the following:

648 729 768 864 972 1024 1152 1296
[9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Hypodorian (impacts on air) or descending Hypophrygian (string lengths) scale.

ζ(7) *729:1458*
There is one possible tone sequence for the octave. It is the following:

729 768 864 972 1024 1152 1296 1458
[256/243 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is STTSTTT. Accordingly, the scale reflected by the octave articulated is either an ascending Mixolydian (impacts on air) or descending Hypolydian (string lengths) scale.

Δ(8) *768:1536*
There is one possible tone sequence for the octave. It is the following, closing out the perfect model disdiapason:

768 864 972 1024 1152 1296 1458 1536
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

Accordingly, the same analysis applies as in the case of octave (1) above.

α(9) *864:1728*
There is one possible tone sequence for the octave. It is the following, closing out a perfect α-string disdiapason:

864 972 1024 1152 1296 1458 1536 1728

Accordingly, the same analysis applies as in the case of octave (2) above.

β(10) *972:1944*
The only possible tone sequence for the octave indicated is the following:

972 1024 1152 1296 1458 1536 1728 1944
[256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8]

The same analysis applies as applies in the case of octave no. (3).

γ(11) *1024:2048*
The only possible tone sequence for the octave indicated is the following:

1024 1152 1296 1458 1536 1728 1944 2048
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The possibilities are the same as those articulated for octave (4). Together with octave (4), octave (11) completes a perfect disdiapason reflecting an ascending Hypolydian (impacts on air) or descending Mixolydian (string lengths) scale.

δ(12) *1152:2304*
The δ-string is the first not to demonstrate a perfect disdiapason prior to the introduction of other possibilities for the scale. The first chromatic element is introduced in the second octave and allows the following additional possibility for the tone sequence not seen in connection with octave (5):

1152 1296 1458 1536 1728 1944 2187 *2304*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Lydian (impacts on air) or descending Dorian (string lengths) scale. It imitates the Δ-string.

ε(13) *1296:2592*
With the introduction of the first chromatic element, a tone sequence in addition to the one specified for octave (6) becomes possible.

(a) *1296 1458 1536 1728 1944 2048 2304 2592*
[9/8 × 256/243 × 9/8 × 9/8 × 256/243× 9/8 × 9/8]

The possibilities for the indicated sequence are identical to those for octave (6).

(b) *1296 1458 1536 1728 1944* 2187 *2304 2592*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern in (b), in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTST. Accordingly, the scale reflected by the octave articulated is either an ascending (impacts on air) or descending Phrygian (string lengths) scale, in imitation of the α-string.

ζ(14) *1458:2916*
The original possibility for this string remains; however, with the introduction of the first chromatic element, the following additional tone sequence becomes possible for the ζ-string:

1458 1536 1728 1944 2187 *2304 2592 2916*
[256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is STTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Dorian (impacts on air) or descending Lydian (string lengths) scale in imitation of the β-string.

Δ(15) *1536:3072*
The possible tone sequences for the octave are two, as follows:

(a) *1536 1728 1944 2048 2304 2592 2916 3072*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern in (a), in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Lydian (impacts on air) or descending Dorian (string lengths) scale, just as one would expect for the Δ-octave string.

(b) *1536 1728 1944* 2187 *2304 2592 2916 3072*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The scale pattern in (b), in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Hypolydian (impacts on air) or descending Mixolydian (string lengths) scale. The latter pattern is, as we have seen, characteristic of the γ-string.

In its second manifestation, i.e., (b), made possible only by the entrance of the chromatic number 2187 into the scale, the Δ-string masquerades as the γ-string. One might say that the Δ-string shifts toward the γ-string with the entrance of the first chromatic element into the original Δ-octave sequence.

α(16) *1728:3456*
There are two possible tone sequences for the octave indicated, as follows:

(a) *1728 1944 2048 2304 2592 2916 3072 3456*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern in (a), in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTST. Accordingly, the scale reflected by the octave articulated is either an ascending Phrygian (impacts on air) or descending Phrygian (string lengths) scale, just as one would expect of the α-string.

(b) *1728 1944* 2187 *2304 2592 2916 3072 3456*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern in (b), in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTTSTT. Accordingly, the scale reflected by this new version of the α-string octave is either an ascending Hypophrygian (impacts on air) or descending Hypodorian (string lengths) scale. The latter pattern, as we have seen, is characteristic of the δ-string.

In its second manifestation, i.e., (b), made possible only by the entrance of the chromatic number 2187 into the scale, the α-string masquerades as the δ-string. One might say that the α-string shifts toward the δ-string with the entrance of the first chromatic element into the original Δ-octave sequence.

β(17) *1944:3888*
There are two possible tone sequences for the octave indicated:

(a) *1944 2048 2304 2592 2916 3072 3456 3888*
[256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 ×9/8]

The same analysis applies for possibility (a) as applies for octave (3).

(b) *1944* 2187 *2304 2592 2916 3072 3456 3888*
[9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Hypodorian (impacts on air) or descending Hypophrygian (string lengths) scale. The latter scale is characteristic of the ε-string.

γ(18) *2048:4096*
Despite the introduction of a chromatic number in this interval (see Table 26, "Fully Annotated Table of Timaeus Numbers Arranged in Numerical Order from Least to Greatest"), there is still only one possible tone sequence for the octave:

2048 2304 2592 2916 3072 3456 3888 4096
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The scale remains, accordingly, an ascending Hypolydian (impacts on air) or descending Mixolydian (string lengths) scale.

δ(19) *2304:4608*
Octave (19) exhibits the same possibilities as octave (12).

ε(20) *2592:5184*
The tone sequence possibilities for octave (20) are the same as for octave (13).

ζ(21) *2916:5832*
Octave (21) exhibits the same patterns as octave (14).

Δ(22) *3072:6144*
This octave exhibits the same features as octave (15).

α(23) *3456:6912*
There are four possible tone sequences for this octave. They are the following:

(a) *3456 3888 4096 4608 5184 5832 6144 6912*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

(b) *3456 3888* 4374 *4608 5184 5832 6144 6912*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8]

Note one: Sequences (a) and (b) reflect the same possibilities as those indicated for octave (16) and share the same analysis. Sequences (c) and (d) allow new interpretations as indicated below.

(c) *3456 3888* 4374 *4608 5184 5832* 6561 *6912*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Lydian (string impacts) or descending Dorian (string lengths) scale in imitation of the Δ-string.

(d) *3456 3888 4096 4608 5184 5832* 6561 *6912*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTTS. The scale reflects an unaccustomed arrangement of the octave. Note that the unaccustomed possibility for octave arrangement emerging at this point for the α-string differs from the unaccustomed possibility native to the progressing Δ-string.

β(24) *3888:7776*
Octave (24) shares the features of octave (17) but allows two additional tone sequences made possible by the introduction of a second chromatic element into the scale. The two additional sequences are the following:

(a) *3888 4096 4608 5184 5832* 6561 *6912 7776*
[256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is STTTTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTTTTS. Neither pattern appears to have corresponded to any recognized ancient arrangement of the octave, diatonic or otherwise. It is an unaccustomed arrangement with no known significance.

(b) *3888* 4374 *4608 5184 5832* 6561 *6912 7776*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTST. Accordingly, the scale reflected by the octave articulated is either an ascending Phrygian (impacts on air) or descending Phrygian (string lengths) scale. The latter scale is, of course, characteristic of the α-string.

γ(25) *4096:8192*
There are two possible tone sequences for the octave indicated, because of the introduction of a second chromatic element, 6561; but the second represents the unaccustomed sequence TTTTSTS:

(a) *4096 4608 5184 5832 6144 6912 7776 8192*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The possibilities for sequence (a) are the same as those for octaves (4), (11), and (18).

(b) *4096 4608 5184 5832* 6561 *6912 7776 8192*
[9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths is TTTTSTS. Neither pattern reflects any recognizable ancient scale.

δ(26) *4608:9216*
The introduction of a second chromatic element into the scale allows the following two tone sequences in addition to those first surfacing with octave (12):

(a) *4608 5184 5832* 6561 *6912 7776 8192 9216*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTSTTT. It appears that the indicated arrangements reflect no scale known to ancient music.

(b) *4608 5184 5832* 6561 *6912 7776* 8748 *9216*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Hypolydian (impacts on air) or descending Mixolydian scale (string lengths) scale. It imitates the γ-string.

ε(27) *5184:10368*
The introduction of the second chromatic element introduces two new possibilities for tone sequences beyond those set forth above in octaves (13) and (20).

(a) *5184 5832* 6561 *6912 7776 8192 9216 10368*
[9/8 × 9/8 × 256/243 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTSTT. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TTSTTST. The indicated arrangement of the octave appears to have been unknown to ancient music.

(b) *5184 5832* 6561 *6912 7776* 8748 *9216 10368*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Hypophrygian (impacts on air) or descending Hypodorian (string lengths) scale, in imitation of the δ-string.

ζ(28) *5832:11664*
Two additional tone sequences become possible for the ζ-string with the introduction of the second chromatic element.

(a) *5832* 6561 *6912 7776 8192 9216 10368 11664*
[9/8 × 256/243 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTSTTT Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TTTSTST. The indicated tone sequence appears to have corresponded to no ancient scale for the octave.

(b) *5832* 6561 *6912 7776* 8748 *9216 10368 11664*
[9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Hypodorian (impacts on air) or descending Hypophrygian (string lengths) scale in imitation of the ε-string.

Δ(29) *6144:12288*
The possibilities for the octave indicated are the same as those indicated for octave (15). The presence of a second chromatic element, 6561, after the first element of the model octave string, does not yield new distortions of the Δ-string pattern.

α(30) *6912:13824*
The octave exhibits the same patterns as octave no. (23).

β(31) *7776:15552*
Octave (31) exhibits the same characteristics as octave (24).

γ(32) *8192:16384*
The possibilities for octave (32) are the same as those for octave (25).

δ(33) *9216:18432*
Octave (33) exhibits the same characteristics as octave (26).

ε(34) *10368:20736*
The introduction of the third chromatic element allows the following two new tone sequence possibilities:

(a) *10368 11664 12288 13824 15552* 17496 **19683** *20736*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTTS. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be STTTTST. The indicated arrangement of the octave appears to have been unknown to ancient music.

(b) *10368 11664* 13122 *13824 15552* 17496 **19683** *20736*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTTS. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be STTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Lydian (impacts on air) or descending Dorian (string lengths) scale, in imitation of the Δ-string.

ζ(35) *11664: 23328*
The third chromatic element permits the following additional tone sequences for the ζ-string:

(a) *11664 12288 13824 15552* 17496 **19683** *20736 23328*
[256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is STTTTST. The patterns indicated appear not to have been recognized as patterns for the octave in ancient music.

(b) *11664* 13122 *13824 15552* 17496 **19683** *20736 23328*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is

TSTTTST. Accordingly, the scale reflected by the octave articulated is either an ascending (impacts on air) or descending Phrygian (string lengths) scale, in imitation of the α-string.

Δ(36) *12288:24576*
The possibilities for the tone sequence of the octave with the introduction of a third chromatic element, **19683**, produces the following very odd additional sequence not reflecting any of the accustomed ancient possibilities for the diatonic scale:

12288 13824 15552 17496 **19683** *20736 23328 24576*
[9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 256/243]

This new distortion of the Δ-string, instead, reflects a scale pattern, TTTTSTS unknown to ancient music as an arrangement of the octave.

α(37) *13824:27648*
All of the possibilities reflected in the analysis of octave (23) exist in the case of octave (37). In addition, due to the emergence of a third chromatic element in the scale, two additional possibilities arise. They are exemplified by the following tone sequences:

(a) *13824 15552* 17496 **19683** *20736 23328* 26244 *27648*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTTS. Accordingly, the scale reflected by the octave articulated is either an ascending Hypolydian (impacts on air) or descending Mixolydian (string lengths) scale in imitation of the γ-string.

(b) *13824 15552* 17496 **19683** *20736 23328 24576 27648*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTSTTT. Accordingly, the scale reflected by the octave articulated is a second unaccustomed arrangement of the octave, differing both from the one emerging at octave (23) upon the introduction of the second chromatic element and from the one that emerges with the progressing Δ-string.

β(38) *15552:31104*
Octave (38) shares the characteristics of Octave (31) but also allows two additional tone sequences due to the introduction of a third chromatic element into the scale. The new tone sequences are the following:

(a) *15552* 17496 **19683** *20736 23328 24576 27648 31104*
[9/8 × 9/8 × 256/243 × 9/8 × 256/243 × 9/8 ×9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTSTT. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TTSTSTT. The sequence, like the Phrygian sequence, is perfectly reciprocal to itself; however, it appears to reflect no known arrangement of the octave known to ancient music.

(b) *15552* 17496 **19683** *20736 23328* 26244 *27648 31104*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTTSTT Accordingly, the scale reflected by the octave articulated is either an ascending Hypophrygian (impacts on air) or descending Hypodorian (string lengths) scale. The latter scale is characteristic of the δ-string.

γ(39) *16384:32768*
The introduction of the third chromatic element does not change the possibilities for the tone sequence of the octave. Rather the same possibilities exist for octave (39) as existed for octaves (32) and (25).

δ(40) *18432:36864*
The introduction of the third chromatic element does not expand the possibilities for the tone sequence beyond those indicated for octaves (26) and (33). The same sequences govern octave (40).

ε(41) *20736:41472*
The same possibilities exist for octave (41) as exist for octave (34).

ζ(42) *23328:46656*
Octave (42) exhibits the same possibilities as octave (35).

Δ(43) *24576:49152*
The octave indicated exhibits the same possibilities as octave (36) above.

α(44) *27648:55296*
The octave exhibits the same patterns as octave (37) above.

β(45) *31104:62208*
The octave indicated exhibits the same characteristics as octave (38) above but allows three additional tone sequences due to the introduction of a fourth chromatic element into the scale. The three new sequences are the following:

(a) *31104 32768 36864 41472 46656* 52488 **59049** *62208*
[256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is STTTTTS. In other words the order is perfectly self-reciprocal. The indicated unaccustomed arrangement of the octave appears to have been unknown to ancient music.

(b) *31104* 34992 *36864 41472 46656* 52488 **59049** *62208*
[9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TSTTTTS. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be STTTTST. The indicated unaccustomed arrangement of the octave was apparently unknown to ancient music.

(c) *31104* 34992 **39366** *41472 46656* 52488 **59049** *62208*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTTTS. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be STTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Lydian (impacts on air) or descending Dorian (string lengths) scale in imitation of the Δ-string.

γ(46) *32768:65536*
The following sequence additional to those listed above for octaves (25), (32), and (39) becomes possible with the introduction in octave (46) of a fourth chromatic element:

32768 36684 41472 46656 52488 **59049** *62208 65536*
[9/8 × 9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTTTSS. Neither of the indicated patterns appears to have recognized as a possible octave scale arrangement in ancient music.

δ(47) *36864:73728*
The introduction of the fourth chromatic element permits the following two tone sequences additional to those extant for octaves (26), (33), and (40):

(a) *36864 41472 46656* 52488 **59049** *62208 65536 73728*
[9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTTSST. The indicated scale arrangement appears to have been unknown to ancient music.

(b) *36864 41472 46656* 52488 **59049** *62208* 69984 *73728*
[9/8 × 9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTTSTS. The indicated scale arrangement appears to have been unknown to ancient music.

ε(48) *41472:82944*
With the fourth chromatic element the following three additional sequencing options beyond those indicated for octaves (34) and (41) become possible:

(a) *41472 46656* 52488 **59049** *62208 65536 73728 82944*
[9/8 × 9/8 × 9/8 × 256/243 × 256/243 ×9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSSTT. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TTSSTTT. The indicated arrangement for the octave appears not to have existed in ancient music.

(b) *41472 46656* 52488 **59049** *62208* 69984 *73728 82944*
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTST. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TSTSTTT. The indicated arrangement of the octave appears to have been unknown to ancient music.

(c) *41472 46656* 52488 **59049** *62208* 69984 **78732** 82944
[9/8 × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTTSTTS. The indicated arrangement of the octave appears to have been unknown to ancient music.

ζ(49) *46656:93312*
A fourth chromatic element allows the following three additional variations in the tone sequence for the ζ-string:

(a) *46656* 52488 **59049** *62208 65536 73728 82944 93312*
[9/8 × 9/8 × 256/243 × 256/243 × 9/8 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSSTTT. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TTTSSTT. The pattern indicated appears to have been unknown to ancient music as an arrangement for the octave.

(b) *46656* 52488 **59049** *62208* 69984 *73728 82944 93312*
[9/8 × 9/8 × 256/243 × 9/8 × 256/243 × 9/8 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths, is TTSTSTT. Descending order of pitch in the event the numbers index string vibrations or impacts on air would be TTSTSTT. The pattern indicated appears to have been unknown to ancient music as an arrangement for the octave.

(c) *46656* 52488 **59049** *62208* 69984 **78732** *82944 93312*
[9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 256/243 × 9/8]

The scale pattern, in ascending order of pitch, if the numbers index string vibrations or impacts on air, and descending order of pitch, if the numbers represent string lengths is TTSTTST. Descending order of pitch, in the event the numbers index string vibrations or impacts on air, would be TSTTSTT. Accordingly, the scale reflected by the octave articulated is either an ascending Hypophrygian (impacts on air) or descending Hypodorian (string lengths) scale.

Δ(50) *49152:98304*
The introduction of a fourth chromatic element produces no new distortions of the model octave. The same possibilities exist for the octave as are exhibited by octave (36) above.

α(51) *55296:110592*
All of the possibilities reflected in the analysis of octaves (37) and (44) exist in the case of octave (51). The appearance of the fourth chromatic element in the scale, does not, in this case, allow the introduction of new tone sequences.

β(52) *62208:124416*
Octave (52) allows the same possibilities as octave (45).

γ(53) *65536:131072*
The same possibilities exist for octave (53) as exist for octave (46).

δ(54) *73728: 147456*
Octave (54) exhibits the same characteristics as octave (47).

ε(55) *82944:165888*
The same possibilities exist as exist for octave (48).

ζ(56) *93312: 186624*
Octave (56) exhibits the same possibilities as octave (49).

Δ(57) *98304:196608*
The same possibilities as exist for octave (50) exist in the case of (57).

α(58) *110592:221184*
The same possibilities exist for octave (58) as exists for octaves (37), (44), and (51).

β(59) *124416:248832*
Octave (59) allows the same possibilities as octaves (45) and (52).

γ(60) *131072:262144*
The same possibilities exist for octave (60) as exist for octaves (46) and (53). There are no more octaves of the γ-string after octave (60). The string ends quite abruptly.

δ(61) *147456:294912*
Octave (61) exhibits the same characteristics as octaves (47) and (54).

ε(62) *165888:331776*
The octave exhibits the same characteristics as octaves (48) and (55).

ζ(63) *186624:373248*
Octave (63) exhibits the same possibilities as octaves (49) and (56).

Δ(64) *196608:393216*
The same possibilities exist for octave (64) as exist for octaves (50) and (57).

α(65) *221184:442368*
The same possibilities exist for octave (65) as exist for octaves (37), (44), (51), and (58).

β(66) *248832:497664*
The same possibilities exist for octave (66) as exist for octaves (45), (52), and (59).

γ(67) *262144:_____*
The ending number is not in the Timaeus set. The γ-string ends.

δ(68) *294912:589824*
Octave (68) marks the beginning of the deterioration of the δ-string. It loses three out of the six patterns possible for octaves (47), (54), and (61), namely: TTSTTST, the descending Hypodorian or ascending Hypophrygian scale proper to the δ-string; TTTSTST, reflecting no known ancient scale; and TTTTSST, also reflecting no known ancient scale.

ε (69) *331776:663552*
With the octave indicated, the possibilities for the arrangement of ε-string sequences begin to diminish. Of the nine different options for arrangement, three disappear, namely: TSTTSTT, the descending Hypophrygian or ascending Hypodorian characteristic of the ε-string; TTSTSTT, one of the odd scales unknown to ancient music, and TTTSSTT, another of the odd scales.

ζ(70) *373248:746496*
Octave (70) marks the beginning of the degeneration of possibilities for the ζ-string. The following three sequences among nine possible drop out: STTSTTT, the descending Hypolydian or ascending Mixolydian scale proper to the ζ-string; TSTSTTT, a sequence for the octave unknown to ancient music, and TTSSTTT, another sequence for the octave unknown to ancient music.

Δ(71) *393216:786432*
At this point, the original diatonic arrangement of the Δ-string is no longer possible; however, an ascending Hypolydian or descending Mixolydian diatonic scale is still possible, together with the unaccustomed arrangement of the octave in the pattern TTTTSTS. No Δ-string octaves, at all, are possible after this point.

α(72) *442368:884736*
With octave (72), two possibilities for the tone sequences of the α-string disappear. The sequences no longer possible are TSTTTST and TSTTTTS. The former sequence reflects either a descending or ascending Phrygian scale, the signature scale of the α-string; and the latter sequence reflects an unaccustomed arrangement of the octave.

β(73) *497664:995328*
Octave (73) marks the beginning of the degeneration of the β-string. Three of the nine possibilities for tone sequences that emerge by the introduction of the fourth chromatic element drop out of existence, namely: (a) the ascending Dorian or descending Lydian scale characteristic of the β-string, marked by the sequence STTTSTT; (b) the unaccustomed arrangement of the octave unknown to ancient music, marked by the sequence STTTTST; and (c) a second unaccustomed arrangement of the octave unknown to ancient music, marked by the sequence STTTTTS.

δ(74) *589824:1179648*
Octave (74) shares the characteristics of octave (68).

ε(75) *663552:1327104*
Octave (75) has the same characteristics as octave (69).

ζ(76) *746496:1492992*
Octave (76) has the same characteristics as octave (70).

α(77) *884736:1769472*
Octave (77) has the same features as octave (72) except that two additional possibilities for the tone sequence of the α-string disappear. Those possibilities are TTSTTST and TTTSTST. The former possibility reflects a descending Hypodorian or ascending Hypophrygian scale. The latter reflects another unaccustomed arrangement of the octave.

β(78) *995328:1990656*
Octave (78) loses the same tone sequences as octave (73) plus two more. The new missing sequences are the following: (a) TSTTSTT, the ascending Hypodorian or descending Hypophrygian scale imitating the ε-string; and (b) TTSTSTT, an unaccustomed arrangement of the octave unknown to ancient music.

δ(79) *1179648:2359296*
Octave (79) loses the following sequence additional to those lost at octave (68): TTSTTTS, the descending Dorian or ascending Lydian scale imitating the Δ-string. It is the last octave of the δ-string.

ε(80) *1327104:2654208*
With the octave indicated, two additional sequences disappear for the ε-string, namely: TSTTTST, the descending or ascending Phrygian arrangement characteristic of the α-string, and TSTTTTS, one of the arrangements unknown to ancient music.

ζ(81) *1492992:2985984*
Octave (81) displays further degeneration of the ζ-string. Tone sequences in addition to those lost at octave (70) drop out as possibilities, namely: STTTSTT, the descending Lydian or ascending Dorian scale imitating the β-string, and STTTTST, a sequence unknown to ancient music as an octave arrangement.

α(82) *1769472:3538944*
The same possibilities exist for octave (82) as existed for octave (77).

β(83) *1990656:3981312*
Octave (83) has the same characteristics as octave (78).

δ(84) *2359296:_____*
There is no complete octave. The δ-string expires.

ε(85) *2654208:5308416*
With the indicated octave, even more ε-string possibilities disappear: TTSTTST, the descending Hypodorian or ascending Hypophrygian scale in imitation of the δ-string and TTTSTST, one of the sequences unknown to ancient music.

ζ(86) *2985984:5971968*
Octave (86) loses the following additional tone sequences as possibilities for the ζ-string: TSTTSTT, the descending Hypophrygian or ascending Hypodorian scale characteristic of the ε-string and TTSTSTT, an arrangement of the octave unknown to ancient music.

α(87) *3538944:7077888*
Octave (87) has the same features as octave (82) except that an additional possibility for the tone sequence of the α-string disappears, namely, TTSTTTS. The latter sequence reflects either a descending Dorian or ascending Lydian scale.

β(88) *3981312:7962624*
Octave (88) shares the characteristics of octaves (78) and (83) but loses two additional tone sequences, namely: (a) TSTTTTS, an unaccustomed arrangement of the octave unknown to ancient music; and (b) TSTTTST, the ascending or descending Phrygian scale imitating the α-string.

ε (89) *5308416:10616832*
The possibilities for octave (89) are the same as those for octave (85).

ζ(90) *5971968: 11943936*
Octave (90) exhibits the same characteristics as octave (86).

α(91) *7077888:*_____
At octave (91), no tone sequence for the α-string remains, based on the *Timaeus* numbers.

β(92) *7962624:15925248*
Octave (92) shares the characteristics of octave (88) but loses one additional tone sequence, namely: TTSTTST, the ascending Hypophrygian or descending Hypodorian scale imitating the δ-string.

ε (93) *10616832:21233664*
With the ninety-third octave, another sequence disappears, namely, TTSTTTS, the ascending Lydian and descending Dorian scale characteristic of the Δ-string.

ζ(94) *11943936:23887872*
Octave (94) loses yet another tone sequence for the ζ-string: TSTTTST, the ascending or descending Phrygian scale characteristic of the α-string.

β(95) *15925248:31850496*
Octave (95) shares the characteristics of octave (92).

ε (96) *21233664:_____*
The ε-string disappears completely.

ζ(97) *23887872:47775744*
Octave (97) loses the remaining tone sequence, TTSTTST, the descending Hypodorian or ascending Hypophrygian scale characteristic of the δ-string, as a complete sequence.

β(98) *31850496:63700992*
Octave (98) loses even the last remaining tone sequence manifested by the β-string, namely: TTSTTTS, the ascending Lydian or descending Dorian scale imitating the Δ-string. The β-string has, thus, completely degenerated. No meaningful octave sequence is articulable.

ζ(99) *47775744:95551488*
Octave (99) has the same characteristics as octave (97).

β(100) *63700992:_____*
No *Timaeus* number fills the void for the end number. It is off the chart, a result that makes sense given the complete degeneration of the β-string in octave (98).

ζ(101) *95551488:191102976*
Octave (101) simply shows further degeneration of the tone sequence lost at octave (97).

ζ(102) *191102976:_____*
No *Timaeus* number fills the void for the end number. It is off the chart, a result that makes sense given the complete degeneration of the ζ-string in the octave (97).

APPENDIX 8

Chromatic Scale Tables

1 Preliminary Notes

Preliminary note one: The first chromatic fourth in the number series is 24 T 27 TS 32, reflecting a descending Mixolydian *ordo*. The first chromatic fifth, arising much later in the series, runs from 1458 to 2187 as follows:

1458 S 1536 T 1728 TS 2048 S′ 2187

It follows a descending Lydian *ordo*.

Preliminary note two: The second chromatic fifth is the sequence running from 1536 to 2304.

1536 T 1728 TS 2048 S′ 2187 S 2304

Preliminary note three: The sequence running from 1536 through 2304 evidences the beginnings of a descending Mixolydian chromatic scale.

Preliminary note four: The sequence running from 1728 through 2304 is the fourth beginning a descending Dorian or Hypodorian chromatic scale.

Preliminary note five: The first entire chromatic scale is a Dorian scale beginning from 3456 in α-string, familiar to readers from the discussion of diatonic scales. The chromatic possibilities within each of the diatonic strings considered in this study are addressed in this Appendix 8.

Preliminary note six: The designation "S′" in the inventory below indicates an interval of proportion 2187/2048 (*apotomē*). The designation "S" indicates an interval of proportions 256/243 (*leimma*).

Preliminary note seven: Many of the chromatic scales utilize true trihemitones. Undivided TS intervals in the chromatic scales capable of internal redivision into trihemitone sequences, designated by numbers in the *Timaeus* set, are marked in bold with the exception noted below in this note. The trihemitone structure of such intervals is also provided in bold. Such intervals give the chromatic scale of which they are a part possibilities for scale ornamentation that would not otherwise exist, while preserving the overall structure of the chromatic scale in which they occur. Note that undivided T intervals, capable of redivision into two S intervals, but independent of and adjacent to S intervals, are not counted as creating a trihemitone with the adjacent S interval, since three such adjacent S intervals cannot function as a musical unit without destroy-

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ing the structure of the chromatic scale in which they occur. Where more than one trihemitone, as defined in this paragraph, occurs in a chromatic scale, the additional trihemitones are marked in italics, rather than in bold.

2 Chromatic Scales Based on Original Descending Dorian Model Octave Scale Numbers

A. 3456 3888 4096 4608 5184 5832 6144 6912

The very first chromatic octave in the *Timaeus* number set begins from 3456. Chains descending from 3456 belong to α-string (division by 2 leads ultimately to 432, the beginning of the diatonic α-string). Note that the descending Dorian scale is the initiating pattern among the chromatic sequences of α-string.

1. 3456 TS 4096 S′ 4374 S 4608 T 5184 TS 6144 S′ 6561 S 6912
   First d.Dorian

2. 6912 TS 8192 S′ 8748 S 9216 T 10368 TS 12288 S′ 13122 S 13824
   Second d.Dorian

3. 13824 TS 16384 S′ 17496 S 18432 T 20736 TS 24576 S′ 26244 S 27648
   Third d.Dorian
   13824 T **15552 TS 18432** S′ 19683 S 20736 TS 24576 S′ 26244 S 27648
   The indicated trihemitone is structured thus: **15552 S 16384 S′ 17496 S 18432.**
   First d.Mixolydian
   13824 TS 16384 T 18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648
   First a.Hypophrygian

4. 27648 TS 32768 S′ 34992 S 36864 T 41472 TS 49152 S′ 52488 S 55296
   Fourth d.Dorian
   27648 T **31104 TS 36864** S′ 39366 S 41472 TS 49152 S′ 52488 S 55296
   The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
   Second d.Mixolydian
   27648 TS 32768 T 36824 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296
   Second a.Hypophrygian

5. **55296 TS 65536** S′ 69984 S 73728 T 82944 TS 98304 S′ 104976 S 110592
   The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
   Fifth d. Dorian

55296 T **62208 TS 73728** S′ 78732 S 82944 TS 98304 S′ 104976 S 110592
62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone.
Third d.Mixolydian

55296 S′ 59049 S **62208 TS 73728** S′ 78732 S 82944 T *93312 TS 110592*
62208 S 65536 S′ 69984 S 73728 is the structure of the first trihemitone; *93312 S 98304 S′ 104976 S 110592* is the structure of the second trihemitone. Observe that with the occurrence of two TS sequences that can be internally rearticulated as trihemitones, one almost gets the entire breakdown of a scale into "semitones" Only the indivisible "T" interval between 82944 and 93312 prevents the result.
First d.Hypophrygian

55296 TS 65536 T 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Third a.Hypophrygian

55296 S′ 59049 S **62208 TS 73728** S′ 78732 S 82944 TS 98304 T 110592
The structure of the trihemitone is **62208 S 65536 S′69984 S 73728.**
First a.Mixolydian

55296 S′ **59049 TS 69984** S 73728 S′ 78732 TS 93312 T 104976 S 110592
The structure of the trihemitone is **59049 S 62208 S 65536 S′ 69984.**
First a.Lydian

6. **110592 TS 131072** S′ 139968 S 147456 T 165888 TS 196608 S′ 209952 S 221184
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Sixth d.Dorian

110592 T **124416 TS 147456** S′ 157464 S 165888 TS 196608 S 209952 S 221184
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Fourth d.Mixolydian

110592 S′ 118098 S **124416 TS 147456** S′ 157464 S 165888 T *186624 TS 221184*
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone. *186624 S 196608 S′ 209952 S 221184* is the structure of the second trihemitone.
Second d.Hypophrygian

110592 TS 131072 T 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184
The structure of the trihemitone is **110592 S′ 118098 S 124416 S 131072.**
Fourth a.Hypophrygian

110592 S′ 118098 S **124416 TS 147456** S′ 157464 S 165888 TS 196608 T 221184
The structure of the trihemitone is **124416 S 131072 S′ 139968 S 147456.**
Second a.Mixolydian

110592 S′ **118098 TS 139968** S 147456 S′ 157464 TS 186624 T 209952 S 221184
The structure of the trihemitone is **118098 S 124416 131072 S′ 139968.**
Second a.Lydian

7. **221184 TS 262144** S′ 279936 S 294912 T 331776 TS 393216 S′ 419904 S 442368
221184 S′ 236196 S 248832 S 262144 is the structure of the indicated trihemitone.
Seventh d.Dorian

221184 T **248832 TS 294912** S′ 314928 S 331776 TS 393216 S′ 419904 S 442368
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fifth d.Mixolydian

221184 S′ 236196 S **248832 TS 294912** S′ 314928 S 331776 T *373248 TS 442368*
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone sequence. *373248 S 393216 S′ 419904 S 442368* is the structure of the second trihemitone.
Third d.Hypophrygian

221184 TS 262144 T 294912 S′ 314928 S 331776 TS 393216 S′ 419904 S 442368
The structure of the trihemitone is **221184 S′ 236196 S 248832 S 262144.**
Fifth a.Hypophrygian

221184 S′ 236196 S **248832 TS 294912** S′ 314928 S 331776 TS 393216 T 442368
The structure of the trihemitone is **248832 S 262144 S′ 279936 S 294912.**
Third a.Mixolydian

221184 S′ **236196 TS 2979936** S 294912 S′ 314928 TS 373248 T 419904 S 442368
The structure of the trihemitone is **236196 S 248832 S 262144 S′ 279936.**
Third a.Lydian

8. d.Dorian and a.Hypophrygian die.

442368 T 497664 TS 589824 S′ 629856 S 663552 TS 786432 S′ 839808 S 884736
Sixth d.Mixolydian

442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 T **746496 TS 884736**
The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
Fourth d.Hypophrygian

442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 TS 786432 T 884736
Fourth a.Mixolydian

442368 S′ 472392 TS 559872 S 589824 S′ 629856 TS 746496 T 839808 S 884736
Fourth a.Lydian

9. d.Mixolydian and a.Mixolydian give out.

884736 S′ 944784 S 995328 TS 1179648 S′ 1259712 S 1327104 T 1492992 TS 1769472
Fifth d.Hypophrygian

884736 S′ 944784 TS 1119744 S 1179648 S′ 1259712 TS 1492992 T 1679616 S 1769472
Fifth a.Lydian

10. 1769472 S′ 1889568 S 1990656 TS 2359296 S′ 2519424 S 2654208 T 2985984 TS 3538944
Sixth d.Hypophrygian
1769472 S′ 1889568 TS 2239488 S 2359296 S′ 2519424 TS 2985984 T 3359232 S 3538944
Sixth a.Lydian

11. 3538944
d.Hypophrygian and a.Lydian die out. No scales remain.

B. 3456 3888 4096 4608 5184 5832 6144 6912

Chains descending from 3888 belong to β-string (division by 2 leads ultimately to 486, the beginning of the diatonic β-string). Note that the a.Dorian scale is the initiating pattern among the chromatic sequences of β-string.

1. 3888 S 4096 S′ 4374 TS 5184 T 5832 S 6144 S′ 6561 TS 7776
First a.Dorian

2. 7776 S 8192 S′ 8748 TS 10368 T 11664 S 12288 S′ 13122 TS 15552
Second a.Dorian

3. 15552 S 16384 S′ 17496 T 19683 TS 23328 S 24576 S′ 26244 TS 31104
First d.Phrygian
15552 TS 18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 T 31104
The indicated trihemitone is structured thus: **15552 S 16384 S′ 17496 S 18432.**
First d.Hypodorian
15552 S 16384 S′ **17496 TS 20736** T 23328 S 24576 S′ 26244 TS 31104
The structure of the trihemitone is **17496 S 18432 S′ 19683 S 20736.**
Third a.Dorian
15552 T 17496 S 18432 S′ 19683 TS 23328 S 24576 S′ 26244 TS 31104
First a.Hypodorian

4. 31104 S 32768 S′ 34992 T 39366 TS 46656 S 49152 S′ **52488 TS 62208**
The structure of the trihemitone indicated is **52488 S 55296 S′ 59049 S 62208.**
Second d.Phrygian
31104 TS 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 T 62208
The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
Second d.Hypodorian

31104 TS 36864 S′ 39366 S 41472 T *46656 TS 55296* S′ 59049 S 62208
The first trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864**; the second trihemitone is structured thus: *46656 S 49152 S′ 52488 S 55296*.
First d.Dorian

31104 S 32768 S′ **34992 TS 41472** T 46656 S 49152 S′ *52488 TS 62208*
The structure of the first trihemitone is **34992 S 36864 S′ 39366 S 41472**. The structure of the second is *52488 S 55296 S′ 59049 S 62208*.
Fourth a.Dorian

31104 T 34992 S 36864 S′ 39366 TS 46656 S 49152 S′ **52488 TS 62208**
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Second a.Hypodorian

31104 TS 36864 S′ 39366 S 41472 TS 49152 T 55296 S′ 59049 S 62208
The structure of the trihemitone is **31104 S 32768 S′ 34992 S 36864.**
First a.Phrygian

5. 62208 S 65536 S′ 69984 T 78732 TS 93312 S 98304 S′ **104976 TS 124416**
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Third d.Phrygian

62208 TS 73728 S′78732 S 82944 T *93312 TS 110592* S′ 118098 S 124416
62208 S 65536 S′ 69984 S 73728 is the structure of the first trihemitone; *93312 S 98304 S′ 104976 S 110592* is the structure of the second trihemitone.
Second d.Dorian

62208 TS 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 T 124416
62208 S 65536 S′ 69984 S 73728 is the structure of the trihemitone.
Third d.Hypodorian

62208 S 65536 S′ **69984 TS 82944** T 93312 S 98304 S′ *104976 TS 124416*
The structure of the first trihemitone is **69984 S 73728 S′ 78732 S 82944**. The structure of the second is *104976 S 110592 S′ 118098 S 124416*.
Fifth a.Dorian

62208 T 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ **104976 TS 124416**
The structure of the trihemitone is **104976 S 110592 S′ 118098 S 124416.**
Third a.Hypodorian

62208 TS 73728 S′ 78732 S 82944 TS 98304 T 110592 S′ 118098 S 124416
The structure of the trihemitone is **62208 S 65536 S′ 69984 S 73728.**
Second a.Phrygian

6. 124416 S 131072 S′ 139968 T 157464 TS 186624 S 196608 S′ **209952 TS 248832**
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fourth d.Phrygian

124416 TS 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 T 248832
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Fourth d.Hypodorian

124416 TS 147456 S′ 157464 S 165888 T *186624 TS 221184* S′ 236196 S 248832
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
186624 S 196608 S′ 209952 S 221184 is the structure of the second trihemitone.
Third d.Dorian

124416 S 131072 S′ **139968 TS 165888** T 186624 S 196608 S′ *209952 TS 248832*
The structure of the first trihemitone is **139968 S 147456 S′ 157464 S 165888**. The structure of the second trihemitone is *209952 S 221184 S′ 236196 S 248832*.
Sixth a.Dorian

124416 T 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ **209952 TS 248832**
The structure of the trihemitone is **209952 S 221184 S′ 236196 S 248832.**
Fourth a.Hypodorian

124416 TS 147456 S′ 157464 S 165888 TS 196608 T 221184 S′ 236196 S 248832
The structure of the trihemitone is **124416 S 131072 S′ 139968 S 147456.**
Third a.Phrygian

7. 248832 S 262144 S′ 279936 T 314928 TS 373248 S 393216 S′ **419904 TS 497664**
497664 S 442368 S′ 472392 S 497664 is the structure of the indicated trihemitone.
Fifth d.Phrygian

248832 TS 294912 S′ 314928 S 331776 TS 393216 S′ 419904 S 442368 T 497664
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fifth d.Hypodorian

248832 TS 294912 S′ 314928 S 331776 T *373248 TS 442368* S′ 472392 S 497664
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
373248 S 393216 S′ 419904 S 442368 is the structure of the second trihemitone.
Fourth d.Dorian

248832 S 262144 S′ **279936 TS 331776** T 373248 S 393216 S′ *419904 TS 497664*
The first trihemitone has the structure **279936 S 294912 S′ 314928 S 331776**. The second trihemitone has the structure *419904 S 442368 S′ 472392 S 497664*.
Seventh a.Dorian

248832 T 279936 S 294912 S′ 314928 TS 373248 S 393216 S′ **419904 TS 497664**
The trihemitone has the structure **419904 S 442368 S′ 472392 S 497664.**
Fifth a.Hypodorian

248832 TS 294912 S′ 314928 S 331776 TS 393216 T 442368 S′ 472392 S 497664
The trihemitone has the structure **248832 S 262144 S′ 279936 S 294912.**
Fourth a.Phrygian

8. d.Phrygian and a.Dorian die.
   497664 TS 589824 S′ 629856 S 663572 TS 786432 S′ 839808 S 884736 T 995328
   Sixth d.Hypodorian
   497664 TS 589824 S′ 629856 S 663552 T **746496 TS 884736** S′ 944784 S 995328
   The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
   Fifth d.Dorian
   497664 T 559872 S 589824 S′ 629856 TS 746496 S 786432 S′ **839808 TS 995328**
   The structure of the trihemitone is **839808 S 884736 S′ 944784 S 995328.**
   Sixth a.Hypodorian
   497664 TS 589824 S′ 629856 S 663552 TS 786432 T 884736 S′ 944784 S 995328
   Fifth a.Phrygian

9. d.Hypodorian, a.Phrygian, and a.Hypodorian die.
   995328 TS 1179648 S′ 1259712 S 1327104 T 1492992 TS 1769472 S′ 1889568 S 1990656
   Sixth d.Dorian alone remains.

10. 1990656 TS 2359296 S′ 2519424 S 2654208 T 2985984 TS 3538944 S′ 3779136 S 3981312
    Seventh d.Dorian

11. 3981312–7962624: Dorian chromatic scale gives out; no scales remain.

C. 3456 3888 <u>4096</u> 4608 5184 5832 6144 6912

Chains descending from 4096 belong to γ-string (division by 2 leads ultimately to 512, the beginning of the diatonic γ-string). Note that the d.Phrygian and a.Hypolydian scales are the initiating patterns among the chromatic sequences of γ-string.

1. 4096 S′ 4374 S 4608 T 5184 TS 6144 S′ 6561 S 6912 TS 8192
   First d.Phrygian
   4096 S′ 4374 TS 5184 T 5832 S 6144 S′ 6561 TS 7776 S 8192
   First a.Hypolydian

2. 8192 S′ 8748 S 9216 T 10368 TS 12288 S′ 13122 S 13824 TS 16384
   Second d.Phrygian
   8192 S′ 8748 TS 10368 T 11664 S 12288 S′ 13122 TS 15552 S 16384
   Second a.Hypolydian

3. 16384 S′ 17496 T 19683 TS 23328 S 24576 S′ 26244 TS 31104 S 32768
   First d.Lydian

16384 S′ 17496 S 18432 T 20736 TS 24576 S′ 26244 S 27648 TS 32768
Third d.Phrygian
16384 S′ **17496 TS 20736** T 23328 S 24576 S′ 26244 TS 31104 S 32768
The structure of the trihemitone is **17496 S 18432 S′ 19683 S 20736.**
Third a.Hypolydian
16384 T 18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 TS 32768
First a.Hypodorian

4. 32768 S′ 34992 T 39366 TS 46656 S 49152 S′ **52488 TS 62208** S 65536
The structure of the trihemitone indicated is **52488 S 55296 S′ 59049 S 62208.**
Second d.Lydian
32768 S′ 34992 S 36864 T 41472 TS 49152 S′ 52488 S **55296 TS 65536**
The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
Fourth d.Phrygian
32768 S′ **34992 TS 41472** T 46656 S 49152 S′ *52488 TS 62208* S 65536
The structure of the first trihemitone is **34992 S 36864 S′ 39366 S 41472**. The structure of the second trihemitone is **52488 S 55296 S′ 59049 S 62208**.
Fourth a.Hypolydian
32768 T 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S **55296 TS 65536**
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Second a.Hypodorian

5. 65536 S′ 69984 T 78732 TS 93312 S 98304 S′ **104976 TS 124416** S 131072
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Third d.Lydian
65536 S′ 69984 S 73728 T 82944 TS 98304 S′ 104976 S **110592 TS 131072**
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Fifth d.Phrygian
65536 S′ **69984 TS 82944** T 93312 S 98304 S′ *104976 TS 124416* S 131072
The structure of the first trihemitone is **69984 S 73728 S′ 78732 S 82944**. The structure of the second trihemitone is *104976 S 110592 S′ 118098 S 124416*.
Fifth a.Hypolydian
65536 T 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S **110592 TS 131072**
The structure of the trihemitone is **110592 S′ 118098 S 124416 S 131072.**
Third a.Hypodorian

6. 131072 S′ 139968 T 157464 TS 186624 S 196608 S′ **209952 TS 248832** S 262144
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fourth d.Lydian

131072 S′ 139968 S 147456 T 165888 TS 196608 S′ 209952 S **221184 TS 262144**
221184 S′ 236196 S 248832 S 262144 is the structure of the trihemitone indicated.
Sixth d.Phrygian

131072 S′ **139968 TS 165888** T 186624 S 196608 S′ *209952 TS 248832 S 262144*
The structure of the first trihemitone is **139968 S 147456 S′ 157464 S 165888**. The structure of the second trihemitone is *209952 S 221184 S′ 236196 S 248832*.
Sixth a.Hypolydian

131072 T 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S **221184 TS 262144**
The structure of the trihemitone is **221184 S′ 236196 S 248832 S 262144**.
Fourth a.Hypodorian

7. 262144-MISSING
No scales.

D. 3456 3888 4096 4608 5184 5832 6144 6912

Chains descending from 4608 belong to 𝔡-string (division by 2 leads ultimately to 576, the beginning of the diatonic 𝔡-string). Note that the d.Mixolydian scale is the initiating pattern among the chromatic sequences of 𝔡-string.

1. 4608 T 5184 TS 6144 S′ 6561 S 6912 TS 8192 S′ 8748 S 9216
First d.Mixolydian

2. 9216 T 10368 TS 12288 S′13122 S 13824 TS 16384 S′ 17496 S 18432
Second d.Mixolydian

3. 18432 T 20736 TS 24576 S′ 26244 S 27648 TS 32768 S′ 34992 S 36864
Third d.Mixolydian

18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 T **31104 TS 36864**
The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
First d.Hypophrygian

18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 TS 32768 T 36864
First a.Mixolydian

18432 S′ 19683 TS 23328 S 24576 S′ 26244 TS 31104 T 34992 S 36864
First a.Lydian

4. 36864 T 41472 TS 49152 S′ 52488 S **55296 TS 65536** S′ 69984 S 73728
The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
Fourth d.Mixolydian

36864 S′ 39366 S 41472 T **46656 TS 55296** S′ 59049 S *62208 TS 73728*
62208 S 65536 S′ 69984 S 73728 is the structure of the first trihemitone; the second trihemitone is structured thus: *46656 S 49152 S′ 52488 S 55296*.
First d.Phrygian

36864 S′ 39366 TS 46656 S 49152 S′ 52488 T **59049 TS 69984** S 73728
59049 S 62208 S 65536 S′ 69984 is the structure of the indicated trihemitone.
First d.Hypolydian

36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 T **62208 TS 73728**
62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone.
Second d.Hypophrygian

36864 S′ 39366 S 41472 TS 49152 S′ 52488 S **55296 TS 65536** T 73728
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Second a.Mixolydian

36864 S′ 39366 TS 46656 S 49152 S′ **52488 TS 62208** T 69984 S 73728
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Second a.Lydian

36864 S′ 39366 S 41472 TS 49152 T 55296 S′ 59049 S **62208 TS 73728**
The structure of the trihemitone is **62208 S 65536 S′ 69984 S 73728.**
First a.Dorian

36864 S′ 39366 TS 46656 T 52488 S 55296 S′ **59049 TS 69984** S 73728
The structure of the trihemitone is **59049 S 62208 S 65536 S′ 69984.**
First a.Hypolydian

5. 73728 T 82944 TS 98304 S′ 104976 S **110592 TS 131072** S′ 139968 S 147456
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Fifth d.Mixolydian

73728 S′ 78732 S 82944 T **93312 TS 110592** S′ 118098 S *124416 TS 147456*
93312 S 98304 S′ 104976 S 110592 is the structure of the second trihemitone.
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Second d.Phrygian

73728 S′ 78732 TS 93312 S 98304 S′ 104976 T **118098 TS 139968** S 147456
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
Second d.Hypolydian

73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 T **124416 TS 147456**
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Third d.Hypophrygian

73728 S′ 78732 S 82944 TS 98304 S′ 104976 S **110592 TS 131072** T 147456
The structure of the trihemitone is **110592 S′ 118098 S 124976 S 131072.**
Third a.Mixolydian

73728 S′ 78732 TS 93312 S 98304 S′ **104976 TS 124416** T 139968 S 147456
The structure of the trihemitone is **104976 S 110592 S′ 118098 S 124416.**
Third a.Lydian

73728 S′ 78732 S 82944 TS 98304 T 110592 S′ 118098 S **124416 TS 147456**
The structure of the trihemitone is **124416 S 131072 S′ 139968 S 147456.**
Second a.Dorian

73728 S′ 78732 TS 93312 T 104976 S 110592 S′ **118098 TS 139968** S 147456
The structure of the trihemitone is **118098 S 124416 S 131072 S′ 139968.**
Second a.Hypolydian

6. 147456 T 165888 TS 196608 S′ 209952 S **221184 TS 262144** S′ 279936 S 294912
221184 S′ 236196 S 248832 S 262144 is the structure of the trihemitone indicated.
Sixth d.Mixolydian

147456 S′ 157464 S 165888 T **186624 TS 221184** S′ 236196 S *248832 TS 294912*
186624 S 196608 S′ 209952 S 221184 is the structure of the second trihemitone.
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Third d.Phrygian

147456 S′ 157464 TS 186624 S 196608 S′ 209952 T **236196 TS 279936** S 294912
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third d.Hypolydian

147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 T **248832 TS 294912**
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fourth d.Hypophrygian

147456 S′ 157464 S 165188 TS 196608 S′209952 S **221184 TS 279936** T 294912
The structure of the trihemitone is **221184 S′ 236196 S 248832 S 262144.**
Fourth a. Mixolydian

147456 S′ 157464 TS 186624 S 196608 S′ **209952 TS 248832** T 279936 S 294912
The structure of the trihemitone is **209952 S 221184 S′ 236196 S 248832.**
Fourth a.Lydian

147456 S′ 157464 S 165888 TS 196608 T 221184 S′ 236196 S 248832 TS 294912
The structure of the trihemitone is **248832 S 262144 S′ 279936 S 294912.**
Third a.Dorian

147456 S′ 157464 TS 186624 T 209952 S 221184 S′ 236196 TS 279936 S 294912
The structure of the trihemitone is **236196 S 248832 S 262144 S′ 279936.**
Third a.Hypolydian

7. Descending and ascending Mixolydian chromatic scales give out.
294912 S′ 314928 S 331776 T **373248 TS 442368** S′ 472392 S 497664 TS 589824
373248 S 393216 S′ 419904 S 442368 is the structure of the trihemitone.
Fourth d.Phrygian

294912 S′ 314928 TS 373248 S 393216 S′ 419904 T 472392 TS 559872 S 589824
Fourth d.Hypolydian
294912 S′ 314928 S 331776 TS 393216 S′ 419904 S 442368 T 497664 TS 589824
Fifth d.Hypophrygian
294912 S′ 314928 TS 373248 S 393216 S′ **419904 TS 497664** T 559872 S 589824
The structure of the trihemitone is **419904 S 442368 S′ 472392 S 497664.**
Fifth a.Lydian
294912 S′ 314928 S 331776 TS 393216 T 442368 S′ 472392 S 497664 TS 589824
Fourth a.Dorian
294912 S′ 314928 TS 373248 T 419904 S 442368 S′ 472392 TS 559872 S 589824
Fourth a.Hypolydian

8. 589824 S′ 629856 S 663552 T **746496 TS 884736** S′ 944784 S 995328 TS 1179648
The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
Fifth d.Phrygian
589824 S′ 629856 TS 746496 S 786432 S′ 839808 T 944784 TS 1119744 S 1179648
Fifth d.Hypolydian
589824 S′ 629856 S 663552 TS 786432 S′ 839808 S 884736 T 995328 TS 1179648
Sixth d.Hypophrygian
589824 S′ 629856 TS 746496 S 786432 S′ **839808 TS 995328** T 1119744 S 1179648
The structure of the trihemitone is **839808 S 884736 S′ 944784 S 995328.**
Sixth a.Lydian
589824 S′ 629856 S 663552 TS 786432 T 884736 S′ 944784 S 995328 TS 1179648
Fifth a.Dorian
589824 S′ 629856 TS 746496 T 839808 S 884736 S′ 944784 TS 1119744 S 1179648
Fifth a.Hypolydian

9. Descending Hypolydian, descending Hypophrygian, ascending Lydian, and ascending Dorian chromatic scales die.
1179648 S′ 1259712 S 1327104 T 1492992 TS 1769472 S′ 1889568 S 1990656 TS 2359296
Sixth d.Phrygian
1179648 S′ 1259712 TS 1492992 T 1679616 S 1769472 S′ 1889568 TS 2239488 S 2359296
Sixth a.Hypolydian

10. 2359296: MISSING
No scales.

E. 3456 3888 4096 4608 **5184** 5832 6144 6912

Chains descending from 5184 belong to ε-string (division by 2 leads ultimately to 648, the beginning of the diatonic ε-string). Note that the d. and a. Hypodorian scales are the initiating patterns among the chromatic sequences of ε-string.

1. 5184 TS 6144 S′ 6561 S 6912 TS 8192 S′ 8748 S 9216 T 10368
   First d.Hypodorian
   5184 T 5832 S 6144 S′ 6561 TS 7776 S 8192 S′ 8748 TS 10368
   First a.Hypodorian

2. 10368 TS 12288 S′ 13122 S 13824 TS 16384 S′ 17496 S 18432 T 20736
   Second d.Hypodorian
   10368 TS 12288 S′ 13122 S 13824 T **15552 TS 18432** S′ 19683 S 20736
   The indicated trihemitone is structured thus: **15552 S 16384 S′ 17496 S 18432.**
   First d.Dorian
   10368 T 11664 S 12288 S′ 13122 TS 15552 S 16384 S′ **17496 TS 20736**
   The structure of the trihemitone is **17496 S 18432 S′ 19683 S 20736.**
   Second a.Hypodorian
   10368 TS 12288 S′ 13122 S 13824 TS 16384 T 18432 S′ 19683 S 20736
   First a.Phrygian

3. 20736 TS 24576 S′ 26244 S 27648 TS 32768 S′ 34992 S 36824 T 41472
   Third d.Hypodorian
   20736 TS 24576 S′ 26244 S 27648 T **31104 TS 36864** S′ 39366 S 41472
   The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
   Second d.Dorian
   20736 T 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ **34992 TS 41472**
   The structure of the trihemitone is **34992 S 36864 S′ 39366 S 41472.**
   Third a.Hypodorian
   20736 TS 24576 S′ 26244 S 27648 TS 32768 T 36864 S′ 39366 S 41472
   Second a.Phrygian

4. 41472 TS 49152 S′ 52488 S **55296 TS 65536** S′ 69984 S 73728 T 82944
   The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
   Fourth d.Hypodorian
   41472 TS 49152 S′ 52488 S 55296 T **62208 TS 73728** S′ 78732 S 82944
   62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone.
   Third d.Dorian

41472 T **46656 TS 55296** S′ 59049 S *62208 TS 73728* S 78732 S 82944
62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone; the second trihemitone is structured thus: *46656 S 49152 S′ 52488 S 55296*.
First d.Mixolydian

41472 T 46656 S 49152 S′ **52488 TS 62208** S 65536 S′ *69984 TS 82944*
The structure of the first trihemitone is **52488 S 55296 S′ 59049 S 62208**. The structure of the second trihemitone is *69984 S 72728 S′ 78732 S 82944*.
Fourth a.Hypodorian

41472 TS 49152 S′ 52488 S **55296 TS 65536** T 73728 S′ 78732 S 82944
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Second a.Phrygian

41472 TS 49152 T 55296 S′ 59049 S **62208 TS 73728** S′ 78732 S 82944
The structure of the trihemitone is **62208 S 65536 S′ 69984 S 73728.**
First a.Hypophrygian

5. 82944 TS 98304 S′ 104976 S **110592 TS 131072** S′ 147456 T 165888
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Fifth d.Hypodorian

82944 TS 98304 S′ 104976 S 110592 T **124416 TS 147456** S′ 157464 S 165888
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Fourth d.Dorian

82944 T **93312 TS 110592** S′ 118098 S **124416 TS 147456** S′ 157464 165888
93312 S 98304 S′ 104976 S 110592 is the structure of the first trihemitone; *124416 S 131072 S′ 139968 S 147456* is the structure of the second trihemitone.
Second d.Mixolydian

82944 T 93312 S 98304 S′ **104976 TS 124416** S 131072 S′ *139968 TS 165888*
The structure of the first trihemitone is **104976 S 110592 S′ 118098 S 124416**. The structure of the second trihemitone is *139968 S 147456 S′ 157464 S 165888*.
Fifth a.Hypodorian

82944 TS 98304 S′ 104976 S **110592 TS 131072** T 147456 S′ 157464 S 165888
The structure of the trihemitone is **110592 S′ 118098 S 124416 S 131072.**
Fourth a.Phrygian

82944 TS 98304 T 110592 S′ 118098 S **124416 TS 147456** S′ 157464 S 165888
The structure of the trihemitone is **124416 S 131072 S′ 139968 S 147456.**
Second a.Hypophrygian

6. 165888 TS 196608 S′ 209952 S **221184 TS 262144** S′ 279936 S 294912 T 331776
221184 S′ 236196 S 248832 S 262144 is the structure of the trihemitone indicated.
Sixth d.Hypodorian

165888 TS 196608 S′ 209952 S 221184 T **248832 TS 294912** S 314928 S 331776
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fifth d.Dorian

165888 T **186624 TS 221184** S′ 236196 S *248832 TS 294912* S′ 314928 S 331776
186624 S 196608 S′ 209952 S 221184 is the structure of the indicated trihemitone sequence. *248832 S 262144 S′ 279936 S 294912* is the structure of the second trihemitone.
Third d.Mixolydian

165888 T 186624 S 196608 S′ **209952 TS 248832** S 262144 S′ *279936 TS 331776*
The structure of the first trihemitone is **209952 S 221184 S′ 236196 S 248832**. The structure of the second trihemitone is *279936 S 294912 S′ 314928 S 331776*.
Sixth a.Hypodorian

165888 TS 196608 S′ 209952 S **221184 TS 262144** T 294912 S′ 314928 S 331776
The structure of the trihemitone is **221184 S′ 236196 S 248832 S 262144.**
Fifth a.Phrygian

165888 TS 196608 T 221184 S′ 236196 S **248832 TS 294912** S′ 314928 S 331776
The structure of the trihemitone is 248832 S 262144 S′ 279936 S 294912.
Third a.Hypophrygian

7. d.Hypodorian, a.Hypodorian, and a.Phrygian die.
331776 TS 393216 S′ 419904 S 442368 T 497664 TS 589824 S′ 629856 S 663552
Sixth d.Dorian

331776 T **373248 TS 442368** S′ 472392 S 497664 TS 589824 S′ 629856 S 663552
373248 S 393216 S′ 419904 S 442368 is the structure of the indicated trihemitone.
Third d.Mixolydian

331776 TS 393216 T 442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552
Fourth a.Hypophrygian

8. 663552 TS 786432 S′ 839808 S 884736 T 995328 TS 1179648 S′ 1259712 S 1327104
Seventh d.Dorian

663552 T **746496 TS 884736** S′ 944784 S 995328 TS 1179648 S′ 1327104
The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
Fourth d.Mixolydian

663552 TS 786432 T 884736 S′ 944784 S 995328 TS 1179648 S′ 1259712 S 1327104
Fifth a.Hypophrygian

9. d.Dorian and a.Hypophrygian die.
1327104 T 1492992 TS 1769472 S′ 1889568 S 1990656 TS 2359296 S′ 2519424 S 2654208
Fifth d.Mixolydian

10. 2654208:5308416
No chromatic scale possibilities remain.

F. 3456 3888 4096 4608 5184 5832 6144 6912

Chains descending from 5832 belong to ζ-string (division by 2 leads ultimately to 729, the beginning of the diatonic ζ-string). Note that the ascending Mixolydian scale is the initiating pattern among the chromatic sequences of ζ-string.

1. 5832 S 6144 S′ 6561 TS 7776 S 8192 S′ 8748 TS 10368 T 11664
First a.Mixolydian

2. 11664 S 12288 S′ 13122 TS 15552 S 16384 S′ 17496 T 19683 TS 23328
First d.Hypophrygian
11664 S 12288 S′ 13122 TS 15552 S 16384 S′ **17496 TS 20736** T 23328
The structure of the trihemitone is **17496 S 18432 S′ 19683.**
Second a.Mixolydian
11664 S 12288 S′ 13122 TS 15552 T 17496 S 18432 S′ 19683 TS 23328
First a.Dorian

3. 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ 34992 T 39366 TS 46656
Second d.Hypophrygian
23328 S 24576 S′ 26244 TS 31104 S 32768 S′ **34992 TS 41472** T 46656
The structure of the trihemitone is **34992 S 36864 S′ 39366 S 41472.**
Third a.Mixolydian
23328 S 24576 S′ 26244 TS 31104 T 34992 S 36864 S′ 39366 TS 46656
Second a.Dorian

4. 46656 S 49152 S′ 52488 T **59049 TS 69984** S 73728 S 78732 TS 93312
59049 S 62208 S 65536 S′ 69984 is the structure of the indicated trihemitone.
First d. Phrygian
46656 S 49152 S′ **52488 TS 62208** S 65536 S′ 69984 T 78732 TS 93312
The structure of the **trihemitone** indicated is **52488 S 55296 S′ 59049 S 62208.**
Third d.Hypophrygian
46656 TS 55296 S′ 59049 S *62208 TS 73728* S′ 78732 S 82944 T 93312
The first trihemitone is structured thus: **46656 S 49152 S′ 52488 S 55296.**
62208 S 65536 S′ 69984 S 73728 is the structure of the second trihemitone.
First d.Hypodorian
46656 S 49152 S′ **52488 TS 62208** S 65536 S′ 69984 TS 82944 T 93312
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Fourth a.Mixolydian

46656 S 49152 S′ **52488 TS 62208** T 69984 S 73728 S′ 78732 TS 93312
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Third a.Dorian

46656 T 52488 S 55296 S′ **59049 TS 69984** S 73728 S′ 78732 TS 93312
The structure of the trihemitone is **59049 S 62208 S 65536 S′ 69984.**
First a.Hypodorian

5. 93312 S 98304 S′ 104976 T **118098 TS 139968** S 147456 S′ 157464 TS 186624
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
First d.Phrygian

93312 S 98304 S′ **104976 TS 124416** S 131072 S′ 139968 T 157464 TS 186624
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Fourth d.Hypophrygian

93312 TS 110592 S′ 118098 S *124416 TS 147456* S′ 157464 S 165888 T 186624
93312 S 98304 S′ 104976 S 110592 is the structure of the indicated trihemitone. *124416 S 131072 S′ 139968 S 147456* is the structure of the second trihemitone.
Second d.Hypodorian

93312 S 98304 S′ **104976 TS 124416** S 131072 S′ *139968 TS 165888* T 186624
The structure of the first trihemitone is **104976 S 110592 S′ 118098 S 124416**. The structure of the second trihemitone is *139968 S 147456 S′ 157464 S 165888*.
Fifth a.Mixolydian

93312 S 98304 S′ **104976 TS 124416** T 139968 S 147456 S′ 156464 TS 186624
The structure of the trihemitone is **104976 S 110592 S′ 118098 S 124416.**
Fourth a.Dorian

93312 T 104976 S 110592 S′ **118098 TS 139968** S 147456 S′ 157464 TS 186624
The structure of the trihemitone is 118098 S 124416 S 131072 S′ 139968.
Second a.Hypodorian

6. 186624 S 196608 S′ 209952 T **236196 TS 279936** S 294912 S 314928 TS 373248
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third descending Phrygian chromatic scale

186624 S 196608 S′ **209952 TS 248832** S 262144 S′ 279936 T 314928 TS 373248
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fifth d.Hypophrygian

186624 TS 221184 S′ 236196 S *248832 TS 294912* S′ 314928 S 331776 T 373298
186624 S 196608 S′ 209952 S 221184 is the structure of the indicated trihemitone. *248832 S 262144 S′ 279936 S 294912* is the structure of the second trihemitone.
Third d.Hypodorian

186624 S 196608 S′ **209952 TS 248832** S 262144 S′ *279936 TS 331776* T 373248
The structure of the first trihemitone is **209952 S 221184 S′ 236196 S 248832**. The structure of the second trihemitone is *279936 S 294912 S′ 314928 S 331776*.
Sixth a.Mixolydian

186624 S 196608 S′ **209952 TS 248832** T 279936 S 294912 S′ 314928 TS 373248
The structure of the trihemitone is **209952 S 221184 S′ 236196 S 248832.**
Fifth a.Dorian

186624 T 209952 S 221184 S′ **236196 TS 279936** S 294912 S′ 314928 TS 373248
The structure of the trihemitone is **236196 S 248832 S 262144 S′ 279936.**
Third a. Hypodorian

7. Descending Hypophrygian and ascending Mixolydian chromatic scales die.

373248 S 393216 S′ 419904 T 472392 TS 559872 S 589824 S 629856 TS 746496
Fourth d. Phrygian

373248 TS 442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 T 746496
373248 S 393216 S′ 419904 S 442368 is the structure of the indicated trihemitone.
Fourth d.Hypodorian

373248 S 393216 S′ **419904 TS 497664** T 529872 S 589824 S′ 629856 TS 746496
The structure of the trihemitone is **419904 S 442368 S′ 472392 S 497664.**
Sixth a. Dorian

373248 T 419904 S 442368 S′ 472392 TS 559872 S 589824 S′ 629856 TS 746496
Fourth a. Hypodorian

8. 746496 S 786432 S′ 839808 T 944784 TS 1119744 S 1179648 S 1259712 TS 1492992
Fifth d. Phrygian

746496 TS 884736 S′ 994784 S 995328 TS 1179648 S′ 1259712 S 1327104 T 1492992
The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
Fifth d.Hypodorian

746496 S 786432 S′ **839808 TS 995328** T 1119744 S 1179648 S′ 1259712 TS 14929912
The structure of the trihemitone is **839808 S 884736 S′ 944784 S 995328.**
Seventh a.Dorian

746496 T 839808 S 884736 S′ 944784 TS 1119744 S 1179648 S′ 1259712 TS 1492992
Fifth a.Hypodorian

9. Descending Phrygian and ascending Dorian chromatic scales die.

1492992 TS 1769472 S′ 1889568 S 1990656 TS 2359296 S′ 2519424 S 2654208 T 2985984
Sixth d. Hypodorian

1492992 T 1679616 S 1769472 S′ 1889568 TS 2239488 S 2359296 S′ 2519424 TS 2985984
Sixth a. Hypodorian

10. 2985984–5971968: No possible chromatic scales

G. 3456 3888 4096 4608 5184 5832 6144 6912

Chains descending from 6144 belong to Δ-string (division by 2 leads ultimately to 384, the beginning of the diatonic Δ-string). Note that the descending Hypophrygian and ascending Lydian scales are the initiating pattern among the chromatic sequences of Δ-string.

1. 6144 S′ 6561 S 6912 TS 8192 S′ 8748 S 9216 T 10368 TS 12288
First d. Hypophrygian
6144 S′ TS 7776 S 8192 S′ 8748 TS 10368 T 11664 S 12288
First a.Lydian

2. 12288 S′ 13122 S 13824 T **15552 TS 18432** S′ 19683 S 20736 TS 24576
The indicated trihemitone is structured thus: **15552 S 16384 S′ 17496 S 18432.**
First d. Phrygian
12288 S′ 13122 TS 15552 S 16384 S′ 17496 T 19683 TS 23328 S 24576
First d. Hypolydian
12288 S′ 13122 S 13824 TS 16384 S′ 17496 S 18432 T 20736 TS 24576
Second d.Hypophrygian
12288 S′ 13122 TS 15552 S′ 16384 S′ **17496 TS 20736** T 23328 S 24576
The structure of the trihemitone is **17496 S 18432 S′ 19683 S 20736.**
Second a. Lydian
12288 S′ 13122 TS 15552 T 17496 S 18432 S′ 19683 TS 23328 S 24576
First a. Hypolydian

3. 24576 S′ 26244 S 27648 T **31104 TS 36864** S′ 39366 S 41472 TS 49152
The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
Second d. Phrygian
24576 S′ 26244 TS 31104 S 32768 S′ 34992 T 39366 TS 46656S 49152
Second d. Hypolydian
24576 S′ 26244 S 27648 TS 32768 S′ 34992 S 36864 T 41472 TS 49152
Third d. Hypophrygian
24576 S′ 26244 TS 31104 S 32768 S′ **34992 TS 41472** T 46656 S 49152
The structure of the trihemitone is **34992 S 36864 S′ 39366 S 41472.**
Third a. Lydian

24576 S′ 26244 TS 31104 T 34992 S 36864 S′ 39366 TS 46656 S 49152
Second a. Hypolydian

24576 S′ 26244 S 27648 TS 32768 T 36864 S′ 39366 S 41472 TS 49152
First a. Dorian

4. 49152 S′ 52488 T **59049 TS 69984** S 73728 S′ 78732 TS 93312 S 98304
59049 S 62208 S 65536 S′ 69984 is the structure of the indicated trihemitone.
First d. Lydian

49152 S′ 52488 S 55296 T **62208 TS 73728** S′ 78732 S 82944 TS 98304
62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone.
Third d. Phrygian

49152 S′ **52488 TS 62208** S 65536 S′ 69984 T 78732 TS 93312 S 98304
The structure of the trihemitone indicated is **52488 S 55296 S′ 59049 S 62208.**
Third d. Hypolydian

49152 S′ 52488 S **55296 TS 65536** S′ 69984 S 73728 T 82944 TS 98304
The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
Fourth d. Hypophrygian

49152 S′ **52488 TS 62208** S 65536 S′ *69984 TS 82944* T 93312 S 98304
The structure of the first trihemitone is **52488 S 55296 S′ 59049 S 62208.**
The structure of the second trihemitone is *69984 S 73728 S′ 78732 S 82944*.
Fourth a. Lydian

49152 S′ **52488 TS 62208** T 69984 S 73728 S′ 78732 TS 93312 S 98304
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Third a. Hypolydian

49152 S′ 52488 S **55296 TS 65536** T 73728 S′ 78732 S 82944 TS 98304
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Second a. Dorian

49152 T 55296 S′ 59049 S **62208 TS 73728** S′ 78732 S 82944 TS 98304
Second a. Hypodorian

5. 98304 S′ 104976 T **118098 TS 139968** S 147456 S′ 157464 TS 186624 S 196608
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
Second d. Lydian

98304 S′ 104976 S 110592 T **124416 TS 147456** S′ 157464 S 165888 TS 196608
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Fourth d.Phrygian

98304 S′ **104976 TS 124416** S 131072 S′ 139968 T 157464 TS 186624 S 196608
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Fourth d. Hypolydian

98304 S′ 104976 S **110592 TS 131072** S′ 139968 S 147456 T 165888 TS 196608
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Fifth d. Hypophrygian

98304 S′ **104976 TS 124416** S 131072 S′ *139968 TS 165888* T 186624 S 196608
The first trihemitone sequence has the structure **104976 S 110592 S′ 118098 S 124416.** The second trihemitone sequence has the structure *139968 S 147456 S′ 157464 S 164888*.
Fifth a. Lydian

98304 S′ **104976 TS 124416** T 139968 S 147456 S′ 157464 TS 186624 S 196608
The first trihemitone sequence has the structure **104976 S 110592 S′ 118098 S 124416.**
Fourth a. Hypolydian

98304 S′ 104976 S **110592 TS 131072** T 147456 S′ 157464 S 165888 TS 196608
The structure of the trihemitone is **110592 S′ 118098 S 124416 S 131072.**
Third a. Dorian

98304 T 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 TS 196608
The structure of the trihemitone is **124416 S 131072 S 139968 S 147456.**
Third a. Hypodorian

6. 196608 S′ 209952 T **236196 TS 279936** S 294912 S 314928 TS 373248 S 393216
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third d.Lydian

196608 S′ 209952 S 221184 T **248832 TS 294912** S′ 314928 S 331776 TS 393216
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fifth d. Phrygian

196608 S′ **209952 TS 248832** S 262144 S′ 279936 T 314928 TS 373248 S 393216
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fifth d. Hypolydian

196608 S′ 209952 S **221184 TS 262144** S′ 279936 S 294912 T 331776 TS 393216
221184 S′ 236196 S 248832 S 262144 is the structure of the trihemitone indicated.
Sixth d. Hypophrygian

196608 S′ **209952 TS 248832** S 262144 S′ *279936 TS 331776* T 373248 S 393216
The structure of the first trihemitone is **209952 S 221184 S 236196 S 248832.**
The structure of the second trihemitone is *279936 S 294912 S′ 314928 S 331776*.
Sixth a. Lydian

196608 S′ 209952 S **221184 TS 262144** T 294912 S′ 314928 S 331776 TS 393216.
The structure of the trihemitone is **221184 S′ 236196 S 248832 S 262144.**
Fourth a. Dorian

196608 T 221184 S′ 236196 S **248832 TS 294912** S′ 314928 S 331776 TS 393216
The structure of the trihemitone is **248832 S 262144 S′ 279936 S 294912.**
Fourth a. Hypodorian

7. d.Hypolydian, d.Hypophrygian, a.Dorian, and a. Lydian patterns die.
   393216 S 419904 T 472392 TS 559872 S 589824 S′ 629856 TS 746496 S 786432
   Fourth d. Lydian
   393216 S′ 419904 S 442368 T 497664 TS 589824 S′ 629856 S 663552 TS 786432
   Sixth d. Phrygian
   393216 S′ 419904 TS 497664 T 559872 S 589824 S′ 629856 TS 746496 S 786432
   Sixth a.Hypolydian
   393216 T 442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 TS 786432
   Fifth a. Hypodorian

8. 786432: MISSING
   No chromatic scale possibilities remain.

3 Chromatic Scales Based on Chromatic Factors of 1719926784

3.1 *First Series*

Chains descending from 4374 belong to string CF1 (division by 2 leads ultimately to 2187, the beginning of the diatonic string CF1). Note that the descending Lydian and ascending Hypophrygian scales are the initiating patterns among the chromatic sequences of string CF1.

1. 2187: No chromatic scale is possible from this starting point.

2. 4374 S 4608 T 5184 TS 6144 S′ 6561 S 6912 TS 8192 S′ 8748
   First d. Lydian
   4374 TS 5184 T 5832 S 6144 S′ 6561 TS 7776 S 8192 S′ 8748
   First a.Hypophrygian

3. 8748 S 9216 T 10368 TS 12288 S′ 13122 S 13824 TS 16384 S′ 17496
   Second d. Lydian
   8748 TS 10368 T 11664 S 12288 S′ 13122 TS 15552 S 16384 S′ 17496
   Second a.Hypophrygian

4. 17496 T 19683 TS 24576 S′ 26244 S 27648 TS 32768 S′ 34992
   First d.Mixolydian
   17496 S 18432 T 20736 TS 24576 S′ 26244 S 27648 TS 32768 S′ 34992
   Third d.Lydian
   17496 TS 20736 T 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ 34992
   The structure of the trihemitone is **17496 S 18432 S′ 19683 S 20736.**
   Third a. Hypophrygian

17496 S 18432 S′ 19683 TS 23328 S 24576 S′ 26244 TS 31104 T 34992
First a. Mixolydian

5. 34992 T 39366 TS 46656 S 49152 S′ **52488 TS 62208** S 65536 S′ 69984.
The structure of the trihemitone indicated is **52488 S 55296 S′ 59049 S 62208.**
Second d.Mixolydian

34992 S 36864 T 41472 TS 49152 S′ 52488 S **55296 TS 65536** S′ 69984.
The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
Fourth d. Lydian

34992 S 36864 S′ 39366 TS 46656 S 49152 S′ 52488 T **59049 TS 69984**
The indicated trihemitone is structured thus: **59049 S 62208 S 65536 S′ 69984.**
First d. Hypophrygian

34992 TS 41472 T 46656 S 49152 S′ *52488 TS 62208* S 65536 S′ 69984
The structure of the first trihemitone is **34992 S 36864 S′ 39366 S 41472**. The structure of the second trihemitone is *52488 S 55296 S′ 59049 S 62208*.
Fourth a.Hypophrygian

34992 S 36864 S′ 39366 TS 46656 S 49152 S′ **52488 TS 62208** T 69984
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Second a.Mixolydian

34992 S 36684 S′ 39366 TS 46656 T 52488 S 55296 S′ 59049 TS 69984
The structure of the trihemitone is **59049 S 62208 S 65536 S′ 69984.**
First a.Dorian

6. 69984 T 78732 TS 93312 S 98304 S′ **104976 TS 124416** S 131072 S′ 139968
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Third d.Mixolydian

69984 S 73728 T 82944 TS 98304 S′ 104976 S **110592 TS 131072** S′ 139968
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Fifth d.Lydian

69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 T **118098 TS 139968.**
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
Second d. Hypophrygian

69984 TS 82944 T 93312 S 98304 S′ *104976 TS 124416* S 131072 S′ 139968
The structure of the first trihemitone is **69984 S73728 S′ 78732 S 82944**. The structure of the second trihemitone is *104976 S110592 S′ 118098 S 124416*.
Fifth a. Hypophrygian

69984 S 73728 S′ 78732 TS 93312 S 98304 S′ **104976 TS 124416** T 139968
The structure of the trihemitone is **104976 S110592 S′ 118098 S 124416.**
Third a.Mixolydian

69984 S 73728 S′ 78732 TS 93312 T 104976 S 110592 S′ **118098 TS 139968**
The structure of the trihemitone is **118098 S 124416 S 131072 S′ 139968.**
Second a.Dorian

7. 139968 T 157464 TS 186624 S 196608 S′ **209952 TS 248832** S 262144 S′ 279936
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fourth d.Mixolydian

139968 S 147456 T 165888 TS 196608 S′ 209952 S **221184 TS 262144** S′ 279936
221184 S′ 236196 S 248832 S 262144 is the structure of the trihemitone indicated.
Sixth d.Lydian

139968 S 147456 S 157464 TS 186624 S 196608 S′ 209952 T **236196 TS 279936**
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third d.Hypophrygian

139968 TS 165888 T 186624 S 196608 S′ *209952 TS 248832* S 262144 S′ 279936
The structure of the first trihemitone is **139968 S 147456 S′ 157464 S 165888**. The structure of the second trihemitone is *209952 S 221184 S′ 236196 S 248832*.
Sixth a.Hypophrygian

139968 S 147456 S′ 157464 TS 186624 S 196608 S′ **209952 TS 248832** T 279936
The structure of the trihemitone is **209952 S 221184 S′ 236196 S 248832.**
Fourth a.Mixolydian

139968 S 147456 S′ 157464 TS 186624 T 209952 S 221184 S′ **236196 TS 279936**
The structure of the trihemitone is **236196 S 248832 S 262144 S′ 279936.**
Third a.Dorian

8. d.Mixolydian, d.Lydian, and a.Hypophrygian die out.

279936 S 294912 S′ 314928 TS 373248 S 393216 S′ 419904 T 472392 TS 559872
Fourth d.Hypophrygian

279936 S 294912 S′ 314928 TS 373248 S 393216 S′ **419904 TS 497664** T 559872
The structure of the trihemitone is **419904 S 442368 S′ 472392 S 497664.**
Fifth a.Mixolydian

279936 S 294912 S′ 314928 TS 373248 T 419904 S 442368 S′ 472392 TS 559872
Fourth a.Dorian

9. 559872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808 T 944784 TS 1119744
Fifth d.Hypophrygian

559872 S 589824 S′ 629856 TS 746496 S 786432 S′ **839808 TS 995328** T 1119744
The structure of the trihemitone is **839808 S 884736 S′ 944784 S 995328.**
Sixth a.Mixolydian

559872 S 589824 S′ 629856 TS 746496 T 839808 S 889736 S′ 944784 TS 1119744
Fifth a.Dorian

10. The tenth chromatic octave loses the d.Hypophrygian, and a.Mixolydian patterns.
1119744 S 1179648 S′ 1259712 TS 1492992 T 1679616 S 1769472 S′ 1889568 TS 2239488
Sixth a.Dorian

11. 2239488 S 2359296 S′ 2519424 TS 2985984 T 3359232 S 3538944 S′ 3779136 TS 4478976
Seventh a.Dorian

3.2 *Second Series*

Chains descending from 6561 belong to string CF2. Note that the d. Hypolydian scale is the initiating pattern among the chromatic sequences of string CF2.

1. 6561 S 6912 TS 8192 S′ 8748 S 9216 T 10368 TS 12288 S 13122
First d.Hypolydian
6561 TS 7776 S 8192 S′ 8748 TS 10368 T 11664 S 12288 S′ 13122
First a.Phrygian

2. 13122 S 13824 T **15552 TS 18432** S′ 19683 S 20736 TS 24576 S 26244
The indicated trihemitone is structured thus: **15552 S 16584 S′ 17496 S 18432.**
First d.Lydian
13122 TS 15552 S 16384 S′ 17496 T 19683 TS 23328 S 24576 S 26244
First d.Dorian
13122 S 13824 TS 16384 S′ 17496 S 18432 T 20736 TS 24576 S 26244
Second d.Hypolydian
13122 TS 15552 S 16384 S′ **17496 TS 20736** T 23328 S 24576 S′ 26244
The structure of the trihemitone is **17496 S 18432 S′ 19683 S 20736.**
Second a.Phrygian
13122 TS 15552 T 17496 S 18432 S′ 19683 T 23328 S 24576 S′ 26244
First a.Hypophrygian
13122 S 13824 TS 16384 T 18432 S′ 19683 S 20736 TS 24576 S′ 26244
First a.Hypolydian

3. 26244 S 27648 T **31104 TS 36864** S′ 39366 S 41472 TS 49152 S′ 52488
The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
Second d.Lydian
26244 TS 31104 S 32768 S′ 34992 T 39366TS 46656 S 49152 S′ 52488
Second d.Dorian
26244 S 27648 TS 32768 S′ 34992 S 36864 T 41472 TS 49152 S 52488
Third d.Hypolydian

26244 TS 31104 S 32768 S′ **34992 TS 41472** T 46656 S 49152 S′ 52488
The structure of the trihemitone is **34992 S 36864 S′ 39366 S 41472.**
Third a.Phrygian

26244 TS 31104 T 34992 S 36864 S′ 39366 TS 46656 S 49152 S′ 52488
Second a.Hypophrygian

26244 S 27648 TS 32768 T 36864 S′ 39366 S 41472 TS 49152 S′ 52488
Second a.Hypolydian

4. 52488 T **59049 TS 69984** S 73728 S′ 78732 TS 93312 S 98304 S′ 104976
59049 S 62208 S 65536 S′ 69984 is the structure of the indicated trihemitone.
First d.Mixolydian

52488 S 55296 T **62208 TS 73728** S′ 78732 S 82944 TS 98304 S 104976
62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone.
Third d.Lydian

52488 TS 62208 S 65536 S′ 69984 T 78732 TS 93312 S 98304 S 104976
The structure of the trihemitone indicated is **52488 S 55296 S′ 59049 S 62208.**
Third d.Dorian

52488 S **55296 TS 65536** S′ 69984 S 73728 T 82944 TS 98304 S 104976
The indicated trihemitone is structured thus: **55296 S′ 59049 S 62208 S 65536.**
Fourth d.Hypolydian

52488 TS 62208 S 65536 S′ *69984 TS 82944* T 93312 S 98304 S′ 104976
The structure of the first trihemitone is **52488 S 55296 S′ 59049 S 62208**. The structure of the second trihemitone is *69984 S 73728 S′ 78732 S 82944*.
Fourth a.Phrygian

52488 TS 62208 T 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Third a.Hypophrygian

52488 S **55296 TS 65536** T 73728 S′ 78732 S 82944 TS 98304 S′ 104976
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Third a.Hypolydian

52488 S 55296 S′ **59049 TS 69984** S 73728 S′ 78732 TS 93312 T 104976
The structure of the trihemitone is **59049 S 62208 S 65536 S 69984.**
First a.Mixolydian

5. 104976 T **118098 TS 139968** S 147456 S′ 157464 TS 186624 S 196608 S 209952
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
Second d.Mixolydian

104976 S 110592 T **124416 TS 147456** S′ 157464 S 165888 TS 196608 S′ 209952
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Fourth d.Lydian

104976 TS 124416 S 131072 S′ 139968 T 157464 TS 186624 S 196608 S′ 209952
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Fourth d.Dorian

104976 S **110592 TS 131072** S′ 139968 S 147456 T 165888 TS 196608 S′ 209952
110592 S′ 118098 S 124416 S 131072 is the structure of the indicated trihemitone.
Fifth d.Hypolydian chromatic scale

104976 TS 124416 S 131072 S′ *139968 TS 165888* T 186624 S 196608 S′ 209952
The structure of the first trihemitone is **104976 S 110592 S′ 118098 S 124416**. The structure of the second trihemitone is *139968 S 147456 S′ 157464 S 165888*.
Fifth a.Phrygian

104976 TS 124416 T 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ 209952
The structure of the trihemitone is **104976 S 110592 S′ 118098 S 124416.**
Fourth a.Hypophrygian

104976 S **110592 TS 131072** T 147456 S′ 157464 S 165888 TS 196608 S′ 209952
The structure of the trihemitone is **110592 S′ 118098 S 124416 S 131072.**
Fourth a.Hypolydian

104976 S 110592 S′ **118098 TS 139968** S 147456 S′ 157464 TS 186624 T 209952
The structure of the trihemitone is **118098 S 124416 S 131072 S′ 139968.**
Second a.Mixolydian

6. 209952 T **236196 TS 279936** S 294912 S′ 314928 TS 373248 S 393216 S′ 419904
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third d.Mixolydian

209952 S 221184 T **248832 TS 294912** S′ 314928 S 331776 TS 393216 S′ 419904
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fifth d.Lydian

209952 TS 248832 S 262144 S′ 279936 T 314928 TS 373248 S 393216 S′ 419904
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fifth d.Dorian

209952 S **221184 TS 262144** S′ 279936 S 294912 T 331776 TS 393216 S′ 419904
221184 S′ 236196 S 248832 S 262144 is the structure of the trihemitone indicated.
Sixth d.Hypolydian

209952 TS 248832 S 262144 S′ *279936 TS 331776* T 373248 S 393216 S′ 419904
The structure of the first trihemitone is **209952 S 221184 S′ 236196 S 248832**. The structure of the second trihemitone is *279936 S 294912 S′ 314928 S 331776*.
Sixth a.Phrygian

209952 TS 248832 T 279936 S 294912 S′ 314928 TS 373248 S 393216 S′ 419904
The structure of the trihemitone is **209952 S 221184 S′ 236196 S 248832.**
Fifth a.Hypophrygian

209952 S **221184 TS 262144** T 294912 S′ 314928 S 331776 TS 393216 S′ 419904
The structure of the trihemitone is **221184 S′ 236196 S 248832 S 262144.**
Fifth a.Hypolydian

209952 S 221184 S′ **236196 TS 279936** S 294912 S′ 314928 TS 373248 T 419904
The structure of the trihemitone is **236196 S 248832 S 262144 S′ 279936.**
Third a.Mixolydian

7. d.Dorian, d.Hypolydian, a.Phrygian and a. Hypolydian die.
419904 T 472392 TS559872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808
Fourth d.Mixolydian

419904 S 442368 T 497664 TS 589824 S′ 629856 S 663552 TS 786432 S′ 839808
Sixth d.Lydian

419904 S 442368 S′ 472392 TS 559872 S 589824 S′ 629856 TS 746496 T 839808
Fourth a.Mixolydian

419904 TS 497664 T 559872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808
The structure of the trihemitone is **419904 S 442368 S′ 472392 S 497664.**
Sixth a.Hypophrygian

8. d.Mixolydian, d.Lydian, and a.Hypophrygian die.
839808 S 884736 S′ 944784 TS 1119744 S 1179648 S′ 1259712 TS 1492992 T 1679616
Fifth a.Mixolydian

9. 1679616 S 1769472 S′ 1889568 TS 2239488 S 2359296 S′ 2519424 TS 2985984 T 3359232
Sixth a.Mixolydian

10. 3359232–6718464
No chromatic octaves remain.

4 Chromatic Scales Based on Chromatic Nonfactors of 1719926784

4.1 *First Series*

Chains descending from 19683 belong to string CNF1. Note that the d.Hypolydian, d.Hypodorian, a.Phrygian, and a.Lydian chromatic patterns are the initiating patterns of CNF1.

1. 19683 S 20736 TS 24576 S′ 26244 S 27648 T **31104 TS 36864** S′ 39366
The indicated trihemitone is structured thus: **31104 S 32768 S′ 34992 S 36864.**
First d.Hypolydian

19683 TS 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ 34992 T 39366
First d.Hypodorian
19683 S 20736 TS 24576 S′ 26244 S 27648 TS 32768 T 36864 S′ 39366
First a.Lydian
19683 TS 23328 S 24576 S′ 26244 TS 31104 T 34992 S 36864 S′ 39366
First a.Phrygian

2. 39366 S 41472 T **46656 TS 55296** S′ 59049 S *62208 TS 73728* S′ 78732
The first trihemitone is structured thus: **46656 S 49152 S′ 52488 S 55296**; *62208 S 65536 S′69984 S 73728* is the structure of the second trihemitone.
First d.Lydian
39366 TS 46656 S 49152 S′ 52488 T **59049 TS 69984** S 73728 S′ 78732
59049 S 62208 S 65536 S′ 69984 is the structure of the indicated trihemitone.
First d.Dorian
39366 S 41472 TS 49152 S′ 52488 S 55296 T **62208 TS 73728** S′ 78732
62208 S 65536 S′ 69984 S 73728 is the structure of the indicated trihemitone.
Second d.Hypolydian
39366 TS 46656 S 49152 S′ **52488 TS 62208** S 65536 S′ 69984 T 78732
The structure of the trihemitone indicated is **52488 S 55296 S′ 59049 S 62208.**
Second d.Hypodorian
39366 S 41472 TS 49152 S′ 52488 S **55296 TS 65536** T 73728 S′ 78732
The structure of the trihemitone is **55296 S′ 59049 S 62208 S 65536.**
Second a.Lydian
39366 TS 46656 S 49152 S′ **52488 TS 62208** T 69984 S 73728 S′ 78732
The structure of the trihemitone is **52488 S 55296 S′ 59049 S 62208.**
Second a.Phrygian
39366 S 41472 TS 49152 T 55296 S′ 59049 S **62208 TS 73728** S′ 78732
The structure of the trihemitone is **62208 S 65526 S′ 69984 S 73728.**
First a.Hypolydian
39366 TS 46656 T 52488 S 55296 S′ **59049 TS 69984** S 73728 S 78732
The structure of the trihemitone is **59049 S 62208 S 65536 S′ 69984.**
First a.Hypophrygian

3. 78732 S 82944 T **93312 TS 110592** S′ 118098 S 124416 TS 147456 S′ 157464
93312 S 98304 S′ 104976 S 110592 is the structure of the first trihemitone.
124416 S 131072 S′ 139968 S 147456 is the structure of the second trihemitone.
Second d.Lydian
78732 TS 93312 S 98304 S′ 104976 T **118098 TS 139968** S 147456 S′ 157464
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
Second d.Dorian

78732 S 82944 TS 98304 S′ 104976 S 110592 T **124416 TS 147456** S′ 157464
124416 S 131072 S′ 139968 S 147456 is the structure of the indicated trihemitone.
Third d.Hypolydian

78732 TS 93312 S 98304 S′ **104976 TS 124416** S 131072 S′ 139968 T 157464
The structure of the trihemitone indicated is **104976 S 110592 S′ 118098 S 124416.**
Third d.Hypodorian

78732 S 82944 TS 98304 S′ 104976 S **110592 TS 131072** T 147456 S′ 157464
The structure of the trihemitone is **110592 S′ 118098 S 124416 S 131072.**
Third a.Lydian

78732 TS 93312 S 98304 S′ 104976 TS 124416 T 139968 S 147456 S′ 157464
The structure of the trihemitone is **104976 S 110592 S′ 118098 S 124416.**
Third a.Phrygian

78732 S 82944 TS 98304 T 110592 S′ 118098 S **124416 TS 147456** S′ 157464
The structure of the trihemitone is **124416 S 131072 S′ 139968 S 147456.**
Second a.Hypolydian

78732 TS 93312 T 104976 S 110592 S′ **118098 TS 139968** S 147456 S′ 157464
The structure of the trihemitone is **118098 S 124416 S 131072 S′ 139968.**
Second a.Hypophrygian

4. 157464 S 165888 T **186624 TS 221184** S′ 236196 S *248832 TS 294912* S 314928
186624 S 196608 S′ 209952 S 221184 is the structure of the first trihemitone.
248832 S 262144 S′ 279936 S 294912 is the structure of the second trihemitone.
Third d.Lydian

157464 TS 186624 S 196608 S′ 209952 T **236196 TS 279936** S 294912 S′ 314928
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third d.Dorian

157464 S 165888 TS 196608 S′ 209952 S 221184 T **248832 TS 294912** S′ 314928
248832 S 262144 S′ 279936 S 294912 is the structure of the indicated trihemitone.
Fourth d.Hypolydian

157464 TS 186624 S 196608 S′ **209952 TS 248832** S 262144 S′ 279936T 314928
209952 S 221184 S′ 236196 S 248832 is the structure of the indicated trihemitone.
Fourth d.Hypodorian

157464 S 165888 TS 196608 S′ 209952 S 221184 TS 262144 T 294912 S′ 314928
The structure of the trihemitone is 221184 S′ 236196 S 248832 S 262144.
Fourth a.Lydian

157464 TS 186624 S 196608 S′ 209952 TS 248832 T 279936 S 294912 S′ 314928
The structure of the trihemitone was 209952 S 221184 S′ 236196 S 248832.
Fourth a.Phrygian

157464 S 165888 TS 196608 T 221184 S′ 236196 S **248832 TS 294912** S′ 314928
The structure of the trihemitone is **248832 S 262144 S′ 279936 S 294912.**
Third a.Hypolydian

157464 TS 186624 T 209952 S 221184 S′ **236196 TS 279936** S 294912 S′ 314928
The structure of the trihemitone is 236196 S 248832 S 262144 S′ 279936.
Third a.Hypophrygian

5. Descending Hypodorian and ascending Lydian die.
314928 S 331776 T **373248 TS 442368** S′ 472392 S 497664 TS 589824 S′ 629856
373248 S 393216 S′ 419904 S 442368 is the structure of the trihemitone.
Fourth d.Lydian
314928 TS 373248 S 393216 S′ 419904 T 472392 TS 559872 S 589824 S′ 629856
Fourth d.Dorian
314928 S 331776 TS 393216 S′ 419904 S 442368 T 497664 TS 589824 S′ 629856
Fifth d.Hypolydian
314928 TS 373248 S 393216 S′ **419904 TS 497664** T 559872 S 589824 S′ 629856
The structure of the trihemitone is **419904 S 442368 S′ 472392 S 497664.**
Fifth a.Phrygian
314928 S 331776 TS 393216 T 442368 S′ 472392 S 497664 TS 589824 S′ 629856
Fourth a.Hypolydian
314928 TS 373248 T 419904 S 442368 S′ 472392 TS 559872 S 589824 S′ 629856
Fourth a.Hypophrygian

6. 629856 S 663552 T **746496 TS 884736** S′ 944784 S 995328 TS 1179648 S′ 1259712
The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
Fifth d.Lydian
629856 TS 746496 S 786432 S′ 839808 T 944784 TS 1119744 S 1179648 S′ 1259712
Fifth d.Dorian
629856 S 663552 TS 786432 S′ 839808 S 884736 T 995328 TS 1179698 S 1259712
Sixth d.Hypolydian
629856 TS 746496 S 786432 S′ **839808 TS 995328** T 1119744 S 11179648 S′ 1259712
The structure of the trihemitone is **839808 S 884736 S′ 944784 S 995328.**
Sixth a.Phrygian
629856 S 663552 TS 786432 T 884736 S′ 944784 S 995328 TS 1179648 S′ 1259712
Fifth a.Hypolydian
629856 TS 746496 T 839808 S 884736 S′ 944784 TS 1119744 S 1179648 S′ 1259712
Fifth a.Hypophrygian

7. d.Dorian, d.Hypolydian, d.Hypophrygian, a.Phrygian, and a.Hypolydian die.
1259712 S 1327104 T 1492992 TS 1769472 S′ 1889568 S 1990656 TS 2359296 S 2519424
Sixth d.Lydian

1259712 TS 1492992 T 1679616 S 1769472 S′ 1889568 TS 2239488 S 2359296 S′ 2519424
Sixth a.Hypophrygian

8. 2519424–5038848
No further chromatic scales are possible.

4.2 *Second Series*

Chains descending from 59049 belong to CNF2-string. Note that the d.Hypolydian, d.Hypodorian, a.Phrygian and a.Lydian chromatic patterns are the initiating patterns among the chromatic sequences of CNF2-string.

1. 59049 S **62208 TS 73728** S′ 78732 S 82944 T 93312 TS 110592 S′ 118098
62208 S 65536 S′ 69984 S 73728 is the structure of the first trihemitone.
scale; *93312 S 98304 S′ 104976 S 110592* is the structure of the second trihemitone.
First d.Hypolydian

59049 TS 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 T 118098
59049 S 62208 S 65536 S′ 69984 is the structure of the indicated trihemitone.
First d.Hypodorian

59049 S 62208 TS 73728 S′ 78732 S 82944 TS 98304 T 110592 S′ 118098
The structure of the trihemitone is **62208 S 65536 S′ 69984 S 73728.**
First a.Lydian

59049 TS 69984 S 73728 S′ 78732 TS 93312 T 104976 S 110592 S′ 118098
The structure of the trihemitone is **59049 S 62208 S 65536 S′ 69984.**
First a.Phrygian

2. 118098 S **124416 TS 147456** S′ 157464 S 165888 T *186624 TS 221184* S′ 236196
124416 S 131072 S′ 139968 S 147456 is the structure of the first trihemitone.
186624 S 196608 S′ 209952 S 221184 is the structure of the second trihemitone.
Second d.Hypolydian

118098 TS 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ 209952 T 236196
118098 S 124416 S 131072 S′ 139968 is the structure of the indicated trihemitone.
Second d.Hypodorian

118098 S **124416 TS 147456** S′ 157464 S 165888 TS 196608 T 221184 S′ 236196
The structure of the trihemitone is **124416 S 131072 S′ 139968 S 147456.**
Second a.Lydian

118098 TS 139968 S 147456 S′ 157464 TS 186624 T 209952 S 221184 S′ 236196
The structure of the trihemitone is **118098 S 124416 S 131072 S′ 139968.**
Second a.Phrygian

3. 236196 S **248832 TS 294912** S′ 314928 S331776 T 373298 TS 442368 S′ 472392
248832 S 262144 S′ 279936 S 294912 is the structure of the trihemitone.
Second d.Hypolydian
236196 TS 279936 S 294912 S′ 314928 TS 373248 S 393216 S 419904 T 472392
236196 S 248832 S 262144 S′ 279936 is the structure of the indicated trihemitone.
Third d.Hypodorian
236196 S **248832 TS 294912** S′ 314928 S 331776 TS 393216 T 442368 S′ 472392
The structure of the trihemitone is **248132 S 262144 S′ 279936 S 294912.**
Third a.Lydian
236196 TS 279936 S 294912 S′ 314928 TS 373248 T 419904 S 442368 S′ 472392
The structure of the trihemitone is **236196 S 248832 S 262144 S′ 279936 S 294912.**
Third a.Phrygian

4. 472392 S 497664 TS 589824 S′ 629856 S 663552 T **746496 TS 884736** S′ 944784
The structure of the trihemitone indicated is **746496 S 786432 S′ 839808 S 884736.**
Fourth d.Hypolydian
472392 TS 559872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808 T 944784
Fourth d.Hypodorian
472392 S 497664 TS 589924 S′ 629856 S 663552 TS 786432 T 884736 S′ 944784
Fourth a.Lydian
472392 TS 559872 S 589824 S′ 629856 TS 746496 T 839808 S 884736 S′ 944784
Fourth a.Phrygian

5. d.Hypodorian and a.Lydian die.
944784 S 995328 TS 1179648 S′ 1259712 S 1327104 T 1492992 TS 1769472 S′ 1889568
Fifth d.Hypolydian
944784 TS 1119744 S 1179648 S′ 1259712 TS 1492992 T 1679616 S 1769472 S′ 1889568
Fifth a.Phrygian

6. 1889568 S 1990656 TS 2359296 S′ 2519424 S 2654208 T 2985984 TS 3538944 S′ 3779136
Sixth d.Hypolydian
1889568 TS 2239488 S 23959296 S′ 2519424 TS 2985984 T 3359232 S 3538949 S′ 3779136
Sixth a.Phrygian

7. 3779136–7558272
No chromatic scale possibilities remain.

APPENDIX 9

Specification of Trihemitones and Chromatic Scales in Which They Manifest

For the purposes of the study and this Appendix 9, a "trihemitone" is an undivided TS sequence, capable of being subdivided into three semitones, comprised of S′ and S elements. There are forty-eight trihemitones in the table below. The forty-eight represent the twelve putative semitones implicit in an octave, progressed to four; therefore, the total number follows a generative grammar ruled by the Decad. The trihemitones come as four types: S S′ S, always color coded in orange below; S′ S S′, always color coded in blue below; S′ S S, always color coded in lavender below; and S S S′, always color coded in olive below.

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set classified by trihemitone usage

Trihemitone	Structure	
1. 1944–2304	1944 S 2048 S′ 2187 S 2304 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
2. 3888–4608	3888 S 4096 S′ 4374 S 4608 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
3. 5832–6912	5832 S 6144 S′ 6561 S 6912 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
4. 7776–9216	7776 S 8192 S′ 8748 S 9216 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
5. 11664–13824	11664 S 12288 S′ 13122 S 13824 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
6. 15552–18432	15552 S 16384 S′ 17496 S 18432 S S′ S	a. 13824 T 15552 TS 18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 (d.Mixolydian) b. 15552 TS 18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 T 31104 (d.Hypodorian)

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TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		c. 10368 TS 12288 S′ 13122 S 13824 T 15552 TS 18432 S′ 19683 S 20736 (d.Dorian)
		d. 12288 S′ 13122 S 13824 T 15552 TS 18432 S′ 19683 S 20736 TS 24576 (d.Phrygian)
		e. 13122 S 13824 T 15552 TS 18432 S′ 19683 S 20736 TS 24576 S′ 26244 (d.Lydian)
7. 16384–19683	16384 S′ 17496 S 18432 S′ 19683 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
8. 17496–20736	17496 S 18432 S′ 19683 S 20736 S S′ S	a. 15552 S 16384 S′ 17496 TS 20736 T 23328 S 24576 S′ 26244 TS 31104 (a.Dorian)
		b. 16384 S′ 17496 TS 20736 T 23328 S 24576 S′ 26244 TS 31104 S 32768 (a.Hypolydian)
		c. 10368 T 11664 S 12288 S′ 13122 TS 15552 S 16384 S′ 17496 TS 20736 (a.Hypodorian)
		d. 11664 S 12288 S′ 13122 TS 15552 S 16384 S′ 17496 TS 20736 T 23328 (a.Mixolydian)
		e. 12288 S′ 13122 TS 15552 S 16384 S′ 17496 TS 20736 T 23328 S 24576 (a.Lydian)
		f. 17496 TS 20736 T 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ 34992 (a.Hypophrygian)
		g. 13122 TS 15552 S 16384 S′ 17496 TS 20736 T 23328 S 24576 S′ 26244 (a.Phrygian)
9. 23328–27648	23328 S 24576 S′ 26244 S 27648 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
10. 31104–36864	31104 S 32768 S′ 34992 S 36864 S S′ S	a. 27648 T 31104 TS 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 (d.Mixolydian)
		b. 31104 TS 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 T 62208 (d.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		c. 31104 TS 36864 S′ 39366 S 41472 T 46656 TS 55296 S′ 59049 S 62208 (d.Dorian)
		d. 18432 S′ 19683 S 20736 TS 24576 S′ 26244 S 27648 T 31104 TS 36864 (d.Hypophrygian)
		e. 20736 TS 24576 S′ 26244 S 27648 T 31104 TS 36864 S′ 39366 S 41472 (d.Dorian)
		f. 24576 S′ 26244 S 27648 T 31104 TS 36864 S′ 39366 S 41472 TS 49152 (d.Phrygian)
		g. 26244 S 27648 T 31104 TS 36864 S′ 39366 S 41472 TS 49152 S′ 52488 (d.Lydian)
		h. 19683 S 20736 TS 24576 S′ 26244 S 27648 T 31104 TS 36864 S′ 39366 (d.Hypolydian)
		i. 31104 TS 36864 S′ 39366 S 41472 TS 49152 T 55296 S′ 59049 S 62208 (a.Phrygian)
11. 32768–39366	32768 S′ 34992 S 36864 S′ 39366 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
12. 34992–41472	34992 S 36864 S′ 39366 S 41472 S S′ S	a. 31104 S 32768 S′ 34992 TS 41472 T 46656 S 49152 S′ 52488 TS 62208 (a.Dorian)
		b. 32768 S′ 34992 TS 41472 T 46656 S 49152 S′ 52488 TS 62208 S 65536 (a.Hypolydian)
		c. 20736 T 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ 34992 TS 41472 (a.Hypodorian)
		d. 23328 S 24576 S′ 26244 TS 31104 S 32768 S′ 34992 TS 41472 T 46656 (a.Mixolydian)
		e. 24576 S′ 26244 TS 31104 S 32768 S′ 34992 TS 41472 T 46656 S 49152 (a.Lydian)
		f. 34992 TS 41472 T 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 (a.Hypophrygian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		g. 26244 TS 31104 S 32768 S′ 34992 TS 41472 T 46656 S 49152 S′ 52488 (a.Phrygian)
13. 46656–55296	46656 S 49152 S′ 52488 S 55296 S S′ S	a. 31104 TS 36864 S′ 39366 S 41472 T 46656 TS 55296 S′ 59049 S 62208 (d.Dorian)
		b. 36864 S′ 39366 S 41472 T 46656 TS 55296 S′ 59049 S 62208 TS 73728 (d.Phrygian)
		c. 41472 T 46656 TS 55296 S′ 59049 S 62208 TS 73728 S 78732 S 82944 (d.Mixolydian)
		d. 46656 TS 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 T 93312 (d.Hypodorian)
		e. 39366 S 41472 T 46656 TS 55296 S′ 59049 S 62208 TS 73728 S′ 78732 (d.Lydian)
14. 49152–59049	49152 S′ 52488 S 55296 S′ 59049 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
15. 52488–62208	52488 S 55296 S′ 59049 S 62208 S S′ S	a. 31104 S 32768 S′ 34992 T 39366 TS 46656 S 49152 S′ 52488 TS 62208 (d.Phrygian)
		b. 32768 S′ 34992 T 39366 TS 46656 S 49152 S′ 52488 TS 62208 S 65536 (d.Lydian)
		c. 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 T 78732 TS 93312 (d.Hypophrygian)
		d. 49152 S′ 52488 TS 62208 S 65536 S′ 69984 T 78732 TS 93312 S 98304 (d.Hypolydian)
		e. 34992 T 39366 TS 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 (d.Mixolydian)
		f. 52488 TS 62208 S 65536 S′ 69984 T 78732 TS 93312 S 98304 S 104976 (d.Dorian)
		g. 39366 TS 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 T 78732 (d.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure
	h. 31104 S 32768 S′ 34992 TS 41472 T 46656 S 49152 S′ 52488 TS 62208 (a.Dorian)
	i. 31104 T 34992 S 36864 S′ 39366 TS 46656 S 49152 S′ 52488 TS 62208 (a.Hypodorian)
	j. 32768 S′ 34992 TS 41472 T 46656 S 49152 S′ 52488 TS 62208 S 65536 (a.Hypolydian)
	k. 36864 S′ 39366 TS 46656 S 49152 S′ 52488 TS 62208 T 69984 S 73728 (a.Lydian)
	l. 41472 T 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 TS 82944 (a.Hypodorian)
	m. 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 TS 82944 T 93312 (a.Mixolydian)
	n. 49152 S′ 52488 TS 62208 S 65536 S′ 69984 TS 82944 T 93312 S 98304 (a.Lydian)
	o. 49152 S′ 52488 TS 62208 T 69984 S 73728 S′ 78732 TS 93312 S 98304 (a.Hypolydian)
	p. 34992 TS 41472 T 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 (a.Hypophrygian)
	q. 34992 S 36864 S′ 39366 TS 46656 S 49152 S′ 52488 TS 62208 T 69984 (a.Mixolydian)
	r. 52488 TS 62208 S 65536 S′ 69984 TS 82944 T 93312 S 98304 S′ 104976 (a.Phrygian)
	s. 52488 TS 62208 T 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 (a.Hypophrygian)
	t. 39366 TS 46656 S 49152 S′ 52488 TS 62208 T 69984 S 73728 S′ 78732 (a.Phrygian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
16. 55296–65536	55296 S′ 59049 S 62208 S 65536 S′ S S	a. 55296 TS 65536 S′ 69984 S 73728 T 82944 TS 98304 S′ 104976 S 110592 (d.Dorian) b. 32768 S′ 34992 S 36864 T 41472 TS 49152 S′ 52488 S 55296 TS 65536 (d.Phrygian) c. 36864 T 41472 TS 49152 S′ 52488 S 55296 TS 65536 S′ 69984 S 73728 (d.Mixolydian) d. 41472 TS 49152 S′ 52488 S 55296 TS 65536 S′ 69984 S 73728 T 82944 (d.Hypodorian) e. 49152 S′ 52488 S 55296 TS 65536 S′ 69984 S 73728 T 82944 TS 98304 (d.Hypophrygian) f. 34992 S 36864 T 41472 TS 49152 S′ 52488 S 55296 TS 65536 S′ 69984 (d.Lydian) g. 52488 S 55296 TS 65536 S′ 69984 S 73728 T 82944 TS 98304 S 104976 (d.Hypolydian) h. 55296 TS 65536 T 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 (a.Hypophrygian) i. 32768 T 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 TS 65536 (a.Hypodorian) j. 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 TS 65536 T 73728 (a.Mixolydian) k. 41472 TS 49152 S′ 52488 S 55296 TS 65536 T 73728 S′ 78732 S 82944 (a.Phrygian) l. 49152 S′ 52488 S 55296 TS 65536 T 73728 S′ 78732 S 82944 TS 98304 (a.Dorian) m. 52488 S 55296 TS 65536 T 73728 S′ 78732 S 82944 TS 98304 S′ 104976 (a.Hypolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		n. 39366 S 41472 TS 49152 S′ 52488 S 55296 TS 65536 T 73728 S′ 78732 (a.Lydian)
17. 59049–69984	59049 S 62208 S 65536 S′ 69984 S S S′	a. 36864 S′ 39366 TS 46656 S 49152 S′ 52488 T 59049 TS 69984 S 73728 (d.Hypolydian)
		b. 46656 S 49152 S′ 52488 T 59049 TS 69984 S 73728 S 78732 TS 93312 (d.Phrygian)
		c. 49152 S′ 52488 T 59049 TS 69984 S 73728 S′ 78732 TS 93312 S 98304 (d.Lydian)
		d. 52488 T 59049 TS 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 (d.Mixolydian)
		e. 39366 TS 46656 S 49152 S′ 52488 T 59049 TS 69984 S 73728 S′ 78732 (d.Dorian)
		f. 59049 TS 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 T 118098 (d.Hypodorian)
		g. 34992 S 36864 S′ 39366 TS 46656 S 49152 S′ 52488 T 59049 TS 69984 (d.Hypophrygian)
		h. 55296 S′ 59049 TS 69984 S 73728 S′ 78732 TS 93312 T 104976 S 110592 (a.Lydian)
		i. 36864 S′ 36366 TS 46656 T 52488 S 55296 S′ 59049 TS 69984 S 73728 (a.Hypolydian)
		j. 46656 T 52488 S 55296 S′ 59049 TS 69984 S 73728 S′ 78732 TS 93312 (a.Hypodorian)
		k. 34992 S 36684 S′ 39366 TS 46656 T 52488 S 55296 S′ 59049 TS 69984 (a.Dorian)
		l. 52488 S 55296 S′ 59049 TS 69984 S 73728 S′ 78732 TS 93312 T 104976 (a.Mixolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		m. 36366 TS 46656 T 52488 S 55296 S′ 59049 TS 69984 S 73728 S 78732 (a.Hypophrygian)
		n. 59049 TS 69984 S 73728 S′ 78732 TS 93312 T 104976 S 110592 S′ 118098 (a.Phrygian)
18. 62208–73728	62208 S 65536 S′ 69984 S 73728 S S′ S	a. 55296 T 62208 TS 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 (d.Mixolydian)
		b. 62208 TS 73728 S′ 78732 S 82944 T 93312 TS 110592 S′ 118098 S 124416 (d.Dorian)
		c. 36864 S′ 39366 S 41472 T 46656 TS 55296 S′ 59049 S 62208 TS 73728 (d.Phrygian)
		d. 36864 S′ 39366 S 41472 TS 49152 S′ 52488 S 55296 T 62208 TS 73728 (d.Hypophrygian)
		e. 41472 TS 49152 S′ 52488 S 55296 T 62208 TS 73728 S′ 78732 S 82944 (d.Dorian)
		f. 41472 T 46656 TS 55296 S′ 59049 S 62208 TS 73728 S 78732 S 82944 (d.Mixolydian)
		g. 46656 TS 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 T 93312 (d.Hypodorian)
		h. 49152 S′ 52488 S 55296 T 62208 TS 73728 S′ 78732 S 82944 TS 98304 (d.Phrygian)
		i. 52488 S 55296 T 62208 TS 73728 S′ 78732 S 82944 TS 98304 S 104976 (d.Lydian)
		j. 39366 S 41472 T 46656 TS 55296 S′ 59049 S 62208 TS 73728 S′ 78732 (d.Lydian)
		k. 59049 S 62208 TS 73728 S′ 78732 S 82944 T 93312 TS 110592 S′ 118098 (d.Hypolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		l. 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 T 93312 TS 110592 (d.Hypophrygian)
		m. 39366 S 41472 TS 49152 S′ 52488 S 55296 T 62208 TS 73728 S′ 78732 (d.Hypolydian)
		n. 62208 TS 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 T 124416 (d.Hypodorian)
		o. 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 TS 98304 T 110592 (a.Mixolydian)
		p. 62208 TS 73728 S′ 78732 S 82944 TS 98304 T 110592 S′ 118098 S 124416 (a.Phrygian)
		q. 36864 S′ 39366 S 41472 TS 49152 T 55296 S′ 59049 S 62208 TS 73728 (a.Dorian)
		r. 41472 TS 49152 T 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 (a.Hypophrygian)
		s. 49152 T 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 TS 98304 (a.Hypodorian)
		t. 39366 S 41472 TS 49152 T 55296 S′ 59049 S 62208 TS 73728 S′ 78732 (a.Hypolydian)
		u. 59049 S 62208 TS 73728 S′ 78732 S 82944 TS 98304 T 110592 S′ 118098 (a.Lydian)
19. 65536–78732	65536 S′ 69984 S 73728 S′ 78732 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
20. 69984–82944	69984 S 73728 S′ 78732 S 82944 S S′ S	a. 62208 S 65536 S′ 69984 TS 82944 T 93312 S 98304 S′ 104976 TS 124416 (a.Dorian)
		b. 65536 S′ 69984 TS 82944 T 93312 S 98304 S′ 104976 TS 124416 S 131072 (a.Hypolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		c. 41472 T 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 TS 82944 (a.Hypodorian)
		d. 46656 S 49152 S′ 52488 TS 62208 S 65536 S′ 69984 TS 82944 T 93312 (a.Mixolydian)
		e. 49152 S′ 52488 TS 62208 S 65536 S′ 69984 TS 82944 T 93312 S 98304 (a.Lydian)
		f. 69984 TS 82944 T 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 (a.Hypophrygian)
		g. 52488 TS 62208 S 65536 S′ 69984 TS 82944 T 93312 S 98304 S′ 104976 (a.Phrygian)
21. 93312–110592	93312 S 98304 S′ 104976 S 110592 S S′ S	a. 55296 S′ 59049 S 62208 TS 73728 S′ 78732 S 82944 T 93312 TS 110592 (d.Hypophrygian)
		b. 62208 TS 73728 S′78732 S 82944 T 93312 TS 110592 S′ 118098 S 124416 (d.Dorian)
		c. 82944 T 93312 TS 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 (d.Mixolydian)
		d. 93312 TS 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 T 186624 (d.Hypodorian)
		e. 78732 S 82944 T 93312 TS 110592 S′ 118098 S 124416 TS 147456 S′ 157464 (d.Lydian)
		f. 59049 S 62208 TS 73728 S′ 78732 S 82944 T 93312 TS 110592 S′ 118098 (d.Hypolydian)
		g. 73728 S′ 78732 S 82944 T 93312 TS 110592 S′ 118098 S 124416 TS 147456 (d.Phrygian)
22. 98304–118098	98304 S′ 104976 S 110592 S′ 118098 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
23. 104976–124416	104976 S 110592 S′ 118098 S 124416 S S′ S	a. 62208 S 65536 S′ 69984 T 78732 TS 93312 S 98304 S′ 104976 TS 124416 (d.Phrygian)
		b. 65536 S′ 69984 T 78732 TS 93312 S 98304 S′ 104976 TS 124416 S 131072 (d.Lydian)
		c. 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 T 157464 TS 186624 (d.Hypophrygian)
		d. 98304 S′ 104976 TS 124416 S 131072 S′ 139968 T 157464 TS 186624 S 196608 (d.Hypolydian)
		e. 104976 TS 124416 S 131072 S′ 139968 T 157464 TS 186624 S 196608 S′ 209952 (d.Dorian)
		f. 78732 TS 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 T 157464 (d.Hypodorian)
		g. 69984 T 78732 TS 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 (d.Mixolydian)
		h. 62208 S 65536 S′ 69984 TS 82944 T 93312 S 98304 S′ 104976 TS 124416 (a.Dorian)
		i. 62208 T 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 TS 124416 (a.Hypodorian)
		j. 65536 S′ 69984 TS 82944 T 93312 S 98304 S′ 104976 TS 124416 S 131072 (a.Hypolydian)
		k. 73728 S′ 78732 TS 93312 S 98304 S′ 104976 TS 124416 T 139968 S 147456 (a.Lydian)
		l. 82944 T 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 TS 165888 (a.Hypodorian)
		m. 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 TS 165888 T 186624 (a.Mixolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		n. 93312 S 98304 S′ 104976 TS 124416 T 139968 S 147456 S′ 157464 TS 186624 (a.Dorian)
		o. 98304 S′ 104976 TS 124416 S 131072 S′ 139968 TS 165888 T 186624 S 196608 (a.Lydian)
		p. 98304 S′ 104976 TS 124416 T 139968 S 147456 S′ 157464 TS 186624 S 196608 (a.Hypolydian)
		q. 69984 TS 82944 T 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 (a.Hypophrygian)
		r. 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 TS 124416 T 139968 (a.Mixolydian)
		s. 104976 TS 124416 S 131072 S′ 139968 TS 165888 T 186624 S 196608 S′ 209952 (a.Phrygian)
		t. 104976 TS 124416 T 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ 209952 (a.Hypophrygian)
		u. 78732 TS 93312 S 98304 S′ 104976 TS 124416 T 139968 S 147456 S′ 157464 (a.Phrygian)
24. 110592–131072	110592 S′ 118098 S 124416 S 131072 S′ S S	a. 110592 TS 131072 S′ 139968 S 147456 T 165888 TS 196608 S′ 209952 S 221184 (d.Dorian)
		b. 65536 S′ 69984 S 73728 T 82944 TS 98304 S′ 104976 S 110592 TS 131072 (d.Phrygian)
		c. 73728 T 82944 TS 98304 S′ 104976 S 110592 TS 131072 S′ 139968 S 147456 (d.Mixolydian)
		d. 82944 TS 98304 S′ 104976 S 110592 TS 131072 S′ 139968 S 147456 T 165888 (d.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		e. 98304 S′ 104976 S 110592 TS 131072 S′ 139968 S 147456 T 165888 TS 196608 (d.Hypophrygian)
		f. 69984 S 73728 T 82944 TS 98304 S′ 104976 S 110592 TS 131072 S′ 139968 (d.Lydian)
		g. 104976 S 110592 TS 131072 S′ 139968 S 147456 T 165888 TS 196608 S′ 209952 (d.Hypolydian)
		h. 110592 TS 131072 T 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 (a.Hypophrygian)
		i. 65536 T 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 TS 131072 (a.Hypodorian)
		j. 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 TS 131072 T 147456 (a.Mixolydian)
		k. 82944 TS 98304 S′ 104976 S 110592 TS 131072 T 147456 S′ 157464 S 165888 (a.Phrygian)
		l. 104976 S 110592 TS 131072 T 147456 S′ 157464 S 165888 TS 196608 S′ 209952 (a.Hypolydian)
		m. 78732 S 82944 TS 98304 S′104976 S 110592 TS 131072 T 147456 S′ 157464 (a.Lydian)
25. 118098–139968	118098 S 124416 S 131072 S′ 139968 S S S′	a. 73728 S′ 78732 TS 93312 S 98304 S′ 104976 T 118098 TS 139968 S 147456 (d.Hypolydian)
		b. 93312 S 98304 S′ 104976 T 118098 TS 139968 S 147456 S′ 157464 TS 186624 (d.Phrygian)
		c. 98304 S′ 104976 T 118098 TS 139968 S 147456 S′ 157464 TS 186624 S 196608 (d.Lydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure
	d. 69984 S 73728 S′ 78732 TS 93312 S 98304 S′ 104976 T 118098 TS 139968 (d.Hypophrygian)
	e. 104976 T 118098 TS 139968 S 147456 S′ 157464 TS 186624 S 196608 S 209952 (d.Mixolydian)
	f. 78732 TS 93312 S 98304 S′ 104976 T 118098 TS 139968 S 147456 S′ 157464 (d.Dorian)
	g. 118098 TS 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ 209952 T 236196 (d.Hypodorian)
	h. 110592 S′ 118098 TS 139968 S 147456 S′ 157464 TS 186624 T 209952 S 221184 (a.Lydian)
	i. 73728 S′ 78732 TS 93312 T 104976 S 110592 S′ 118098 TS 139968 S 147456 (a.Hypolydian)
	j. 93312 T 104976 S 110592 S′ 118098 TS 139968 S 147456 S′ 157464 TS 186624 (a.Hypodorian)
	k. 98304 S′ 104976 S 110592 TS 131072 T 147456 S′ 157464 S 165888 TS 196608 (a.Dorian)
	l. 69984 S 73728 S′ 78732 TS 93312 T 104976 S 110592 S′ 118098 TS 139968 (a.Dorian)
	m. 104976 S 110592 S′ 118098 TS 139968 S 147456 S′ 157464 TS 186624 T 209952 (a.Mixolydian)
	n. 78732 TS 93312 T 104976 S 110592 S′ 118098 TS 139968 S 147456 S′ 157464 (a.Hypophrygian)
	o. 118098 TS 139968 S 147456 S′ 157464 TS 186624 T 209952 S 221184 S′ 236196 (a.Phrygian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
26. 124416–147456	124416 S 131072 S′ 139968 S 147456 S S′ S	a. 110592 T 124416 TS 147456 S′ 157464 S 165888 TS 196608 S 209952 S 221184 (d.Mixolydian) b. 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 T 186624 TS 221184 (d.Hypophrygian) c. 124416 TS 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 T 248832 (d.Hypodorian) d. 124416 TS 147456 S′ 157464 S 165888 T 186624 TS 221184 S′ 236196 S 248832 (d.Dorian) e. 73728 S′ 78732 S 82944 T 93312 TS 110592 S′ 118098 S 124416 TS 147456 (d.Phrygian) f. 73728 S′ 78732 S 82944 TS 98304 S′ 104976 S 110592 T 124416 TS 147456 (d.Hypophrygian) g. 82944 TS 98304 S′ 104976 S 110592 T 124416 TS 147456 S′ 157464 S 165888 (d.Dorian) h. 82944 T 93312 TS 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 (d.Mixolydian) i. 93312 TS 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 T 186624 (d.Hypodorian) j. 98304 S′ 104976 S 110592 T 124416 TS 147456 S′ 157464 S 165888 TS 196608 (d.Phrygian) k. 104976 S 110592 T 124416 TS 147456 S′ 157464 S 165888 TS 196608 S′ 209952 (d.Lydian) l. 78732 S 82944 T 93312 TS 110592 S′ 118098 S 124416 TS 147456 S′ 157464 (d.Lydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		m. 78732 S 82944 TS 98304 S′ 104976 S 110592 T 124416 TS 147456 S′ 157464 (d.Hypolydian)
		n. 118098 S 124416 TS 147456 S′ 157464 S 165888 T 186624 TS 221184 S′ 236196 (d.Hypolydian)
		o. 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 TS 196608 T 221184 (a.Mixolydian)
		p. 124416 TS 147456 S′ 157464 S 165888 TS 196608 T 221184 S′ 236196 S 248832 (a.Phrygian)
		q. 73728 S′ 78732 S 82944 TS 98304 T 110592 S′ 118098 S 124416 TS 147456 (a.Dorian)
		r. 82944 TS 98304 T 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 (a.Hypophrygian)
		s. 98304 T 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 TS 196608 (a.Hypodorian)
		t. 78732 S 82944 TS 98304 T 110592 S′ 118098 S 124416 TS 147456 S′ 157464 (a.Hypolydian)
		u. 118098 S 124416 TS 147456 S′ 157464 S 165888 TS 196608 T 221184 S′ 236196 (a.Lydian)
27. 131072–157464	131072 S′ 139968 S 147456 S′ 157464 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
28. 139968–165888	139968 S 147456 S′ 157464 S 165888 S S′ S	a. 124416 S 131072 S′ 139968 TS 165888 T 186624 S 196608 S′ 209952 TS 248832 (a.Dorian)
		b. 131072 S′ 139968 TS 165888 T 186624 S 196608 S′ 209952 TS 248832 S 262144 (a.Hypolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		c. 82944 T 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 TS 165888 (a.Hypodorian)
		d. 93312 S 98304 S′ 104976 TS 124416 S 131072 S′ 139968 TS 165888 T 186624 (a.Mixolydian)
		e. 98304 S′ 104976 TS 124416 S 131072 S′ 139968 TS 165888 T 186624 S 196608 (a.Lydian)
		f. 139968 TS 165888 T 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 (a.Hypophrygian)
		g. 104976 TS 124416 S 131072 S′ 139968 TS 165888 T 186624 S 196608 S′ 209952 (a.Phrygian)
29. 186624–221184	186624 S 196608 S′ 209952 S 221184 S S′ S	a. 110592 S′ 118098 S 124416 TS 147456 S′ 157464 S 165888 T 186624 TS 221184 (d.Hypophrygian)
		b. 124416 TS 147456 S′ 157464 S 165888 T 186624 TS 221184 S′ 236196 S 248832 (d.Dorian)
		c. 147456 S′ 157464 S 165888 T 186624 TS 221184 S′ 236196 S 248832 TS 294912 (d.Phrygian)
		d. 165888 T 186624 TS 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 (d.Mixolydian)
		e. 186624 TS 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 T 373298 (d.Hypodorian)
		f. 157464 S 165888 T 186624 TS 221184 S′ 236196 S 248832 TS 294912 S 314928 (d.Lydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		g. 118098 S 124416 TS 147456 S′ 157464 S 165888 T 186624 TS 221184 S′ 236196 (d.Hypolydian)
30. 196608–236196	196608 S′ 209952 S 221184 S′ 236196 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
31. 209952–248832	209952 S 221184 S′ 236196 S 248832 S S′ S	a. 124416 S 131072 S′ 139968 T 157464 TS 186624 S 196608 S′ 209952 TS 248832 (d.Phrygian)
		b. 131072 S′ 139968 T 157464 TS 186624 S 196608 S′ 209952 TS 248832 S 262144 (d.Lydian)
		c. 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 T 314928 TS 373248 (d.Hypophrygian)
		d. 196608 S′ 209952 TS 248832 S 262144 S′ 279936 T 314928 TS 373248 S 393216 (d.Hypolydian)
		e. 139968 T 157464 TS 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 (d.Mixolydian)
		f. 209952 TS 248832 S 262144 S′ 279936 T 314928 TS 373248 S 393216 S′ 419904 (d.Dorian)
		g. 157464 TS 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 T 314928 (d.Hypodorian)
		h. 124416 S 131072 S′ 139968 TS 165888 T 186624 S 196608 S′ 209952 TS 248832 (a.Dorian)
		i. 124416 T 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ 209952 TS 248832 (a.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure
	j. 131072 S′ 139968 TS 165888 T 186624 S 196608 S′ 209952 TS 248832 S 262144 (a.Hypolydian)
	k. 147456 S′ 157464 TS 186624 S 196608 S′ 209952 TS 248832 T 279936 S 294912 (a.Lydian)
	l. 165888 T 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 TS 331776 (a.Hypodorian)
	m. 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 TS 331776 T 373248 (a.Mixolydian)
	n. 186624 S 196608 S′ 209952 TS 248832 T 279936 S 294912 S′ 314928 TS 373248 (a.Dorian)
	o. 196608 S′ 209952 TS 248832 S 262144 S′ 279936 TS 331776 T 373248 S 393216 (a.Lydian)
	p. 196608 S′ 209952 TS 248832 T 279936 S 294912 S′ 314928 TS 373248 S 393216 (a.Hypolydian)
	q. 139968 TS 165888 T 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 (a.Hypophrygian)
	r. 139968 S 147456 S′ 157464 TS 186624 S 196608 S′ 209952 TS 248832 T 279936 (a.Mixolydian)
	s. 209952 TS 248832 S 262144 S′ 279936 TS 331776 T 373248 S 393216 S′ 419904 (a.Phrygian)
	t. 209952 TS 248832 T 279936 S 294912 S′ 314928 TS 373248 S 393216 S′ 419904 (a.Hypophrygian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		u. 157464 TS 186624 S 196608 S′ 209952 TS 248832 T 279936 S 294912 S′ 314928 (a.Phrygian)
32. 221184–262144	221184 S′ 236196 S 248832 S 262144 S′ S S	a. 221184 TS 262144 S′ 279936 S 294912 T 331776 TS 393216 S′ 419904 S 442368 (d.Dorian)
		b. 131072 S′ 139968 S 147456 T 165888 TS 196608 S′ 209952 S 221184 TS 262144 (d.Phrygian)
		c. 147456 T 165888 TS 196608 S′ 209952 S 221184 TS 262144 S′ 279936 S 294912 (d.Mixolydian)
		d. 165888 TS 196608 S′ 209952 S 221184 TS 262144 S′ 279936 S 294912 T 331776 (d.Hypodorian)
		e. 196608 S′ 209952 S 221184 TS 262144 S′ 279936 S 294912 T 331776 TS 393216 (d.Hypophrygian)
		f. 139968 S 147456 T 165888 TS 196608 S′ 209952 S 221184 TS 262144 S′ 279936 (d.Lydian)
		g. 209952 S 221184 TS 262144 S′ 279936 S 294912 T 331776 TS 393216 S′ 419904 (d.Hypolydian)
		h. 221184 TS 262144 T 294912 S′ 314928 S 331776 TS 393216 S′ 419904 S 442368 (a.Hypophrygian)
		i. 131072 T 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 TS 262144 (a.Hypodorian)
		j. 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 TS 262144 T 294912 (a.Mixolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		k. 165888 TS 196608 S′ 209952 S 221184 TS 262144 T 294912 S′ 314928 S 331776 (a.Phrygian)
		l. 196608 S′ 209952 S 221184 TS 262144 T 294912 S′ 314928 S 331776 TS 393216 (a.Dorian)
		m. 209952 S 221184 TS 262144 T 294912 S′ 314928 S 331776 TS 393216 S′ 419904 (a.Hypolydian)
		n. 157464 S 165888 TS 196608 S′ 209952 S 221184 TS 262144 T 294912 S′ 314928 (a.Lydian)
33. 236196–279936	236196 S 248832 S 262144 S′ 279936 S S S′	a. 147456 S′ 157464 TS 186624 S 196608 S′ 209952 T 236196 TS 279936 S 294912 (d.Hypolydian)
		b. 186624 S 196608 S′ 209952 T 236196 TS 279936 S 294912 S 314928 TS 373248 (d.Phrygian)
		c. 196608 S′ 209952 T 236196 TS 279936 S 294912 S 314928 TS 373248 S 393216 (d.Lydian)
		d. 139968 S 147456 S 157464 TS 186624 S 196608 S′ 209952 T 236196 TS 279936 (d.Hypophrygian)
		e. 209952 T 236196 TS 279936 S 294912 S′ 314928 TS 373248 S 393216 S′ 419904 (d.Mixolydian)
		f. 157464 TS 186624 S 196608 S′ 209952 T 236196 TS 279936 S 294912 S′ 314928 (d.Dorian)
		g. 236196 TS 279936 S 294912 S′ 314928 TS 373248 S 393216 S 419904 T 472392 (d.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		h. 221184 S′ 236196 TS 279936 S 294912 S′ 314928 TS 373248 T 419904 S 442368 (a.Lydian)
		i. 147456 S′ 157464 TS 186624 T 209952 S 221184 S′ 236196 TS 279936 S 294912 (a.Hypolydian)
		j. 186624 T 209952 S 221184 S′ 236196 TS 279936 S 294912 S′ 314928 TS 373248 (a.Hypodorian)
		k. 139968 S 147456 S′ 157464 TS 186624 T 209952 S 221184 S′ 236196 TS 279936 (a.Dorian)
		l. 209952 S 221184 S′ 236196 TS 279936 S 294912 S′ 314928 TS 373248 T 419904 (a.Mixolydian)
		m. 157464 TS 186624 T 209952 S 221184 S′ 236196 TS 279936 S 294912 S′ 314928 (a.Hypophrygian)
		n. 236196 TS 279936 S 294912 S′ 314928 TS 373248 T 419904 S 442368 S′ 472392 (a.Phrygian)
34. 248832–294912	248832 S 262144 S′ 279936 S 294912 S S′ S	a. 221184 T 248832 TS 294912 S′ 314928 S 331776 TS 393216 S′ 419904 S 442368 (d.Mixolydian)
		b. 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 T 373248 TS 442368 (d.Hypophrygian)
		c. 248832 TS 294912 S′ 314928 S 331776 TS 393216 S′ 419904 S 442368 T 497664 (d.Hypodorian)
		d. 248832 TS 294912 S′ 314928 S 331776 T 373248 TS 442368 S′ 472392 S 497664 (d.Dorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure
	e. 147456 S′ 157464 S 165888 T 186624 TS 221184 S′ 236196 S 248832 TS 294912 (d.Phrygian)
	f. 147456 S′ 157464 S 165888 TS 196608 S′ 209952 S 221184 T 248832 TS 294912 (d.Hypophrygian)
	g. 165888 TS 196608 S′ 209952 S 221184 T 248832 TS 294912 S 314928 S 331776 (d.Dorian)
	h. 165888 T 186624 TS 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 (d.Mixolydian)
	i. 186624 TS 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 T 373298 (d.Hypodorian)
	j. 196608 S′ 209952 S 221184 T 248832 TS 294912 S′ 314928 S 331776 TS 393216 (d.Phrygian)
	k. 209952 S 221184 T 248832 TS 294912 S′ 314928 S 331776 TS 393216 S′ 419904 (d.Lydian)
	l. 157464 S 165888 T 186624 TS 221184 S′ 236196 S 248832 TS 294912 S 314928 (d.Lydian)
	m. 157464 S 165888 TS 196608 S′ 209952 S 221184 T 248832 TS 294912 S′ 314928 (d.Hypolydian)
	n. 236196 S 248832 TS 294912 S′ 314928 S 331776 T 373298 TS 442368 S′ 472392 (d.Hypolydian)
	o. 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 TS 393216 T 442368 (a.Mixolydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		p. 248832 TS 294912 S′ 314928 S 331776 TS 393216 T 442368 S′ 472392 S 497664 (a.Phrygian)
		q. 147456 S′ 157464 S 165888 TS 196608 T 221184 S′ 236196 S 248832 TS 294912 (a.Dorian)
		r. 165888 TS 196608 T 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 (a.Hypophrygian)
		s. 196608 T 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 TS 393216 (a.Hypodorian)
		t. 157464 S 165888 TS 196608 T 221184 S′ 236196 S 248832 TS 294912 S′ 314928 (a.Hypolydian)
		u. 236196 S 248832 TS 294912 S′ 314928 S 331776 TS 393216 T 442368 S′ 472392 (a.Lydian)
35. 262144–314928	262144 S′ 279936 S 294912 S′ 314928 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
36. 279936–331776	279936 S 294912 S′ 314928 S 331776 S S′ S	a. 248832 S 262144 S′ 279936 TS 331776 T 373248 S 393216 S′ 419904 TS 497664 (a.Dorian)
		b. 165888 T 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 TS 331776 (a.Hypodorian)
		c. 186624 S 196608 S′ 209952 TS 248832 S 262144 S′ 279936 TS 331776 T 373248 (a.Mixolydian)
		d. 196608 S′ 209952 TS 248832 S 262144 S′ 279936 TS 331776 T 373248 S 393216 (a.Lydian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		e. 209952 TS 248832 S 262144 S′ 279936 TS 331776 T 373248 S 393216 S′ 419904 (a.Phrygian)
37. 373248–442368	373248 S 393216 S′ 419904 S 442368 S S′ S	a. 221184 S′ 236196 S 248832 TS 294912 S′ 314928 S 331776 T 373248 TS 442368 (d.Hypophrygian)
		b. 248832 TS 294912 S′ 314928 S 331776 T 373248 TS 442368 S′ 472392 S 497664 (d.Dorian)
		c. 294912 S′ 314928 S 331776 T 373248 TS 442368 S′ 472392 S 497664 TS 589824 (d.Phrygian)
		d. 331776 T 373248 TS 442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 (d.Mixolydian)
		e. 373248 TS 442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 T 746496 (d.Hypodorian)
		f. 314928 S 331776 T 373248 TS 442368 S′ 472392 S 497664 TS 589824 S′ 629856 (d.Lydian)
		g. 236196 S 248832 TS 294912 S′ 314928 S 331776 T 373298 TS 442368 S′ 472392 (d.Hypolydian)
38. 393216–472392	393216 S′ 419904 S 442368 S′ 472392 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
39. 419904–497664	419904 S 442368 S′ 472392 S 497664 S S′ S	a. 248832 S 262144 S′ 279936 T 314928 TS 373248 S 393216 S′ 419904 TS 497664 (d.Phrygian)
		b. 248832 S 262144 S′ 279936 TS 331776 T 373248 S 393216 S′ 419904 TS 497664 (a.Dorian)
		c. 248832 T 279936 S 294912 S′ 314928 TS 373248 S 393216 S′ 419904 TS 497664 (a.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		d. 294912 S′ 314928 TS 373248 S 393216 S′ 419904 TS 497664 T 559872 S 589824 (a.Lydian)
		e. 373248 S 393216 S′ 419904 TS 497664 T 529872 S 589824 S′ 629856 TS 746496 (a.Dorian)
		f. 279936 S 294912 S′ 314928 TS 373248 S 393216 S′ 419904 TS 497664 T 559872 (a.Mixolydian)
		g. 419904 TS 497664 T 558872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808 (a.Hypophrygian)
		h. 314928 TS 373248 S 393216 S′ 419904 TS 497664 T 559872 S 589824 S′ 629856 (a.Phrygian)
40. 559872–663552	559872 S 589824 S′ 629856 S 663552 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
41. 746496–884736	746496 S 786432 S′ 839808 S 884736 S S′ S	a. 442368 S′ 472392 S 497664 TS 589824 S′ 629856 S 663552 T 746496 TS 884736 (d.Hypophrygian)
		b. 497664 TS 589824 S′ 629856 S 663552 T 746496 TS 884736 S′ 944784 S 995328 (d.Dorian)
		c. 589824 S′ 629856 S 663552 T 746496 TS 884736 S′ 944784 S 995328 TS 1179648 (d.Phrygian)
		d. 663552 T 746496 TS 884736 S′ 944784 S 995328 TS 1179648 S′ 1259712 S 1327104 (d.Mixolydian)
		e. 746496 TS 884736 S′ 994784 S 995328 TS 1179648 S′ 1259712 S 1327104 T 1492992 (d.Hypodorian)

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
		f. 629856 S 663552 T 746496 TS 884736 S′ 944784 S 995328 TS 1179648 S′ 1259712 (d.Lydian)
		g. 472392 S 497664 TS 589824 S′ 629856 S 663552 T 746496 TS 884736 S′ 944784 (d.Hypolydian)
42. 786432–944784	786432 S′ 839808 S 884736 S′ 944784 S′ S S′	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
43. 839808–995328	839808 S 884736 S′ 944784 S 995328 S S′ S	a. 497664 T 559872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808 TS 995328 (a.Hypodorian)
		b. 589824 S′ 629856 TS 746496 S 786432 S′ 839808 TS 995328 T 1119749 S 1179648 (a.Lydian)
		c. 746496 S 786432 S′ 839808 TS 995328 T 1119744 S 1179648 S′ 1259712 TS 1492992 (a.Dorian)
		d. 559872 S 589824 S′ 629856 TS 746496 S 786432 S′ 839808 TS 995328 T 1119744 (a.Mixolydian)
		e. 629856 TS 746496 S 786432 S′ 839808 TS 995328 T 1119744 S 11179648 S′ 1259712 (a.Phrygian)
44. 1119744–1327104	1119744 S 1179648 S′ 1259712 S 1327104 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
45. 1679616–1990656	1679616 S 1769472 S′ 1889568 S 1990656 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
46. 2239488–2654208	2239488 S 2359296 S′ 2519424 S 2654208 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.
47. 3359232–3981312	3359232 S 3538944 S′ 3779136 S 3981312 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.

TABLE 57 Catalogue of chromatic scales of the *Timaeus* number set (*cont.*)

Trihemitone	Structure	
48. 6718464–7962624	6718464 S 7077888 S′ 7558272 S 7962624 S S′ S	No chromatic scale among the *Timaeus* numbers uses the trihemitone indicated.

As Appendix 8, "Chromatic Scale Tables," shows, there are 385 chromatic scales, in all, in the *Timaeus* number set. Two hundred fifty-six of them (256) use trihemitones. One hundred twenty-nine (129) do not. The ratio of 385/256 is an approximation of the 3/2 ratio. The ratio 256/129 is an approximation of the double ratio; and the ratio 385/129 is an approximation of the triple ratio.

There are three hundred two (302) usages of trihemitones in the chromatic scales of the *Timaeus* set in total. The number of total usages of trihemitones is not the same as the number of chromatic scales using trihemitones because some chromatic scales use two trihemitones. There are two hundred eighteen (218) usages of the S S′ S type; forty-three (43) of the S S S′ type; forty-one (41) of the S′ S S type; and zero (0) of the S′ S S′ type. All scales with two trihemitones have trihemitones only of the type S S′ S.

The four recurring trihemitone patterns in the chromatic scales of the *Timaeus* number set are indicated as follows in Table 58 below: S S′ S is designated "O" for "one" *apotomē* in the center; S′ S S′ is designated "B" for one *apotomē* on both ends; S′ S S is "P" for one *apotomē* preceding, and S S S′ is designated "F" for one *apotomē* following. These four sequences arise in the *Timaeus* number set according to the order set forth in Table 58, as one can verify by a comparison with Table 57.

If one proceeds simultaneously down Table 58 from the first trihemitone, in the first column, and up the table from the last trihemitone, in the second column, making pairs, one finds that the semitone patterns of trihemitones in Table 58 are almost perfectly symmetrical. The pairs (P.16, F.33) and (F.17, P.32), as well as (P.24, F.25) disrupt the symmetry slightly with the placement of the single S′ at the opposite ends of their patterns.

There are thirty-two (32) "O" patterns; three (3) "F" patterns; three (3) "P" patterns; and ten "B" patterns. The sterile patterns of the "B" type are very symmetrically distributed in the trihemitone inventory of Table 58. Table 59 charts the distribution. Trihemitone usages are also symmetrically distributed around patterns of the "P" type, as indicated in Table 60. The distribution of trihemitone usages around patterns of the "F" type is symmetrically distributed, too, if one counts them somewhat differently from patterns of the "P" type, as Table 61 shows.

TABLE 58 Inventory of scales using trihemitones of each type

1.	S S′ S	O	Sterile	25.	S S S′	F	15 Scales
2.	S S′ S	O	Sterile	26.	S S′ S	O	21 Scales
3.	S S′ S	O	Sterile	27.	S′ S S′	B	Sterile
4.	S S′ S	O	Sterile	28.	S S′ S	O	7 Scales
5.	S S′ S	O	Sterile	29.	S S′ S	O	7 Scales
6.	S S′ S	O	5 Scales	30.	S′ S S′	B	Sterile
7.	S′ S S′	B	Sterile	31.	S S′ S	O	21 Scales
8.	S S′ S	O	7 Scales	32.	S′ S S	P	14 Scales
9.	S S′ S	O	Sterile	33.	S S S′	F	14 Scales
10.	S S′ S	O	9 Scales	34.	S S′ S	O	21 Scales
11.	S′ S S′	B	Sterile	35.	S′ S S′	B	Sterile
12.	S S′ S	O	7 Scales	36.	S S′ S	O	5 Scales
13.	S S′ S	O	5 Scales	37.	S S′ S	O	7 Scales
14.	S′ S S′	B	Sterile	38.	S′ S S′	B	Sterile
15.	S S′ S	O	20 Scales	39.	S S′ S	O	8 Scales
16.	S′ S S	P	14 Scales	40.	S S′ S	O	Sterile
17.	S S S′	F	14 Scales	41.	S S′ S	O	7 Scales
18.	S S′ S	O	21 Scales	42.	S′ S S′	B	Sterile
19.	S′ S S′	B	Sterile	43.	S S′ S	O	5 Scales
20.	S S′ S	O	7 Scales	44.	S S′ S	O	Sterile
21.	S S′ S	O	7 Scales	45.	S S′ S	O	Sterile
22.	S′ S S′	B	Sterile	46.	S S′ S	O	Sterile
23.	S S′ S	O	21 Scales	47.	S S′ S	O	Sterile
24.	S′ S S	P	13 Scales	48.	S S′ S	O	Sterile

TABLE 59 Distribution of trihemitone usages around patterns of the "B" type

Intervals	Up to B.7	B.7–B.11	B.11–B.14	B.14–B.19	B.19–B.22	B.22–B.27	B.27–B.30	B.30–B.35	B.35–B.38	B.38–B.42	After B.42
Trihemitone usages	5	16	12	69	14	70	14	70	12	15	5

TABLE 60 Distribution of trihemitone usages around patterns of the "P" type, where "P" rows end every interval except the last

Intervals	Up through P.16	F.17–P.24	F.25–P.32	F.33 to end
Trihemitone usages	67	83	85	67

TABLE 61 Distribution of trihemitone usages around patterns of the "F" type, where "F" rows begin every interval except the first

Intervals	Up through P.16	F.17–P.24	F.25–P.32	F.33 to end
Trihemitone usages	67	83	85	67

Note that Table 61 counts the distribution of trihemitone usages around patterns of the "F" type as not including F.17 in the first column. The count stops right before F.17 and the next column stops right before F.25 and the next right before F.33. Each column, other than the first, in other words, counts an "F" row as the beginning of an interval. Table 60 counts each "P" occurrence as the end of an interval, except for the last.

If one, instead, counts the distribution of trihemitone usages around "P" patterns, such that "P" patterns are treated as the beginnings, instead of the ends of intervals, and counts the "F" patterns, so that "F" patterns are treated as the ends of intervals instead of the beginnings, then one gets an identical asymmetrical distribution of trihemitone occurrences around patterns of both types, as Tables 62 and 63, below, indicate, but in reverse order.

TABLE 62 Distribution of trihemitone usages around patterns of the "P" type, where "P" rows begin every interval except the first

Intervals	Up through row 15	P.16–row 23	P.24–row 31	P.32 to end
Trihemitone usages	53	84	84	81

TABLE 63 Distribution of trihemitone usages around patterns of the "F" type, where "F" rows end every interval except the last

Intervals	Up through row F.17	Row 18–F.25	Row 36–F.33	Row 34 to end
Trihemitone usages	81	84	84	53

The reasons for the symmetry of trihemitone usages around the three least common patterns, separately considered, are not straightforward. It is an interesting feature of chromatic development in the *Timaeus* number set and warrants further research. The fact that there are four patterns among the trihemitones is not surprising, given the fourfold pattern of generation that governs the *Timaeus*, generally.

APPENDIX 10

Enharmonic Scale Tables

1 Preliminary Notes

1. TT in the catalogue below indicates a ditone.
2. Numbers in orange are not *Timaeus* numbers but values derived from *Timaeus* numbers to define quarter tone intervals.

2 Enharmonic Scale Tables

2.1 *Enharmonic Scales Based on* d.Dorian/a.Lydian *Model Diatonic Octave Scale Numbers*

2.1.1 384 432 486 512 576 648 729 768

This section articulates the Δ-string enharmonic octave chains beginning from 384.

1. 384 TT 486 QT 499 QT 512 T 576 TT 729 S (No even split) 768

   No enharmonic QT fourth: just descending Dorian ST enharmonic fourth

   First Enharmonic QT and ST Fourth
   Because interval splits evenly:
   d.Dorian; also fits pattern
   for the d.Hypo-
   Dorian and a.Phrygian.

   If one argues that TT S was a sufficient scale for an enharmonic fourth in the oldest variety of enharmonic scale, then the scale from 384 to 768 already articulates a d.Dorian and a.Phrygian ST enharmonic octave as follows.[1]

   a. 384 TT 486 S 512 T 576 TT 729 S 768
      d.Dorian ST enharmonic scale

1 See Hagel, *Ancient Greek Music*, 135 and 413–419 for his discussion of the evolution from a semitone enharmonic to a QT enharmonic in the second half of the fifth century and related matters.

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b. 384 TT 486 S 512 TT 648 T 729 S 768
a.Phrygian ST enharmonic scale

2. a. 768 TT 972 QT 998 QT 1024 T 1152 TT 1458 QT 1497 QT 1536
First d.Dorian QT enharmonic scale
b. 768 TT 972 QT 998 Q 1024 TT 1296 T 1458 QT 1497 QT 1536
First a.Phrygian QT enharmonic scale

3. a. 1536 TT 1944 QT 1996 QT 2048 T 2304 TT 2916 QT 2994 QT 3072
Second d.Dorian QT enharmonic scale
b. 1536 TT 1944 1996 QT 2048 TT 2592 T 2916 QT 2994 QT 3072
Second a.Phrygian QT enharmonic scale
c. 1536 TT 1944 T 2187 S 2304 TT 2916 S 3072
a.Hypophrygian ST enharmonic scale
d. 1536 T 1728 TT 2187 S 2304 TT 2916 S 3072
d.Mixolydian ST enharmonic scale
e. 1497 QT 1536 TT 1944 QT 1996 QT 2048 T 2304 TT 2916 QT 2994
First associated d.Hypolydian QT enharmonic scale
f. 1497 QT 1536 TT 1944 QT 1996 QT 2048 TT 2592 T 2916 QT 2994
First associated a.Lydian QT enharmonic scale

4. a. 3072 TT 3888 QT 3992 QT 4096 T 4608 TT 5832 QT 5988 QT 6144
Third d.Dorian QT enharmonic scale
b. 3072 TT 3888 QT 3992 QT 4096 TT 5184 T 5832 QT 5988 QT 6144
Third a.Phrygian QT enharmonic scale
c. 3072 T 3456 TT 4374 QT 4491 QT 4608 TT 5832 QT 5988 QT 6144
First d.Mixolydian QT enharmonic scale
d. 3072 TT 3888 T 4374 QT 4491 QT 4608 TT 5832 QT 5988 QT 6144
First a.Hypophrygian QT enharmonic scale
e. 2994 QT 3072 TT 3888 QT 3992 QT 4096 T 4608 TT 5832 QT 5988
Second associated d.Hypolydian QT enharmonic scale
f. 2994 QT 3072 TT 3888 QT 3992 QT 4096 TT 5184 T 5832 QT 5988
Second associated a.Lydian QT enharmonic scale
g. 2994 QT 3072 T 3456 TT 4374 QT 4782 QT 4608 TT 5832 QT 5988
First associated d.Lydian QT enharmonic scale
h. 2994 QT 3072 TT 3888 T 4374 QT 4782 QT 4608 TT 5832 QT 5988
First associated a.Hypolydian QT enharmonic scale

5. a. 6144 TT 7776 QT 7984 QT 8192 T 9216 TT 11664 QT 11976 QT 12288
Fourth d.Dorian QT enharmonic scale

b. 6144 TT 7776 QT 7984 QT 8192 TT 10368 T 11664 QT 11976 QT 12288
Fourth a.Phrygian QT enharmonic scale
c. 6144 T 6912 TT 8748 QT 8982 QT 9216 TT 11664 QT 11976 QT 12288
Second d.Mixolydian QT enharmonic scale
d. 6144 TT 7776 T 8748 QT 8982 QT 9216 TT 11664 QT 11976 QT 12288
Second a.Hypophrygian QT enharmonic scale
e. 5988 QT 6144 TT 7776 QT 7984 QT 8192 T 9216 TT 11664 QT 11976
Third associated d.Hypolydian QT enharmonic scale
f. 5988 QT 6144 TT 7776 QT 7984 QT 8192 TT 10368 T 11664 QT 11976
Third associated a.Lydian QT enharmonic scale
g. 5988 QT 6144 T 6912 TT 8748 QT 8982 QT 9216 TT 11664 QT 11976
Second associated d.Lydian QT enharmonic scale
h. 5988 QT 6144 TT 7776 T 8748 QT 8982 QT 9216 TT 11664 QT 11976
Second associated a.Hypolydian QT enharmonic scale

6. a. 12288 TT 15552 QT 15968 QT 16384 T 18432 TT 23328 QT 23952 QT 24576
Fifth d.Dorian QT enharmonic scale
b. 12288 TT 15552 QT 15968 QT 16384 TT 20736 T 23328 QT 23952 QT 24576
Fifth a.Phrygian QT enharmonic scale
c. 12288 T 13824 TT 17496 QT 17964 QT 18432 TT 23328 QT 23952 QT 24576
Third d.Mixolydian QT enharmonic scale
d. 12288 TT 15552 T 17496 QT 17964 QT 18432 TT 23328 QT 23952 QT 24596
Third a.Hypophrygian QT enharmonic scale
e. 11976 QT 12288 TT 15552 QT 15968 QT 16384 T 18432 TT 23328 QT 23952
Fourth associated d.Hypolydian QT enharmonic scale
f. 11976 QT 12288 TT 15552 QT 15968 QT 16384 TT 20736 T 23328 QT 23952
Fourth associated a.Lydian QT enharmonic scale
g. 11976 QT 12288 T 13824 TT 17496 QT 17964 QT 18432 TT 23328 QT 23952
Third associated d.Lydian QT enharmonic scale
h. 11976 QT 12288 TT 15552 T 17496 QT 17964 QT 18432 TT 23328 QT 23952
Third associated a.Hypolydian QT enharmonic scale

7. a. 24576 TT 31104 QT 31936 QT 32768 T 36864 TT 46656 QT 47904 QT 49152
Sixth d.Dorian QT enharmonic scale
b. 24576 TT 31104 QT 31986 QT 32768 TT 41472 T 46656 QT 47904 QT 49152
Sixth a.Phrygian QT enharmonic scale
c. 24576 T 27648 TT 34992 QT 35928 QT 36864 TT 46656 QT 47904 QT 49152
Fourth d.Mixolydian QT enharmonic scale
d. 24576 TT 31104 T 34992 QT 35928 QT 36864 TT 46656 QT 47904 QT 49152
Fourth a.Hypophrygian QT enharmonic scale

e. 23952 QT 24576 TT 31104 QT 31936 QT 32768 T 36864 TT 46656 QT 47904
Fifth associated d.Hypolydian QT enharmonic scale

f. 23952 QT 24576 TT 31104 QT 31936 QT 32768 TT 41472 T 46656 QT 47904
Fifth associated a.Lydian QT enharmonic scale

g. 23952 QT 24576 T 27648 TT 34992 QT 35928 QT 36864 TT 46656 QT 47904
Fourth associated d.Lydian QT enharmonic scale

h. 23952 QT 24576 TT 31104 T 34992 QT 35928 QT 36864 TT 46656 QT 47904
Fourth associated a.Hypolydian QT enharmonic scale

8. a. 49152 TT 62208 QT 63872 QT 65536 T 73728 TT 93312 QT 95808 QT 98304
Seventh d.Dorian QT enharmonic scale

b. 49152 TT 62208 QT 63872 QT 65536 TT 82944 T 93312 QT 95508 QT 98304
Seventh a.Phrygian QT enharmonic scale

c. 49152 T 55296 TT 69984 QT 71856 QT 73728 TT 93312 QT 95808 QT 98304
Fifth d.Mixolydian QT enharmonic scale

d. 49152 TT 62208 T 69984 QT 71856 QT 73728 TT 93312 QT 95808 QT 98304
Fifth a.Hypophrygian QT enharmonic scale

e. 47904 QT 49152 TT 62208 QT 63872 QT 65536 T 73728 TT 93312 QT 95508
Sixth associated d.Hypolydian QT enharmonic scale

f. 47904 QT 49152 TT 62208 QT 63872 QT 65536 TT 82944 T 93312 QT 95808
Sixth associated a.Lydian QT enharmonic scale

g. 47904 QT 49152 T 55296 TT 69984 QT 71856 QT 73728 TT 93312 QT 95808
Fifth associated d.Lydian QT enharmonic scale

h. 47904 QT 49152 TT 62208 T 69984 QT 71856 QT 73728 TT 93312 QT 95508
Fifth associated a.Hypolydian QT enharmonic scale

9. a. 98304 TT 124416 QT 127744 QT 131072 T 147456 TT 186624 QT 191616 QT 196608
Eighth d.Dorian QT enharmonic scale

b. 98304 TT 124416 QT 127744 QT 131072 TT 165888 T 186624 QT 191616 QT 196608
Eighth a.Phrygian QT enharmonic scale

c. 98304 T 110592 TT 139968 QT 143712 QT 147456 TT 186624 QT 191616 QT 196608
Sixth d.Mixolydian QT enharmonic scale

d. 98304 TT 124416 T 139968 QT 143712 QT 147456 TT 186624 QT 191616 QT 196608
Sixth a.Hypophrygian QT enharmonic scale

e. 95808 QT 98304 TT 124416 QT 127744 QT 131072 T 147456 TT 186624 QT 191616
Seventh associated d.Hypolydian QT enharmonic scale

f. 95808 QT 98304 TT 124416 QT 127744 QT 131072 TT 165888 T 186624 QT 191616
Seventh associated a.Lydian QT enharmonic scale

g. 95808 QT 98304 T 110592 TT 139968 QT 143712 QT 147456 TT 186624 QT 191616
Sixth associated d.Lydian QT enharmonic scale

h. 95808 QT 98304 TT 124416 T 139968 QT 143712 QT 147456 TT 186624 QT 191616
Sixth associated a.Hypolydian QT enharmonic scale

10. a. 196608 TT 248832 QT 255488 QT 262144 T 294912 TT 373248 QT 383232 QT 393216
Ninth d.Dorian QT enharmonic scale

b. 196608 TT 248832 QT 379904 QT 262144 TT 331776 T 373248 QT 383232 QT 393216
Ninth a.Phrygian QT enharmonic scale

c. 196608 T 221184 TT 279936 QT 287424 QT 294912 TT 373248 QT 383232 QT 393216
Seventh d.Mixolydian QT enharmonic scale

d. 196608 TT 248832 T 279936 QT 287424 QT 294912 TT 373248 QT 383232 QT 393216
Seventh a.Hypophrygian QT enharmonic scale

e. 191616 QT 196608 TT 248832 QT 255488 QT 262144 T 294912 TT 373248 QT 383232
Eighth d.Hypolydian QT enharmonic scale

f. 191616 QT 196608 TT 248832 QT 255488 QT 262144 TT 331776 T 373248 QT 383232
Eighth a.Lydian QT enharmonic scale

g. 191616 QT 196608 T 221184 TT 279936 QT 287424 QT 294912 TT 373248 QT 383232
Seventh d.Lydian QT enharmonic scale

h. 191616 QT 196608 TT 248832 T 279936 QT 287424 QT 294912 TT 373248 QT 383232
Seventh a.Hypolydian enharmonic scale

11. d.Dorian, a.Phrygian, associated d.Hypolydian, and associated a.Lydian enharmonic scales die.

a. 393216 T 442368 TT 559872 QT 574848 QT 589824 TT 746496 QT 766464 QT 786432
Eighth d.Mixolydian QT enharmonic scale

b. 393216 TT 497664 T 559872 QT 574848 QT 589824 TT 746496 QT 766464 QT 786432
Eighth a.Hypophrygian QT enharmonic scale

c. 383232 QT 393216 T 442368 TT 559872 QT 574848 QT 589824 TT 746496 QT 766464
Eighth associated d.Lydian QT enharmonic scale

d. 383232 QT 393216 TT 497664 T 559872 QT 574848 QT 589824 TT 746496 QT 766464
Eighth associated a.Hypolydian QT enharmonic scale

12. The d.Mixolydian and a.Hypophrygian enharmonic patterns die out.

a. 766464 QT 786432 T 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992 QT 1539648
Ninth associated d.Lydian QT enharmonic scale
Note that the last quarter tone is calculated from two splits of the tone between 1492992 and 1679616 as follows: [(1492992 + 1679616/2) + 1492992]/2. The size of the resulting QT is 1.03125.

b. 766464 QT 786432 TT 995328 T 1119744 QT 1149696 QT 1179648 TT 1492992 QT 1539648
Ninth associated a.Hypolydian QT enharmonic scale
Note that the last quarter tone is calculated from two splits of the tone between 1492992 and 1679616 as follows: [(1492992 + 1679616/2) + 1492992]/2. The size of the resulting QT is 1.03125.

The Δ-string enharmonic octave chains end at this point.

2.1.2 384 432 486 512 576 648 729 768

This section articulates the α-string enharmonic octave chains beginning from 1728.

1. 432: No enharmonic scale is possible from this number.

2. 864: No enharmonic scale is possible from this number.

3. 1728 TT 2187 S 2304 TT 2916 S 3072 T 3456
d.Hypodorian ST enharmonic scale

4. a. 3456 TT 4374 QT 4491 QT 4608 TT 5832 QT 5988 QT 6144 T 6912
First d.Hypodorian QT enharmonic scale

b. 3456 TT 4374 S 4608 T 5184 TT 6561 S 6912
d.Dorian ST enharmonic scale

c. 3456 TT 4374 S 4608 TT 5832 T 6561 S 6912
a.Phrygian ST enharmonic scale

5. a. 6912 TT 8748 QT 8982 QT 9216 TT 11664 QT 11976 QT 12288 T 13824
Second d.Hypodorian QT enharmonic scale

b. 6912 TT 8748 QT 8982 QT 9216 T 10368 TT 13122 QT 13473 QT 13824
First d.Dorian QT enharmonic scale

c. 6912 TT 8748 QT 8982 QT 9216 TT 11664 T 13122 QT 13473 QT 13824
First a.Phrygian QT enharmonic scale

6. a. 13824 TT 17496 QT 17964 QT 18432 TT 23328 QT 23952 QT 24576 T 27648
Third d.Hypodorian QT enharmonic scale
b. 13824 TT 17496 QT 17964 QT 18432 T 20736 TT 26244 QT 26946 QT 27648
Second d.Dorian QT enharmonic scale
c. 13824 TT 17496 QT 17964 QT 18432 TT 23328 T 26244 QT 26946 QT 27648
Second a.Phrygian QT enharmonic scale
d. 13824 T 15552 TT 19683 S 20736 TT 26244 S 27648
d.Mixolydian ST enharmonic scale
e. 13824 TT 17496 T 19683 S 20736 TT 26244 S 27648
a.Hypophrygian ST enharmonic scale
f. 13473 QT 13824 TT 17496 QT 17964 QT 18432 T 20736 TT 26244 QT 26946
First associated d.Hypolydian QT enharmonic scale
g. 13473 QT 13824 TT 17496 QT 17964 QT 18432 TT 23328 T 26244 QT 26946
First associated a.Lydian QT enharmonic scale

7. a. 27648 TT 34992 QT 35928 QT 36864 TT 46656 QT 47904 QT 49152 T 55296
Fourth d.Hypodorian QT enharmonic scale
b. 27648 TT 34992 QT 35928 QT 36864 T 41472 TT 52488 QT 53892 QT 55296
Third d.Dorian QT enharmonic scale
c. 27648 TT 34992 QT 35928 QT 36864 TT 46656 T 52488 QT 53892 QT 55296
Third a.Phrygian QT enharmonic scale
d. 27648 T 31104 TT 39366 QT 40419 QT 41472 TT 52488 QT 53892 QT 55296
First d.Mixolydian QT enharmonic scale
e. 27648 TT 34992 T 39366 QT 40419 QT 41472 TT 52488 QT 53892 QT 55296
First a.Hypophrygian QT enharmonic scale
f. 26946 QT 27648 TT 34992 QT 35928 QT 36864 T 41472 TT 52488 QT 53892
Second associated d.Hypolydian QT enharmonic scale
g. 26946 QT 27648 TT 34992 QT 35928 QT 36864 TT 46656 T 52488 QT 53892
Second associated a.Lydian QT enharmonic scale
h. 26946 QT 27648 T 31104 TT 39366 QT 40419 QT 41472 TT 52488 QT 53892
First associated d.Lydian QT enharmonic scale
i. 26946 QT 27648 TT 34992 T 39366 QT 40419 QT 41472 TT 52488 QT 53892
First associated a.Hypolydian QT enharmonic scale

8. a. 55296 TT 69984 QT 71856 QT 73728 TT 93312 QT 95808 QT 98304 T 110592
Fifth d.Hypodorian QT enharmonic scale

b. 55296 TT 69984 QT 71856 QT 73728 T 82944 TT 104976 QT 107784 QT 110592
Fourth d.Dorian QT enharmonic scale

c. 55296 TT 69984 QT 71856 QT 73728 TT 93312 T 104976 QT 107784 QT 110592
Fourth a.Phrygian QT enharmonic scale

d. 55296 T 62208 TT 78732 QT 80838 QT 82944 TT 104976 QT 107784 QT 110592
Second d.Mixolydian QT enharmonic scale

e. 55296 TT 69984 T 78732 QT 80838 QT 82944 TT 104976 QT 107784 QT 110592
Second a.Hypophrygian QT enharmonic scale

f. 53892 QT 55296 TT 69984 QT 71856 QT 73728 T 82944 TT 104976 QT 107784
Third associated d.Hypolydian QT enharmonic scale

g. 53892 QT 55296 TT 69984 QT 71856 QT 73728 TT 93312 T 104976 QT 107784
Third associated a.Lydian QT enharmonic scale

h. 53892 QT 55296 T 62208 TT 78732 QT 80838 QT 82944 TT 104976 QT 107884
Second associated d.Lydian QT enharmonic scale

i. 53892 QT 55296 TT 69984 T 78732 QT 80838 QT 82944 TT 104976 QT 107784
Second associated a.Hypolydian QT enharmonic scale

9. a. 110592 TT 139968 QT 143712 QT 147456 TT 186624 QT 191616 QT 196608 T 221184
Sixth d.Hypodorian QT enharmonic scale

b. 110592 TT 139968 QT 143712 QT 147456 T 165888 TT 209952 QT 215568 QT 221184
Fifth d.Dorian QT enharmonic scale

c. 110592 TT 139968 QT 143712 QT 147456 TT 186624 T 209952 QT 215568 QT 221184
Fifth a.Phrygian QT enharmonic scale

d. 110592 T 124416 TT 157464 QT 161676 QT 165888 TT 209952 QT 215568 QT 221184
Third d.Mixolydian QT enharmonic scale

e. 110592 TT 139968 T 157464 QT 161676 QT 165888 TT 209952 QT 215568 QT 221184
Third a.Hypophrygian QT enharmonic scale

f. 107784 QT 110592 TT 139968 QT 143712 QT 147456 T 165888 TT 209952 QT 215568
Fourth associated d.Hypolydian QT enharmonic scale

g. 107784 QT 110592 TT 139968 QT 143712 QT 147456 TT 186624 T 209952 QT 215568
Fourth associated a.Lydian QT enharmonic scale

h. 107784 QT 110592 T 124416 TT 157464 QT 161726 QT 165888 TT 209952 QT 215568
Third associated d.Lydian QT enharmonic scale

i. 107784 QT 110592 TT 139968 QT 143712 QT 147456 TT 186624 T 209952 QT 215568
Third associated a.Hypolydian QT enharmonic scale

10. a. 221184 TT 279936 QT 287424 QT 294912 TT 373248 QT 383232 QT 393216 T 442368
Seventh d.Hypodorian QT enharmonic scale
b. 221184 TT 279936 QT 287424 QT 294912 T 331776 TT 419904 QT 431136 QT 442368
Sixth d.Dorian QT enharmonic
c. 221184 TT 279936 QT 287424 QT 294912 TT 373248 T 419904 QT 431136 QT 442368
Sixth a.Phrygian QT enharmonic scale
d. 221184 T 248832 TT 314928 QT 323352 QT 331776 TT 419904 QT 431136 QT 442368
Fourth d.Mixolydian QT enharmonic scale
e. 221184 TT 279936 T 314928 QT 323352 QT 331776 TT 419904 QT 431136 QT 442368
Fourth a.Hypophrygian QT enharmonic scale
f. 215568 QT 221184 TT 279936 QT 287424 QT 294912 T 331776 TT 419904 QT 431136
Fifth associated d.Hypolydian QT enharmonic scale
g. 215568 QT 221184 TT 279936 QT 287424 QT 294912 TT 373248 T 419904 QT 431136
Fifth associated a.Lydian QT enharmonic scale
h. 215568 QT 221184 T 248832 TT 314928 QT 323352 QT 331776 TT 419904 QT 431136
Fourth associated d.Lydian QT enharmonic scale
i. 215568 QT 221184 TT 279936 T 314928 QT 323352 QT 331776 TT 419904 QT 431136
Fourth associated a.Hypolydian QT enharmonic scale

11. a. 442368 TT 559872 QT 574848 QT 589824 TT 746496 QT 766464 QT 786432 T 884736
Eighth d.Hypodorian QT enharmonic scale
b. 442368 TT 559872 QT 574848 QT 589824 T 663552 TT 839808 QT 862272 QT 884736
Seventh d.Dorian QT enharmonic scale
c. 442368 TT 559872 QT 574848 QT 589824 TT 746496 T 839808 QT 862272 QT 884736
Seventh a.Phrygian QT enharmonic scale

d. 442368 T 497664 TT 629856 QT 646704 QT 663552 TT 839808 QT 862272 QT 884736
Fifth d.Mixolydian QT enharmonic scale

e. 442368 TT 559872 T 629856 QT 646704 QT 663552 TT 839808 QT 862272 QT 884736
Fifth a.Hypophrygian QT enharmonic scale

f. 431136 QT 442368 TT 559872 QT 574848 QT 589824 T 663552 TT 839808 QT 862272
Sixth associated d.Hypolydian QT enharmonic scale

g. 431136 QT 442368 TT 559872 QT 574848 QT 589824 TT 746496 T 839808 QT 862272
Sixth associated a.Lydian QT enharmonic scale

h. 431136 QT 442368 T 497664 TT 629856 QT 646704 QT 663552 TT 839808 QT 862272
Fifth associated d.Lydian QT enharmonic scale

i. 431136 QT 442368 TT 559872 T 629856 QT 646704 QT 663552 TT 839808 QT 862272
Fifth associated a.Hypolydian QT enharmonic scale

12. d.Hypodorian enharmonic scale types cease.

a. 884736 TT 1119744 QT 1149696 QT 1179648 T 1327104 TT 1679616 QT 1724544 QT 1769472
Eighth d.Dorian QT enharmonic scale

b. 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992 T 1679616 QT 1724544 QT 1769472
Eighth a.Phrygian QT enharmonic scale

c. 884736 T 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472
Sixth d.Mixolydian QT enharmonic scale

d. 884736 TT 1119744 T 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472
Sixth a.Hypophrygian QT enharmonic scale

e. 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 T 1327104 TT 1679616 QT 1724544
Seventh associated d.Hypolydian QT enharmonic scale

f. 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992 T 1679616 QT 1724544
Seventh associated a.Lydian QT enharmonic scale

g. 862272 QT 884736 T 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544
Sixth associated d.Lydian QT enharmonic scale

h. 862272 QT 884736 TT 1119744 T 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544
Sixth associated a.Hypolydian QT enharmonic scale

13. a. 1769472 TT 2239488 QT 2299392 QT 2359296 T 2654208 TT 3359232 QT 3449088 QT 3538944
Ninth d.Dorian QT enharmonic scale
b. 1769472 TT 2239488 QT 2299392 QT 2359296 TT 2985984 T 3359232 QT 3449088 QT 3538944
Ninth a.Phrygian QT enharmonic scale
c. 1769472 T 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944
Seventh d.Mixolydian QT enharmonic scale
d. 1769472 TT 2239488 T 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944
Seventh a.Hypophrygian QT enharmonic scale
e. 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 T 2654208 TT 3359232 QT 3449088
Eighth associated d.Hypolydian QT enharmonic scale
f. 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 TT 2985984 T 3359232 QT 3449088
Eighth associated a.Lydian QT enharmonic scale
g. 1724544 QT 1769472 T 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088
Seventh associated d.Lydian QT enharmonic scale
h. 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088
Seventh associated a.Hypolydian QT enharmonic scale

14. d.Dorian, a.Phrygian, associated d.Hypolydian and associated a.Lydian enharmonic scale types vanish.
a. 3538944 T 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888
Eighth d.Mixolydian QT enharmonic scale
b. 3538944 TT 4478976 T 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888
Eighth a.Hypophrygian QT enharmonic scale
c. 3449088 QT 3538944 T 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176
Eighth associated d.Lydian QT enharmonic scale

d. 3449088 QT 3538944 TT 4478976 T 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176
Eighth associated a.Hypolydian QT enharmonic scale

15. d.Mixolydian and a.Hypophrygian scale types disappear.
a. 6898176 QT 7077888 T 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436928 QT 13856832
Ninth associated d.Lydian QT enharmonic scale
Note that the last quarter tone is calculated from two splits of the tone between 13436928 and 15116544 as follows: [(13436928 + 15116544/2) + 13436928]/2. The size of the resulting QT is 1.03125.
b. 6898176 QT 7077888 TT 8957952 T 10077696 QT 10347264 QT 10616832 TT 13436928 QT 13856832
Ninth associated a.Hypolydian QT enharmonic scale
Note that the last quarter tone is calculated from two splits of the tone between 13436928 and 15116544 as follows: [(13436928 + 15116544 /2) + 13436928]/2. The size of the resulting QT is 1.03125.

The α-string enharmonic octave chains end at this point.

2.1.3 384 432 486 512 576 648 729 768
This section articulates the β-string enharmonic octave chains beginning from 486.

1. a. 486 S 512 T 576 TT 729 S 768 TT 972
d.Phrygian ST enharmonic scale
b. 486 S 512 TT 648 T 729 S 768 TT 972
a.Dorian ST enharmonic scale

2. a. 972 QT 998 QT 1024 T 1152 TT 1458 QT 1497 QT 1536 TT 1944
First d.Phrygian QT enharmonic scale
b. 972 QT 998 QT 1024 TT 1296 T 1458 QT 1497 QT 1536 TT 1944
First a.Dorian QT enharmonic scale
c. 998 QT 1024 T 1152 TT 1458 QT 1497 QT 1536 TT 1944 QT 1996
First associated d.Lydian QT enharmonic scale
d. 998 QT 1024 TT 1296 T 1458 QT 1497QT 1536 TT 1944 QT 1996
First associated a.Hypolydian enharmonic scale

3. a. 1944 QT 1996 QT 2048 T 2304 TT 2916 QT 2994 QT 3072 TT 3888
Second d.Phrygian QT enharmonic scale
b. 1944 Q 1996 QT 2048 TT 2592 T 2916 QT 2994 QT 3072 TT 3888
Second a.Dorian QT enharmonic scale

c. 1996 QT 2048 T 2304 TT 2916 QT 2994 QT 3072 TT 3888 QT 3992
Second associated d.Lydian QT enharmonic scale
d. 1996 QT 2048 TT 2592 T 2916 QT 2994 QT 3072 TT 3888 QT 3992
Second associated a.Hypolydian QT enharmonic scale
e. 1944 T 2187 S 2304 TT 2916 S 3072 TT 3888
First a.Hypodorian ST enharmonic scale

4. a. 3888 QT 3992 QT 4096 T 4608 TT 5832 QT 5988 QT 6144 TT 7776
Third d.Phrygian QT enharmonic scale
b. 3888 QT 3992 QT 4096 TT 5184 T 5832 QT 5988 QT 6144 TT 7776
Third a.Dorian QT enharmonic scale
c. 3992 QT 4096 T 4608 TT 5832 QT 5988 QT 6144 TT 7776 QT 7984
Third associated d.Lydian QT enharmonic scale
d. 3992 QT 4096 TT 5184 T 5832 QT 5988 QT 6144 TT 7776 QT 7984
Third associated a.Hypolydian QT enharmonic scale
e. 3888 T 4374 QT 4491 QT 4608 TT 5832 QT 5988 QT 6144 TT 7776
First a.Hypodorian QT enharmonic scale

5. a. 7776 QT 7984 QT 8192 T 9216 TT 11664 QT 11976 QT 12288 TT 15552
Fourth d.Phrygian QT enharmonic scale
b. 7776 QT 7984 QT 8192 TT 10368 T 11664 QT 11976 QT 12288 TT 15552
Fourth a.Dorian QT enharmonic scale
c. 7984 QT 8192 T 9216 TT 11664 QT 11976 QT 12288 TT 15552 QT 15968
Fourth associated d.Lydian QT enharmonic scale
d. 7984 QT 8192 TT 10368 T 11664 QT 11976 QT 12288 TT 15552 QT 15968
Fourth associated a.Hypolydian QT enharmonic scale
e. 7776 T 8748 QT 8982 QT 9216 TT 11664 QT 11976 QT 12288 TT 15552
Second a.Hypodorian QT enharmonic scale

6. a. 15552 15968 16384 T 18432 TT 23328 QT 23952 QT 24576 TT 31104
Fifth d.Phrygian QT enharmonic scale
b. 15552 QT 15968 QT 16384 TT 20736 T 23328 QT 23952 QT 24576 TT 31104
Fifth a.Dorian QT enharmonic scale
c. 15968 QT 16384 T 18432 TT 23328 QT 23952 QT 24576 TT 31104 QT 31936
Fifth associated d.Lydian QT enharmonic scale
d. 15968 QT 16384 TT 20736 T 23328 QT 23952 QT 24576 TT 31104 QT 31936
Fifth associated a.Hypolydian QT enharmonic scale
e. 15552 T 17496 QT 17964 QT 18432 TT 23328 QT 23952 QT 24576 TT 31104
Third a.Hypodorian QT enharmonic scale
f. 15552 TT 19683 S 20736 TT 26244 S 27648 T 31104
d.Hypodorian ST enharmonic scale

7. a. 31104 QT 31936 QT 32768 T 36864 TT 46656 QT 47904 QT 49152 TT 62208
Sixth d.Phrygian QT enharmonic scale
b. 31104 QT 31936 QT 32768 TT 41472 T 46656 QT 47904 QT 49152 TT 62208
Sixth a.Dorian QT enharmonic scale
c. 31936 QT 32768 T 36864 TT 46656 QT 47904 QT 49152 TT 62208 QT 63872
Sixth associated d.Lydian QT enharmonic scale
d. 31936 QT 32768 TT 41472 T 46656 QT 47904 QT 49152 TT 62208 QT 63872
Sixth associated a.Hypolydian QT enharmonic scale
e. 31104 T 34992 QT 35928 QT 36864 TT 46656 QT 47904 QT 49152 TT 62208
Fourth a.Hypodorian QT enharmonic scale
f. 31104 TT 39366 QT 40419 QT 41472 TT 52488 QT 53892 QT55296 T 62208
First d.Hypodorian QT enharmonic scale
g. 31104 TT 39366 S 41472 TT 52488 T 59049 S 62208
a.Phrygian ST enharmonic scale
h. 31104 TT 39366 S 41472 T 46656 TT 59049 S 62208
d.Dorian ST enharmonic scale

8. a. 62208 QT 63872 QT 65536 T 73728 TT 93312 QT 95808 QT 98304 TT 124416
Seventh d.Phrygian QT enharmonic scale
b. 62208 QT 63872 QT 65536 TT 82944 T 93312 QT 95808 QT 98304 TT 124416
Seventh a.Dorian QT enharmonic scale
c. 63872 QT 65536 T 73728 TT 93312 QT 95808 QT 98304 TT 124416 QT 127744
Seventh associated d.Lydian QT enharmonic scale
d. 63872 QT 65536 TT 82944 T 93312 QT 95808 QT 98304 TT 124416 QT 127744
Seventh associated a.Hypolydian enharmonic scale
e. 62208 T 69984 QT 71856 QT 73728 TT 93312 QT 95808 QT 98304 TT 124416
Fifth a.Hypodorian QT enharmonic scale
f. 62208 TT 78732 QT 80838 QT 82944 T 93312 TT 118098 QT 121257 QT 124416
First d.Dorian QT enharmonic scale
g. 62208 TT 78732 QT 80838 QT 82944 TT 104976 T 118098 QT 121257 QT 124416
First a.Phrygian QT enharmonic scale
h. 62208 TT 78732 QT 80838 QT 82944 TT 104976 QT 107784 QT 110592 T 124416
Second d.Hypodorian QT enharmonic scale

9. a. 124416 QT 127744 QT 131072 T 147456 TT 186624 QT 191616 QT 196608 TT 248832
Eighth d.Phrygian QT enharmonic scale
b. 124416 QT 127744 QT 131072 TT 165888 T 186624 QT 191616 QT 196608 TT 248832
Eighth a.Dorian QT enharmonic scale
c. 127744 QT 131072 T 147456 TT 186624 QT 191616 QT 196608 TT 248832 QT 255488
Eighth associated d.Lydian QT enharmonic scale

d. 127744 QT 131072 TT 165888 T 186624 QT 191616 QT 196608 TT 248832 QT 255488
Eighth associated a.Hypolydian QT enharmonic scale

e. 124416 T 139968 QT 143712 QT 147456 TT 186624 QT 191616 QT 196608 TT 248832
Sixth a.Hypodorian QT enharmonic scale

f. 124416 TT 157464 QT 161676 QT 165888 T 186624 TT 236196 QT 242514 QT 248832
Second d.Dorian QT enharmonic scale

g. 124416 TT 157464 QT 161676 QT 165888 TT 209952 T 236196 QT 242514 QT 248832
Second a.Phrygian QT enharmonic scale

h. 124416 TT 157464 QT 161676 QT 165888 TT 209952 QT 215568 QT 221184 T 248832
Third d.Hypodorian QT enharmonic scale

i. 121257 QT 124416 TT 157464 QT 161676 QT 165888 TT 209952 T 236196 QT 242514
First associated a.Lydian QT enharmonic scale

j. 121257 QT 124416 TT 157464 QT 161676 QT 165888 T 186624 TT 236196 QT 242514
First associated d.Hypolydian QT enharmonic scale

10. a. 248832 QT 255488 QT 262144 T 294912 TT 373248 QT 383232 QT 393216 TT 497664
Ninth d.Phrygian QT enharmonic scale

b. 248832 QT 255488 QT 262144 TT 331776 T 373248 QT 383232 QT 393216 TT 497664
Ninth a.Dorian QT enharmonic scale

c. 255488 QT 262144 T 294912 TT 373248 QT 383232 QT 393216 TT 497664 QT 510976
Ninth associated d.Lydian QT enharmonic scale

d. 255488 QT 262144 TT 331776 T 373248 QT 383232 QT 393216 TT 497664 QT 528768
Ninth associated a.Hypolydian enharmonic scale

e. 248832 T 279936 QT 287424 QT 294912 TT 373248 QT 383232 QT 393216 TT 497664
Seventh a.Hypodorian QT enharmonic scale

f. 248832 TT 314928 QT 323352 QT 331776 T 373248 TT 472392 QT 485028 QT 497664
Third d.Dorian QT enharmonic scale

g. 248832 TT 314928 QT 323352 QT 331776 TT 419904 T 472392 QT 485028 QT 497664
Third a.Phrygian QT Enharmonic scale

h. 248832 TT 314928 QT 323352 QT 331776 TT 419904 QT 431136 QT 442368 T 497664
Fourth d.Hypodorian QT enharmonic scale

i. 242514 QT 248832 TT 314928 QT 323352 QT 331776 T 373248 TT 472392 QT 485028
Second associated d.Hypolydian QT enharmonic scale

j. 242514 QT 248832 TT 314928 QT 323352 QT 331776 TT 419904 T 472392 QT 485028
Second associated a.Lydian QT enharmonic scale

11. d.Phrygian, a.Dorian, associated d.Lydian, and associated a.Hypolydian scale types give out.

a. 497664 TT 629856 QT 646704 QT 663552 T 746496 TT 944784 QT 970056 QT 995328
Fourth d.Dorian QT enharmonic scale

b. 497664 TT 629856 QT 646704 QT 663552 TT 839808 T 944784 QT 970056 QT 995328
Fourth a.Phrygian QT enharmonic scale

c. 497664 T 559872 QT 574888 QT 589824 TT 746496 QT 766464 QT 786432 TT 995328
Eighth a.Hypodorian QT enharmonic scale

d. 497664 TT 629856 QT 646704 QT 663552 TT 839808 QT 862272 QT 884736 T 995328
Fifth d.Hypodorian QT enharmonic scale

e. 485028 QT 497664 TT 629856 QT 646704 QT 663552 T 746496 TT 944784 QT 970056
Third associated d.Hypolydian QT enharmonic scale

f. 485028 QT 497664 TT 629856 QT 646704 QT 663552 TT 839808 T 944784 QT 970056
Third associated a.Lydian QT enharmonic scale

12. a.Hypodorian enharmonic scale types die out.

a. 995328 TT 1259712 QT 1293408 QT 1327104 T 1492992 TT 1889568 QT 1940112 QT 1990656
Fifth d.Dorian QT enharmonic scale

b. 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 T 1889568 QT 1940112 QT 1990656
Fifth a.Phrygian QT enharmonic scale

c. 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 T 1990656
Sixth d.Hypodorian QT enharmonic scale

d. 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 T 1492992 TT 1889568 QT 1940112
Fourth associated d.Hypolydian QT enharmonic scale

e. 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 T 1889568 QT 1940112
Fourth associated a.Lydian QT enharmonic scale

13. a. 1990656 TT 2519424 QT 2586816 QT 2654208 T 2985984 TT 3779136 QT 3880224 QT 3981312
Sixth d.Dorian QT enharmonic scale

b. 1990656 TT 2519424 QT 2586725 QT 2654208 TT 3359232 T 3779136 QT 3880224 QT 3981312
Sixth a.Phrygian QT enharmonic scale

c. 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 T 3981312
Seventh d.Hypodorian QT enharmonic scale

d. 1940112 QT 1990656 TT 2519424 QT 2586725 QT 3654208 T 2985984 TT 3779136 QT 3880224
Fifth associated d.Hypolydian QT enharmonic scale

e. 1940112 QT 1990656 TT 2519424 QT 2586725 QT 2654208 TT 3359232 T 3779136 QT 3880224
Fifth associated a.Lydian QT enharmonic scale

14. a. 3981312 TT 5038848 QT 5173632 QT 5308416 T 5971968 TT 7558272 QT 7760448 QT 7962624
Seventh d.Dorian QT enharmonic scale

b. 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 T 7558272 QT 7760448 QT 7962624
Seventh a.Phrygian QT enharmonic scale

c. 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 T 7962624
Eighth d.Hypodorian QT enharmonic scale

d. 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 T 5971968 TT 7558272 QT 7760448
Sixth associated d.Hypolydian QT enharmonic scale

e. 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 T 7558272 QT 7760448
Sixth associated a.Lydian QT enharmonic scale

15. d.Hypodorian enharmonic scale vanishes.
    a. 7962624 TT 10077696 QT 10347264 QT 10616832 T 11943936 TT 15116544 QT 15520896 QT 15925248
    Eighth d.Dorian QT enharmonic scale
    b. 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436928 T 15116544 QT 15520896 QT 15925248
    Eighth a.Phrygian QT enharmonic scale
    c. 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832 T 11943936 TT 15116544 QT 15520896
    Seventh associated d.Hypolydian QT enharmonic scale
    d. 7760448 QT 7962624 TT 10077696 Q 10347264 QT 10616832 TT 13436928 T 15116544 QT 15520896
    Seventh associated a.Lydian QT enharmonic scale

16. a. 15925248 TT 20155392 QT 20694528 QT 21233664 T 23887872 TT 30233088 QT 31041792 QT 31850496
    Ninth d.Dorian QT enharmonic scale
    b. 15925248 TT 20155392 QT 20694528 QT 21233664 TT 26873856 T 30233088 QT 31041792 QT 31850496
    Ninth a.Phrygian QT enharmonic scale
    c. 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 T 23887872 TT 30233088 QT 31041792
    Eighth associated d.Hypolydian QT enharmonic scale
    d. 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 TT 26873856 T 30233088 QT 31041792
    Eighth associated a.Lydian QT enharmonic scale

The β-string enharmonic octave chains end at this point.

2.1.4 384 432 486 512 576 648 729 768

This section articulates the γ-string enharmonic octave chains beginning from 512.

1. a. 512 T 576 TT 729 S 768 TT 972 S 1024
    d.Mixolydian ST enharmonic octave
    b. 512 TT 648 T 729 S 768 TT 972 S 1024
    a.Hypophrygian ST enharmonic octave

2. a. 1024 T 1152 TT 1458 QT 1497 QT 1536 TT 1944 QT 1996 QT 2048
    First d.Mixolydian QT enharmonic octave
    b. 1024 TT 1296 T 1458 QT 1497 QT 1536 TT 1944 QT 1996 QT 2048
    First a.Hypophrygian QT enharmonic octave

c. 998 QT 1024 T 1152 TT 1458 QT 1497 QT 1536 TT 1944 QT 1996
First associated d.Lydian QT enharmonic octave
d. 998 QT 1024 TT 1296 T 1458 QT 1497 QT 1536 TT 1944 QT 1996
First associated a.Hypolydian QT enharmonic octave

3. a. 2048 T 2304 TT 2916 QT 2994 QT 3072 TT 3888 QT 3992 QT 4096
Second d.Mixolydian QT enharmonic scale
b. 2048 TT 2592 T 2916 QT 2994 QT 3072 TT 3888 QT 3992 QT 4096
Second a.Hypophrygian QT enharmonic scale
c. 1996 QT 2048 T 2304 TT 2916 QT 2994 QT 3072 TT 3888 QT 3992
Second associated d.Lydian QT enharmonic scale
d. 1996 QT 2048 TT 2592 T 2916 QT 2994 QT 3072 TT 3888 QT 3992
Second associated a.Hypolydian QT enharmonic scale

4. a. 4096 T 4608 TT 5832 QT 5988 QT 6144 TT 7776 QT 7984 QT 8192
Third d.Mixolydian QT enharmonic scale
b. 4096 TT 5184 T 5832 QT 5988 QT 6144 TT 7776 QT 7984 QT 8192
Third a.Hypophrygian QT enharmonic scale
c. 3992 QT 4096 T 4608 TT 5832 QT 5988 QT 6144 TT 7776 QT 7984
Third associated d.Lydian QT enharmonic scale
d. 3992 QT 4096 TT 5184 T 5832 QT 5988 QT 6144 TT 7776 QT 7984
Third associated a.Hypolydian QT enharmonic scale

5. a. 8192 T 9216 TT 11664 QT 11976 QT 12288 TT 15552 QT 15968 QT 16384
Fourth d.Mixolydian QT enharmonic scale
b. 8192 TT 10368 T 11664 QT 11976 QT 12288 TT 15552 QT 15968 QT 16384
Fourth a.Hypophrygian QT enharmonic scale
c. 7984 QT 8192 T 9216 TT 11664 QT 11976 QT 12288 TT 15552 QT 15968
Fourth associated d.Lydian QT enharmonic scale
d. 7984 QT 8192 TT 10368 T 11664 QT 11976 QT 12288 TT 15552 QT 15968
Fourth associated a.Hypolydian QT enharmonic scale

6. a. 16384 T 18432 TT 23328 QT 23952 QT 24576 TT 31104 QT 31936 QT 32768
Fifth d.Mixolydian QT enharmonic scale
b. 16384 TT 20736 T 23328 QT 23952 QT 24576 TT 31104 QT 31936 QT 32768
Fifth a.Hypophrygian QT enharmonic scale
c. 15968 QT 16384 T 18432 TT 23328 QT 23952 QT 24576 TT 31104 QT 31936
Fifth associated d.Lydian QT enharmonic scale
d. 15968 QT 16384 TT 20736 T 23328 QT 23952 QT 24576 TT 31104 QT 31936
Fifth associated a.Hypolydian QT enharmonic scale

7. a. 32768 T 36864 TT 46656 QT 47904 QT 49152 TT 62208 QT 63872 QT 65536
   Sixth d.Mixolydian QT enharmonic scale
   b. 32768 TT 41472 T 46656 QT 95808 QT 49152 TT 62208 QT 63872 QT 65536
   Sixth a.Hypophrygian QT enharmonic scale
   c. 31936 QT 32768 T 36864 TT 46656 QT 47904 QT 49152 TT 62208 QT 63872
   Sixth associated d.Lydian QT enharmonic scale
   d. 31936 QT 32768 TT 41472 T 46656 QT 47904 QT 49152 TT 62208 QT 63872
   Sixth associated a.Hypolydian QT enharmonic scale

8. a. 65536 T 73728 TT 93312 QT 95808 QT 98304 TT 124416 QT 127744 QT 131072
   Seventh d.Mixolydian QT enharmonic scale
   b. 65536 TT 82944 T 93312 QT 95808 QT 98304 TT 124416 QT 127744 QT 131072
   Seventh a.Hypophrygian QT enharmonic scale
   c. 63872 QT 65536 T 73728 TT 93312 QT 95808 QT 98304 TT 124416 QT 127744
   Seventh associated d.Lydian QT enharmonic scale
   d. 63872 QT 65536 TT 82944 T 93312 QT 95808 QT 98304 TT 124416
   Seventh associated a.Hypolydian QT enharmonic scale

9. a. 131072 T 147456 TT 186624 QT 191616 QT 196608 TT 248832 QT 255488 QT 262144
   Eighth d.Mixolydian QT enharmonic scale
   b. 131072 TT 165888 T 186624 QT 191616 QT 196608 TT 248832 QT 255488 QT 262144
   Eighth a.Hypophrygian QT enharmonic scale
   c. 127744 QT 131072 T 147456 TT 186624 QT 191616 QT 196608 TT 248832 QT 255488
   Eighth associated d.Lydian QT enharmonic scale
   d. 127744 QT 131072 TT 165888 T 186624 QT 191616 QT 198808 TT 248832 QT 255488
   Eighth associated a.Hypolydian QT enharmonic scale

10. d.Mixolydian and a.Hypophrygian scale types give out.
   a. 255488 QT 262144 T 294912 TT 373248 QT 383232 QT 393216 TT 497664 QT 513216
   Ninth associated d.Lydian QT enharmonic scale
   Note that the last quarter tone is calculated from two splits of the tone between 497664 and 559872 as follows: [(497664 + 559872/2) + 497664]/2. The size of the resulting QT is 1.03125.
   b. 255488 QT 262144 TT 331776 T 373248 QT 383232 QT 393216 TT 497664 QT 513216
   Ninth associated a.Hypolydian QT enharmonic scale

Note that the last quarter tone is calculated from two splits of the tone between 497664 and 559872 as follows: [(497664 + 559872/2) + 497664]/2. The size of the resulting QT is 1.03125.

The γ-string enharmonic octave chains end at this point.

2.1.5 384 432 486 512 576 648 729 768

This section articulates the δ-string enharmonic octave chains beginning from 576.

1. 576 TT 729 S 768 TT 972 S 1024 T 1152
   d.Hypodorian ST enharmonic scale

2. a. 1152 TT 1458 QT 1497 QT 1536 TT 1944 QT 1996 QT 2048 T 2304
      First d.Hypodorian QT enharmonic scale
   b. 1152 TT 1458 S 1536 T 1728 TT 2187 S 2304
      d.Dorian ST enharmonic scale
   c. 1152 TT 1458 S 1536 TT 1944 T 2187 S 2304
      a.Phrygian ST enharmonic scale

3. a. 2304 TT 2916 QT 2994 QT 3072 T 3456 TT 4374 QT 4491 QT 4608
      First d.Dorian QT enharmonic scale
   b. 2304 TT 2916 QT 2994 QT 3072 TT 3888 T 4374 QT 4491 QT 4608
      First a.Phrygian QT enharmonic scale
   c. 2304 TT 2916 QT 2994 QT 3072 TT 3888 QT 3992 QT 4096 T 4608
      Second d.Hypodorian QT enharmonic scale

4. a. 4608 TT 5832 QT 5988 QT 6144 T 6912 TT 8748 QT 8982 QT 9216
      Second d.Dorian QT enharmonic scale
   b. 4608 TT 5832 QT 5988 QT 6144 TT 7776 T 8748 QT 8982 QT 9216
      Second a.Phrygian QT enharmonic scale
   c. 4608 TT 5832 QT 5988 QT 6144 TT 7776 T 8748 QT 8982 QT 9216
      Third d.Hypodorian QT enharmonic scale
   d. 4608 T 5184 TT 6561 S 6912 TT 8748 S 9216
      d.Mixolydian ST enharmonic scale
   e. 4608 TT 5832 T 6561 S 6912 TT 8748 S 9216
      a.Hypophrygian ST enharmonic scale
   h. 4491 QT 4608 TT 5832 QT 5988 QT 6144 T 6912 TT 8748 QT 8982
      First associated d.Hypolydian QT enharmonic scale
   i. 4491 QT 4608 TT 5832 QT 5988 QT 6144 TT 7776 T 8748 QT 8982
      First associated a.Lydian QT enharmonic scale

5. a. 9216 T 10368 TT 13122 QT 13473 QT 13824 TT 17496 QT 17964 QT 18432
First d.Mixolydian QT enharmonic scale
b. 9216 TT 11664 T 13122 QT 13473 QT 13824 TT 17496 QT 17964 QT 18432
First a.Hypophrygian QT enharmonic scale
c. 9216 TT 11664 QT 11976 QT 12288 T 13824 TT 17496 QT 17964 QT 18432
Third d.Dorian QT enharmonic scale
d. 9216 TT 11664 QT 11976 QT 12288 TT 15552 T 17496 QT 17964 QT 18432
Third a.Phrygian QT enharmonic scale
e. 9216 TT 11664 QT 11976 QT 12288 TT 15552 QT 15968 QT 16384 T 18432
Fourth d.Hypodorian QT enharmonic scale
f. 8982 QT 9216 TT 11664 QT 11976 QT 12288 T 13824 TT 17496 QT 17964
Second associated d.Hypolydian QT enharmonic scale
g. 8982 QT 9216 TT 11664 QT 11976 QT 12288 TT 15552 T 17496 QT 17964
Second associated a.Lydian QT enharmonic scale
h. 8982 QT 9216 T 10368 TT 13122 QT 13473 QT 13824 TT 17496 QT 17964
First associated d.Lydian QT enharmonic scale
i. 8982 QT 9216 TT 11664 T 13122 QT 13473 QT 13824 TT 17496 QT 17964
First associated a.Hypolydian QT enharmonic scale

6. a. 18432 T 20736 TT 26244 QT 26946 QT 27648 TT 34992 QT 35928 QT 36864
Second d.Mixolydian QT enharmonic scale
b. 18432 TT 23328 T 26244 QT 26946 QT 27648 TT 34992 QT 35928 QT 36864
Second a.Hypophrygian QT enharmonic scale
c. 18432 TT 23328 QT 23952 QT 24576 T 27648 TT 34992 QT 35928 QT 36864
Fourth d.Dorian QT enharmonic scale
d. 18432 TT 23328 QT 23952 QT 24576 TT 31104 T 34992 QT 35928 QT 36864
Fourth a.Phrygian QT enharmonic scale
e. 18432 TT 23328 QT 23952 QT 24576 TT 31104 QT 31936 QT 32768 T 36864
Fifth d.Hypodorian QT enharmonic scale
f. 17964 QT 18432 T 20736 TT 26244 QT 26946 QT 27648 TT 34992 QT 35928
Second associated d.Lydian QT enharmonic scale
g. 17964 QT 18432 TT 23328 T 26244 QT 26946 QT 27648 TT 34992 QT 35928
Second associated a.Hypolydian QT enharmonic scale
h. 17964 QT 18432 TT 23328 QT 23952 QT 24576 T 27648 TT 34992 QT 35928
Third associated d.Hypolydian QT enharmonic scale
i. 17964 QT 18432 TT 23328 QT 23952 QT 24576 TT 31104 T 34992 QT 35928
Third associated a.Lydian QT enharmonic scale

7. a. 36864 T 41472 TT 52488 QT 53892 QT 55296 TT 69984 QT 71856 QT 73728
Third d.Mixolydian QT enharmonic scale

b. 36864 TT 46656 T 52488 QT 53892 QT 55296 TT 69984 QT 71856 QT 73728
Third a.Hypophrygian QT enharmonic scale

c. 36864 TT 46656 47904 49152 T 55296 TT 69984 QT 71856 QT 73728
Fifth d.Dorian QT enharmonic scale

d. 36864 TT 46656 QT 47904 QT 49152 TT 62208 T 69984 QT 71856 QT 73728
Fifth a.Phrygian QT enharmonic scale

e. 36864 TT 46656 QT 47904 QT 49152 TT 62208 QT 63872 QT 65536 T 73728
Sixth d.Hypodorian QT enharmonic scale

f. 35838 QT 36864 T 41472 TT 52488 QT 53892 QT 55296 TT 69984 QT 71856
Third associated d.Lydian QT enharmonic scale

g. 35838 QT 36864 TT 46656 T 52488 QT 53892 QT 55296 TT 69984 QT 71856
Third associated a.Hypolydian QT enharmonic scale

h. 35838 QT 36864 TT 46656 QT 47904 QT 49152 T 55296 TT 69984 QT 71856
Fourth associated d.Hypolydian QT enharmonic scale

i. 35838 QT 36864 TT 46656 QT 47904 QT 49152 TT 62208 T 69984 QT 71856
Fourth associated a.Lydian QT enharmonic scale

8. a. 73728 T 82944 TT 104976 QT 107784 QT 110592 TT 139968 QT 143712 QT 147456
Fourth d.Mixolydian QT enharmonic scale

b. 73728 TT 93312 T 104976 QT 107784 QT 110592 TT 139968 QT 143712 QT 147456
Fourth a.Hypophrygian QT enharmonic scale

c. 73728 TT 93312 QT 95808 QT 98304 T 110592 TT 139968 QT 143712 QT 147456
Sixth d.Dorian QT enharmonic scale

d. 73728 TT 93312 QT 95808 QT 98304 TT 124416 T 139968 QT 143712 QT 147456
Sixth a.Phrygian QT enharmonic scale

e. 73728 TT 93312 QT 95808 QT 98304 TT 124416 QT 127744 QT 131072 T 147456
Seventh d.Hypodorian QT enharmonic scale

f. 71856 QT 73728 T 82944 TT 104976 QT 107784 QT 110592 TT 139968 QT 143712
Fourth associated d.Lydian QT enharmonic scale

g. 71856 QT 73728 TT 93312 T 104976 QT 107784 QT 110592 TT 139968 QT 143712
Fourth associated a.Hypolydian QT enharmonic scale

h. 71856 QT 73728 TT 93312 QT 95808 QT 98304 T 110592 TT 139968 QT 143712
Fifth associated d.Hypolydian QT enharmonic scale

i. 71856 QT 73728 TT 93312 QT 95808 QT 98304 TT 124416 T 139968 QT 143712
Fifth associated a.Lydian QT enharmonic scale

9. a. 147456 T 165888 TT 209952 QT 215568 QT 221184 TT 279936 QT 287424 QT 294912
Fifth d.Mixolydian QT enharmonic scale

b. 147456 TT 186624 T 209952 QT 215568 QT 221184 TT 279936 QT 287424 QT 294912
Fifth a.Hypophrygian QT enharmonic scale

c. 147456 TT 186624 QT 191616 QT 196608 T 221184 TT 279936 QT 287424 QT 294912
Seventh d.Dorian QT enharmonic scale

d. 147456 TT 186624 QT 191616 QT 196608 TT 248832 T 279936 QT 287424 QT 294912
Seventh a.Phrygian QT enharmonic scale

e. 147456 TT 186624 QT 191616 QT 196608 TT 248832 QT 255488 QT 262144 T 294912
Eighth d.Hypodorian QT enharmonic scale

f. 143712 QT 147456 T 165888 TT 209952 QT 215568 QT 221184 TT 279936 QT 287424
Fifth associated d.Lydian QT enharmonic scale

g. 143712 QT 147456 TT 186624 T 209952 QT 215568 QT 221184 TT 279936 QT 287424
Fifth associated a.Hypolydian QT enharmonic scale

h. 143712 QT 147456 TT 186624 QT 191616 QT 196608 T 221184 TT 279936 QT 287424
Sixth associated d.Hypolydian QT enharmonic scale

i. 143712 QT 147456 TT 186624 QT 191616 QT 196608 TT 248832 T 279936 QT 287424
Sixth associated a.Lydian QT enharmonic scale

10. d.Hypodorian enharmonic scale types give out.

a. 294912 T 331776 TT 419904 QT 431136 QT 442368 TT 559872 QT 574848 QT 589824
Sixth d.Mixolydian QT enharmonic scale

b. 294912 TT 373248 T 419904 QT 431136 QT 442368 TT 559872 QT 574848 QT 589824
Sixth a.Hypophrygian QT enharmonic scale

c. 294912 TT 373248 QT 383232 QT 393216 T 442368 TT 559872 QT 574848 QT 589824
Eighth d.Dorian QT enharmonic scale

d. 294912 TT 373248 QT 383232 QT 393216 TT 497664 T 559872 QT 574848 QT 589824
Eighth a.Phrygian QT enharmonic scale

e. 287424 QT 294912 T 331776 TT 419904 QT 431136 QT 442368 TT 559872 QT 574848
Sixth associated d.Lydian QT enharmonic scale

f. 287424 QT 294912 TT 373248 T 419904 QT 431136 QT 442368 TT 559872 QT 574848
Sixth associated a.Hypolydian QT enharmonic scale

g. 287424 QT 294912 TT 373248 QT 383232 QT 393216 T 442368 TT 559872 QT 574848
Seventh associated d.Hypolydian QT enharmonic scale

h. 287424 QT 294912 TT 373248 QT 383232 QT 393216 TT 497664 T 559872 QT 574848
Seventh associated a.Lydian QT enharmonic scale

11. a. 589824 T 663552 TT 839808 QT 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648
Seventh d.Mixolydian QT enharmonic scale

b. 589824 TT 746496 T 839808 QT 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648
Seventh a.Hypophrygian QT enharmonic scale

c. 589824 TT 746496 QT 766464 QT 786432 T 884736 TT 1119744 QT 1149696 QT 1179648
Ninth d.Dorian QT enharmonic scale

d. 589824 TT 746496 QT 766464 QT 786432 TT 995328 T 1119744 QT 1149696 QT 1179648
Ninth a.Phrygian QT enharmonic scale

e. 574848 QT 589824 T 663552 TT 839808 QT 862272 QT 884736 TT 1119744 QT 1149696
Seventh d.Lydian QT enharmonic scale

f. 574848 QT 589824 TT 746496 T 839808 QT 862272 QT 884736 TT 1119744 QT 1149696
Seventh a.Hypolydian QT enharmonic scale

g. 574848 QT 589824 TT 746496 QT 766464 QT 786432 T 884736 TT 1119744 QT 1149696
Eighth d.Hypolydian QT enharmonic scale

h. 574848 QT 589824 TT 746496 QT 766464 QT 786432 TT 995328 T 1119744 QT 1149696
Eighth a.Lydian QT enharmonic scale

12. d.Dorian, a.Phrygian, associated d.Hypolydian, and associated a.Lydian scales give out.

a. 1179648 T 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296
Eighth d.Mixolydian QT enharmonic scale

b. 1179648 TT 1492992 T 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296
Eighth a.Hypophrygian QT enharmonic scale

c. 1149696 QT 1179648 T 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392
Eighth associated d.Lydian QT enharmonic scale

d. 1149696 QT 1179648 TT 1492992 T 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392
Eighth associated a.Hypolydian QT enharmonic scale

The ϑ-string enharmonic octave chains end at this point.

2.1.6 384 432 486 512 576 648 729 768

This section articulates the ε-string enharmonic octave chains beginning from 648.

1. 648 T 729 S 768 TT 972 S 1024 TT 1296
a.Hypodorian ST enharmonic scale

2. 1296 T 1458 QT 1497 QT 1536 TT 1944 QT 1996 QT 2048 TT 2592
First a.Hypodorian QT enharmonic scale

3. 2592 T 2916 QT 2994 QT 3072 TT 3888 QT 3992 QT 4096 TT 5184
Second a.Hypodorian QT enharmonic scale

4. a. 5184 T 5832 QT 5988 QT 6144 TT 7776 QT 7984 QT 8192 TT 10368
Third a.Hypodorian QT enharmonic scale

b. 5184 TT 6561 S 6912 TT 8748 S 9216 T 10368
D.Hypodorian ST enharmonic scale

5. a. 10368 T 11664 QT 11976 QT 12288 TT 15552 QT 15968 QT 16384 TT 20736
Fourth a.Hypodorian QT enharmonic scale

b. 10368 TT 13122 QT 13473 QT 13824 TT 17496 QT 17964 QT 18432 T 20736
First d.Hypodorian QT enharmonic scale

c. 10368 TT 13122 S 13824 T 15552 TT 19683 S 20736
d.Dorian ST enharmonic scale

d. 10368 TT 13122 S 13824 TT 17496 T 19683 S 20736
a.Phrygian ST enharmonic scale

6. a. 20736 T 23328 QT 23952 QT 24576 TT 31104 QT 31936 QT 32768 TT 41472
Fifth a.Hypodorian QT enharmonic scale

b. 20736 TT 26244 QT 26946 QT 27648 T 31104 TT 39366 QT 40419 QT 41472
First d.Dorian QT enharmonic scale
c. 20736 TT 26244 QT 26946 QT 27648 TT 34992 T 39366 QT 40419 QT 41472
First a.Phrygian QT enharmonic scale
d. 20736 TT 26244 QT 26946 QT 27648 TT 34992 QT 35928 QT 36864 T 41472
Second d.Hypodorian QT enharmonic scale

7. a. 41472 T 46656 QT 47904 QT 49152 TT 62208 QT 63872 QT 65536 TT 82944
Sixth a.Hypodorian QT enharmonic scale
b. 41472 TT 52488 QT 53892 QT 55296 T 62208 TT 78732 QT 80838 QT 82944
Second d.Dorian QT enharmonic scale
c. 41472 TT 52488 QT 53892 QT 55296 TT 69984 T 78732 QT 80838 QT 82944
Second a.Phrygian QT enharmonic scale
d. 41472 TT 52488 QT 53892 QT 55296 TT 69984 QT 71856 QT 73728 T 82944
Third d.Hypodorian QT enharmonic scale
e. 41472 T 46656 TT 59049 S 62208 TT 78732 S 82944
d.Mixolydian ST enharmonic scale
f. 41472 TT 52488 T 59049 S 62208 TT 78732 S 82944
a.Hypophrygian ST enharmonic scale
g. 40419 QT 41472 TT 52488 QT 53892 QT 55296 T 62208 TT 78732 QT 80838
First associated d.Hypolydian QT enharmonic scale
h. 40419 QT 41472 TT 52488 QT 53892 QT 55296 TT 69984 T 78732 QT 80838
First associated a.Lydian QT enharmonic scale

8. a. 82944 T 93312 QT 95808 QT 98304 TT 124416 QT 127744 QT 131072 TT 165888
Seventh a.Hypodorian QT enharmonic scale
b. 82944 T 93312 TT 118098 QT 121257 QT 124416 TT 157464 QT 161676 QT 165888
First d.Mixolydian QT enharmonic scale
c. 82944 TT 104976 T 118098 QT 121257 QT 124416 TT 157464 QT 161676 QT 165888
First a.Hypophrygian QT enharmonic scale
d. 82944 TT 104976 QT 107784 QT 110592 T 124416 TT 157464 QT 161676 QT 165888
Third d.Dorian QT enharmonic scale
e. 82944 TT 104976 QT 107784 QT 110592 TT 139968 T 157464 QT 161676 QT 165888
Third a.Phrygian QT enharmonic scale
f. 82944 TT 104976 QT 107784 QT 110592 TT 139968 QT 143712 QT 147456 T 165888
Fourth d.Hypodorian QT enharmonic scale
g. 80838 QT 82944 TT 104976 QT 107784 QT 110592 T 124416 TT 157464 QT 161676
Second associated d.Hypolydian QT enharmonic scale
h. 80838 QT 82944 TT 104976 QT 107784 QT 110592 TT 139968 T 157464 QT 161676
Second associated a.Lydian QT enharmonic scale

i. 80838 QT 82944 T 93312 TT 118098 QT 121257 QT 124416 TT 157464 QT 161676
First associated d.Lydian QT enharmonic scale

j. 80838 QT 82944 TT 104976 T 118098 QT 121257 QT 124416 TT 157464 QT 161676
First associated a.Hypolydian QT enharmonic scale

9. a. 165888 T 186624 QT 191616 QT 196608 TT 248832 QT 255488 QT 262144 TT 331776
Eighth a.Hypodorian QT enharmonic scale

b. 165888 T 186624 TT 236196 QT 242514 QT 248832 TT 314928 QT 323352 QT 331776
Second d.Mixolydian QT enharmonic scale

c. 165888 TT 209952 T 236196 QT 242514 QT 248832 TT 314928 QT 323352 QT 331776
Second a.Hypophrygian QT enharmonic scale

d. 165888 TT 209952 QT 215568 QT 221184 T 248832 TT 314928 QT 323352 QT 331776
Fourth d.Dorian QT enharmonic scale

e. 165888 TT 209952 QT 215568 QT 221184 TT 279936 T 314928 QT 323352 QT 331776
Fourth a.Phrygian QT enharmonic scale

f. 165888 TT 209952 QT 215568 QT 221184 TT 279936 QT 287424 QT 294912 T 331776
Fifth d.Hypodorian enharmonic scale

g. 161676 QT 165888 TT 209952 QT 215568 QT 221184 T 248832 TT 314928 QT 323352
Third associated d.Hypolydian QT enharmonic scale

h. 161676 QT 165888 TT 209952 QT 215568 QT 221184 TT 279936 T 314928 QT 323352
Third associated a.Lydian QT enharmonic scale

i. 161676 QT 165888 TT 209952 T 236196 QT 242514 QT 248832 TT 314928
Second associated a.Hypolydian QT enharmonic scale

j. 161676 QT 165888 T 186624 TT 236196 QT 242514 QT 248832 TT 314928 QT 323352
Second associated d.Lydian QT enharmonic scale

10. The a.Hypodorian enharmonic scale types cease.

a. 331776 T 373248 TT 472392 QT 485028 QT 497664 TT 629856 QT 646704 QT 663552
Third d.Mixolydian QT enharmonic scale

b. 331776 TT 419904 T 472392 QT 485028 QT 497664 TT 629856 QT 644204 QT 663552
Third a.Hypophrygian QT enharmonic scale

c. 331776 TT 419904 QT 431136 QT 442368 T 497664 TT 629856 QT 646704 QT 663552
Fifth d.Dorian QT enharmonic scale

d. 331776 TT 419904 QT 431136 QT 442368 TT 559872 T 629856 QT 646704 QT 663552
Fifth a.Phrygian QT enharmonic scale

e. 331776 TT 419904 QT 431136 QT 442368 TT 559872 QT 574848 QT 589824 T 663552
Sixth d.Hypodorian QT enharmonic scale

f. 323352 QT 331776 TT 419904 QT 431136 QT 442368 TT 559872 T 629856 QT 646704
Fourth associated a.Lydian QT enharmonic scale

g. 323352 QT 331776 T 373248 QT 383232 QT 393216 T 442368 TT 559872 QT 574848
Fourth associated d.Hypolydian QT enharmonic scale

h. 323352 QT 331776 T 373248 TT 472392 QT 485028 QT 497664 TT 629856 QT 646704
Third associated d.Lydian QT enharmonic scale

i. 323352 QT 331776 TT 419904 T 472392 QT 485028 QT 497664 TT 629856 QT 646704
Third associated a.Hypolydian QT enharmonic scale

11. a. 663552 T 746496 TT 944784 QT 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104
Fourth d.Mixolydian QT enharmonic scale

b. 663552 TT 839808 T 944784 QT 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104
Fourth a.Hypophrygian QT enharmonic scale

c. 663552 TT 839808 QT 862272 QT 884736 T 995328 TT 1259712 QT 1293408 QT 1327104
Sixth d.Dorian QT enharmonic scale

d. 663552 TT 839808 QT 862272 QT 884736 TT 1119744 T 1259712 QT 1293408 QT 1327104
Sixth a.Phrygian QT enharmonic scale

e. 663552 TT 839808 QT 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 T 1327104
Seventh d.Hypodorian QT enharmonic scale

f. 646704 QT 663552 TT 839808 QT 862272 QT 884736 T 995328 TT 1259712 QT 1293408
Fifth associated d.Hypolydian QT enharmonic scale

g. 646704 QT 663552 TT 839808 QT 862272 QT 884736 TT 1119744 T 1259712 QT 1293408
Fifth associated a.Lydian QT enharmonic scale

h. 646704 QT 663552 T 746496 TT 944784 QT 970056 QT 995328 TT 1259712 QT 1293408
Fourth associated d.Lydian QT enharmonic scale

i. 646704 QT 663552 TT 839808 T 944784 QT 970056 QT 995328 TT 1259712 QT 1293408
Fourth associated a.Hypolydian QT enharmonic scale

12. a. 1327104 T 1492992 TT 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208
Fifth d.Mixolydian QT enharmonic scale

b. 1327104 TT 1679616 T 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208
Fifth a.Hypophrygian QT enharmonic scale

c. 1327104 TT 1679616 QT 1724544 QT 1769472 T 1990656 TT 2519424 QT 2586816 QT 2654208
Seventh d.Dorian QT enharmonic scale

d. 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816 QT 2654208
Seventh a.Phrygian QT enharmonic scale

e. 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 T 2654208
Eighth d.Hypodorian QT enharmonic scale

f. 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 T 1990656 TT 2519424 QT 2586816
Sixth associated d.Hypolydian QT enharmonic scale

g. 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816
Sixth associated a.Lydian QT enharmonic scale

h. 1293408 QT 1327104 T 1492992 TT 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816
Fifth associated d.Lydian QT enharmonic scale

i. 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816
Fifth associated a.Hypolydian QT enharmonic scale

13. d.Hypodorian enharmonic scale types die out.
    a. 2654208 T 2985984 TT 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416
       Sixth d.Mixolydian QT enharmonic scale
    b. 2654208 TT 3359232 T 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416
       Sixth a.Hypophrygian QT enharmonic scale
    c. 2654208 TT 3359232 QT 3449088 QT 3538944 T 3981312 TT 5038848 QT 5173632 QT 5308416
       Eighth d.Dorian QT enharmonic scale
    d. 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976 T 5038848 QT 5173632 QT 5308416
       Eighth a.Phrygian QT enharmonic scale
    e. 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 T 3981312 TT 5038848 QT 5173632
       Seventh associated d.Hypolydian QT enharmonic scale
    f. 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976 T 5038848 QT 5173632
       Seventh associated a.Lydian QT enharmonic scale
    g. 2586816 QT 2654208 T 2985984 TT 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632
       Sixth associated. d.Lydian QT enharmonic scale
    h. 2586816 QT 2654208 TT 3359232 T 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632
       Sixth associated a.Hypolydian QT enharmonic scale

14. a. 5308416 T 5971968 TT 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832
       Seventh d.Mixolydian QT enharmonic scale
    b. 5308416 TT 6718464 T 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832
       Seventh a.Hypophrygian QT enharmonic scale
    c. 5308416 TT 6718464 QT 6898176 QT 7077888 T 7962624 TT 10077696 QT 10347264 QT 10616832
       Ninth d.Dorian QT enharmonic scale
    d. 5308416 TT 6718464 QT 6898176 QT 7077888 TT 8957952 T 10077696 QT 10347264 QT 10616832
       Ninth a.Phrygian QT enharmonic scale
    e. 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 T 7962624 TT 10077696 QT 10347264
       Eighth associated d.Hypolydian QT enharmonic scale

f. 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 TT 8957952 T 10077696 QT 10347264
Eighth associated a.Lydian QT enharmonic scale

g. 5173632 QT 5308416 T 5971968 TT 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264
Seventh associated d.Lydian QT enharmonic scale

h. 5173632 QT 5308416 TT 6718464 T 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264
Seventh associated a.Hypolydian QT enharmonic scale

15. The d.Dorian, a.Phrygian, associated d.Hypolydian and associated a.Lydian scale types die out.

a. 10616832 T 11943936 TT 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664
Eighth d.Mixolydian QT enharmonic scale

b. 10616832 TT 13436928 T 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664
Eighth a.Hypophrygian QT enharmonic scale

c. 10347264 QT 10616832 T 11943936 TT 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528
Eighth d.Lydian QT enharmonic scale

d. 10347264 QT 10616832 TT 13436928 T 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528
Eighth a.Hypolydian QT enharmonic scale

The ε-string enharmonic octave chains end at this point.

2.1.7 384 432 486 512 576 648 729 768

This section articulates the ζ-string enharmonic octave chains beginning from 729.

1. a. 729 S 768 TT 972 S 1024 TT 1296 T 1458
a.Mixolydian ST enharmonic scale

b. 729 S 768 TT 972 S 1024 TT 1296 T 1458
d.Hypophrygian ST enharmonic scale

2. a. 1458 QT 1497 QT 1536 TT 1944 QT 1996 QT 2048 T 2304 TT 2916
First d.Hypophrygian QT enharmonic scale

b. 1458 QT 1497 QT 1536 TT 1944 QT 1996 QT 2048 TT 2592 T 2916
First a.Mixolydian QT enharmonic scale

c. 1458 S 1536 T 1728 TT 2187 S 2304 TT 2916
d.Phrygian ST enharmonic scale

d. 1458 S 1536 TT 1944 T 2187 S 2304 TT 2916
a.Dorian ST enharmonic scale

e. 1497 QT 1536 TT 1944 QT 1996 QT 2048 T 2304 TT 2916 QT 2994
First associated d.Hypolydian QT enharmonic scale

f. 1497 QT 1536 TT 1944 QT 1996 QT 2048 TT 2592 T 2916 QT 2994
First associated a.Lydian QT enharmonic scale

3. a. 2916 QT 2994 QT 3072 T 3456 TT 4374 QT 4491 QT 4608 TT 5832
First d.Phrygian QT enharmonic scale

b. 2916 QT 2994 QT 3072 TT 3888 T 4374 QT 4491 QT 4608 TT 5832
First a.Dorian QT enharmonic scale

c. 2994 QT 3072 T 3456 TT 4374 QT 4491 QT 4608 TT 5832 QT 5988
First associated d.Lydian QT enharmonic scale

d. 2994 QT 3072 TT 3888 T 4374 QT 4491 QT 4608 TT 5832 QT 5988
First associated a.Hypolydian QT enharmonic scale

e. 2916 QT 2994 QT 3072 TT 3888 QT 3992 QT 4096 T 4608 TT 5832 QT 5988
Second d.Hypophrygian QT enharmonic scale

f. 2916 QT 2994 QT 3072 TT 3888 QT 3992 QT 4096 TT 5184 T 5832
Second a.Mixolydian QT enharmonic scale

g. 2994 QT 3072 TT 3888 QT 3992 QT 4096 T 4608 TT 5832 QT 5988
Second associated d.Hypolydian QT enharmonic scale

h. 2994 QT 3072 TT 3888 QT 3992 QT 4096 TT 5184 T 5832 QT 5988
Second associated a.Lydian QT enharmonic scale

4. a. 5832 QT 5988 QT 6144 T 6912 TT 8748 QT 8982 QT 9216 TT 11664
Second d.Phrygian QT enharmonic scale

b. 5832 QT 5988 QT 6144 TT 7776 T 8748 QT 8982 QT 9216 TT 11664
Second a.Dorian QT enharmonic scale

c. 5988 QT 6144 T 6912 TT 8748 QT 8982 QT 9216 TT 11664 QT 11976
Second associated d.Lydian QT enharmonic scale

d. 5988 QT 6144 TT 7776 T 8748 QT 8982 QT 9216 TT 11664 QT 11976
Second associated a.Hypolydian QT enharmonic scale

e. 5832 QT 5988 QT 6144 TT 7776 QT 7984 QT 8192 T 9216 TT 11664
Third d.Hypophrygian QT enharmonic scale

f. 5832 QT 5988 QT 6144 TT 7776 QT 7984 TT 10368 T 11664
Third a.Mixolydian QT enharmonic scale

g. 5988 QT 6144 TT 7776 QT 7984 QT 8192 T 9216 TT 11664 QT 11976
Third associated d.Hypolydian QT enharmonic scale

h. 5988 QT 6144 TT 7776 QT 7984 QT 8192 TT 10368 T 11664 QT 11976
Third associated a.Lydian QT enharmonic scale

i. 5832 T 6561 S 6912 TT 8748 S 9216 TT 11664
a.Hypodorian ST enharmonic scale

5.
a. 11664 11976 12288 13824 17496 17964 18432 23328
Third d.Phrygian QT enharmonic scale
b. 11664 QT 11976 QT 12288 TT 15552 T 17496 QT 17964 QT 18432 TT 23328
Third a.Dorian QT enharmonic scale
c. 11976 QT 12288 T 13824 TT 17496 QT 17964 QT 18432 TT 23328 QT 23952
Third associated d.Lydian QT enharmonic scale
d. 11976 QT 12288 TT 15552 T 17496 QT 17964 QT 18432 TT 23328 QT 23952
Third associated a.Hypolydian QT enharmonic scale
e. 11664 QT 11976 QT 12288 TT 15552 QT 15968 QT 16384 T 18432 TT 23328
Fourth d.Hypophrygian QT enharmonic scale
f. 11664 QT 11976 QT 12288 TT 15552 QT 15968 QT 16384 TT 20736 T 23328
Fourth a.Mixolydian QT enharmonic scale
g. 11976 QT 12288 TT 15552 QT 15968 QT 16384 T 18432 TT 23328 QT 23952
Fourth associated d.Hypolydian QT enharmonic scale
h. 11976 QT 12288 TT 15552 QT 15968 QT 16384 TT 20736 T 23328 QT 23952
Fourth associated a.Lydian QT enharmonic scale
i. 11664 T 13122 QT 13473 QT 13824 TT 17496 QT 17964 QT 18432 TT 23328
First a.Hypodorian QT enharmonic scale

6.
a. 23328 QT 23952 QT 24576 T 27648 TT 34992 QT 35928 QT 36864 TT 46656
Fourth d.Phrygian QT enharmonic scale
b. 23328 QT 23952 QT 24576 TT 31104 T 34992 QT 35928 QT 36864 TT 46656
Fourth a.Dorian QT enharmonic scale
c. 23952 QT 24576 T 27648 TT 34992 QT 35928 QT 36864 TT 46656 QT 47904
Fourth associated d.Lydian QT enharmonic scale
d. 23952 QT 24576 TT 31104 T 34992 QT 35928 QT 36864 TT 46656 QT 47904
Fourth associated a.Hypolydian QT enharmonic scale
e. 23328 QT 23952 QT 24576 TT 31104 QT 31936 QT 32768 T 36864 TT 46656
Fifth d.Hypophrygian QT enharmonic scale
f. 23328 QT 23952 QT 24576 TT 31104 QT 31936 QT 32768 TT 41472 T 46656
Fifth a.Mixolydian QT enharmonic scale
g. 23952 QT 24576 TT 31104 QT 31936 QT 32768 T 36864 TT 46656 QT 47904
Fifth associated d.Hypolydian QT enharmonic scale
h. 23952 QT 24576 TT 31104 QT 31936 QT 32768 TT 41472 T 46656 QT 47904
Fifth associated a.Lydian QT enharmonic scale
i. 23328 T 26244 QT 26946 Q 27648 TT 34992 QT 35928 QT 36864 TT 46656
Second a.Hypodorian QT enharmonic scale

7. a. 46656 QT 47904 QT 49152 T 55296 TT 69984 QT 71856 QT 73728 TT 93312
Fifth d.Phrygian QT enharmonic scale
b. 46656 QT 47904 QT 49152 TT 62208 T 69984 QT 71856 QT 73728 TT 93312
Fifth a.Dorian QT enharmonic scale
c. 47904 QT 49152 T 55296 TT 69984 QT 71856 QT 73728 TT 93312 QT 95808
Fifth associated d.Lydian QT enharmonic scale
d. 47904 QT 49152 TT 62208 T 69984 QT 71856 QT 73728 TT 93312 QT 95808
Fifth associated a.Hypolydian QT enharmonic scale
e. 46656 QT 47904 QT 49152 TT 62208 QT 63872 QT 65536 T 73728 TT 93312
Sixth d.Hypophrygian QT enharmonic scale
f. 46656 QT 47904 QT 49152 TT 62208 QT 63872 QT 65536 TT 82944 T 93312
Sixth a.Mixolydian QT enharmonic scale
g. 47904 QT 49152 TT 62208 QT 63872 QT 65536 T 73728 TT 93312 QT 95808
Sixth associated d.Hypolydian QT enharmonic scale
h. 47904 QT 49152 TT 62208 S 65536 TT 82944 T 93312
Sixth associated a.Lydian QT enharmonic scale
i. 46656 T 52488 S 55296 TT 69984 S 73728 TT 93312
Third a.Hypodorian QT enharmonic scale
j. 46656 TT 59049 S 62208 TT 78732 S 82944 T 93312
d.Hypodorian ST enharmonic scale

8. a. 93312 TT 118098 QT 121257 QT 124416 TT 157464 QT 161676 QT 165888 T 186624
First d.Hypodorian QT enharmonic scale
b. 93312 QT 95808 QT 98304 T 110592 TT 139968 QT 143712 QT 147456 TT 186624
Sixth d.Phrygian QT enharmonic scale
c. 93312 QT 95808 QT 98304 TT 124416 T 139968 QT 143712 QT 147456 TT 186624
QT 191616 QT 196608
Sixth a.Dorian QT enharmonic scale
d. 95808 QT 98304 T 110592 TT 139968 QT 143712 QT 147456 TT 186624 QT 191616
Sixth associated d.Lydian QT enharmonic scale
e. 95808 QT 98304 TT 124416 T 139968 QT 143712 QT 147456 TT 186624 QT 191616
Sixth associated a.Hypolydian QT enharmonic scale
f. 93312 QT 95808 QT 98304 TT 124416 QT 127744 QT 131072 T 147456 TT 186624
Seventh d.Hypophrygian QT enharmonic scale
g. 93312 QT 95808 QT 98304 TT 124416 QT 127744 QT 131072 TT 165888 T 186624
Seventh a.Mixolydian QT enharmonic scale
h. 95808 QT 98304 TT 124416 QT 127744 QT 131072 T 147456 TT 186624 QT 191616
Seventh associated d.Hypolydian QT enharmonic scale
i. 95808 QT 98304 TT 124416 QT 127744 QT 131072 TT 165888 T 186624 QT 191616
Seventh associated a.Lydian QT enharmonic scale

j. 93312 T 104976 QT 107784 QT 110592 TT 139968 QT 143712 QT 147456 TT 186624
Fourth a.Hypodorian QT enharmonic scale

9. a. 186624 TT 236196 QT 242514 QT 248832 TT 314928 QT 323352 QT 331776 T 373248
Second d.Hypodorian QT enharmonic scale

b. 186624 QT 191616 QT 196608 T 221184 TT 279936 QT 287424 QT 294912 TT 373248
Seventh d.Phrygian QT enharmonic scale

c. 186624 QT 191616 QT 196608 TT 248832 T 279936 QT 287424 QT 294912 TT 373248
Seventh a.Dorian QT enharmonic scale

d. 191616 QT 196608 T 221184 TT 279936 QT 287424 QT 294912 TT 373248 QT 383232
Seventh associated d.Lydian QT enharmonic scale

e. 191616 QT 196608 TT 248832 T 279936 QT 287424 QT 294912 TT 373248 QT 383232
Seventh associated a.Hypolydian QT enharmonic scale

f. 186624 QT 191616 QT 196608 TT 248832 QT 255488 QT 262144 T 294912 TT 373248
Eighth d.Hypophrygian QT enharmonic scale

g. 186624 QT 191616 QT 196608 TT 248832 QT 255488 QT 262144 TT 331776 T 373248
Eighth a.Mixolydian QT enharmonic scale

h. 191616 QT 196608 TT 248832 QT 255488 QT 262144 T 294912 TT 373248 QT 383232
Eighth associated d.Hypolydian QT enharmonic scale

i. 191616 QT 196608 TT 248832 QT 255488 QT 262144 TT 331776 T 373248 QT 383232
Eighth associated a.Lydian QT enharmonic scale

j. 186624 T 209952 QT 215568 QT 221184 TT 279936 QT 287424 QT 294912 TT 373248
Fifth a.Hypodorian QT enharmonic scale

10. d.Hypophrygian, a.Mixolydian, d.Hypolydian, and a.Lydian enharmonic scale types die.

a. 373248 TT 472392 QT 485028 QT 497664 TT 629856 QT 646704 QT 663552 T 746496
Third d.Hypodorian QT enharmonic scale

b. 373248 QT 383232 QT 393216 T 442368 TT 559872 QT 574848 QT 589824 TT 746496
Eighth d.Phrygian QT enharmonic scale

c. 373248 QT 383232 QT 393216 TT 497664 T 559872 QT 574888 QT 589824 TT 746496
Eighth a.Dorian QT enharmonic scale

d. 383232 QT 393216 T 442368 TT 559872 QT 574848 QT 589824 TT 746496 QT 766464
Eighth associated d.Lydian QT enharmonic scale

e. 383232 QT 393216 TT 497664 T 559872 QT 574848 QT 589824 TT 746496 QT 766464
Eighth associated a.Hypolydian beginning

f. 373248 T 419904 QT 431136 QT 442368 TT 559872 QT 574848 QT 589824 TT 746496
Sixth a.Hypodorian QT enharmonic scale

11. a. 746496 TT 944784 QT 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 T 1492992
Fourth d.Hypodorian QT enharmonic scale

b. 746496 QT 766464 QT 786432 T 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992
Ninth d.Phrygian QT enharmonic scale

c. 746496 QT 766464 QT 786432 TT 995328 T 1119744 QT 1149696 QT 1179648 TT 1492992
Ninth a.Dorian QT enharmonic scale

d. 766464 QT 786432 T 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992 QT 1539648
Ninth associated d.Lydian QT enharmonic scale
Note that the final quarter tone is achieved by splitting tone as follows: [(1492992 + 1679616)/2 + 1492992]/2 = 1539648. The size of the quarter tone is 1.03125.

e. 766464 QT 786432 TT 995328 T 1119744 QT 1149696 QT 1179648 TT 1492992 QT 1539648
Ninth associated a.Hypolydian QT enharmonic scale
Note that the final quarter tone is achieved by splitting tone as follows: [(1492992 + 1679616)/2 + 1492992]/2 = 1539648. The size of the quarter tone is 1.03125.

f. 746496 T 839808 QT 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992
Seventh a.Hypodorian QT enharmonic scale

12. d.Phrygian, a.Dorian, d.Lydian, and a.Hypolydian scale types vanish.
    a. 1492992 TT 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208 T 2985984
    Fifth d.Hypodorian QT enharmonic scale
    b. 1492992 T 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 TT 2985984
    Eighth a.Hypodorian QT enharmonic scale

13. a.Hypodorian QT enharmonic scale vanishes.
    2985984 TT 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 T 5971968
    Sixth d.Hypodorian QT enharmonic scale

14. 5971968 TT 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832 T 11943936
    Seventh d.Hypodorian QT enharmonic scale

15. 11943936 TT 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 T 23887872
    Eighth d.Hypodorian QT enharmonic scale

The ζ-string enharmonic octave chains end at this point.

2.2 *Enharmonic Scales Based on Chromatic Numbers*

2.2.1 Enharmonic Scales Beginning from Chromatic Factors of 1719926784

2.2.1.1 *Series 1*

This section articulates the CF1-string enharmonic octave chains beginning from 2187.

1. a. 2187 S 2304 TT 2916 S 3072 T 3456 TT 4374
   d.Hypophrygian ST enharmonic scale
   b. 2187 S 2304 TT 2916 S 3072 TT 3888 T 4374
   a.Mixolydian ST enharmonic scale

2. a. 4374 QT 4491 QT 4608 TT 5832 QT 5988 QT 6144 T 6912 TT 8748
   First d.Hypophrygian QT enharmonic scale
   b. 4374 QT 4491 QT 4608 TT 5832 QT 5988 QT 6144 TT 7776 T 8748
   First a.Mixolydian QT enharmonic scale
   c. 4374 S 4608 T 5184 TT 6561 S 6912 TT 8748
   d.Phrygian ST enharmonic scale
   d. 4374 S 4608 TT 5832 T 6144 S 6912 TT 8748
   a.Dorian ST enharmonic scale

e. 4491 QT 4608 TT 5832 QT 5988 QT 6144 T 6912 TT 8748 QT 8982
First associated d.Hypolydian QT enharmonic scale
f. 4491 QT 4608 TT 5832 QT 5988 QT 6144 TT 7776 T 8748 QT 8982
First associated a.Lydian QT enharmonic scale

3. a. 8748 QT 8982 QT 9216 TT 11664 QT 11976 QT 12288 T 13824 TT 17496
Second d.Hypophrygian QT enharmonic scale
b. 8748 QT 8982 9216 TT 11664 QT 11976 QT 12288 TT 15552 T 17496
Second a.Mixolydian QT enharmonic scale
c. 8748 QT 8982 QT 9216 T 10368 TT 13122 QT 13473 QT 13824 TT 17496
First d.Phrygian QT enharmonic scale
d. 8748 QT 8982 QT 9216 TT 11664 T 13122 QT 13473 QT 13824 TT 17496
First a.Dorian QT enharmonic scale
e. 8982 QT 9216 T 10368 TT 13122 QT 13473 QT 13824 TT 17496 QT 17964
First associated d.Lydian QT enharmonic scale
f. 8982 QT 9216 TT 11664 T 13122 QT 13473 QT 13824 TT 17496 QT 17964
First associated a.Hypolydian QT enharmonic scale
g. 8982 QT 9216 TT 11664 QT 11976 QT 12288 T 13824 TT 17496 QT 17964
Second associated d.Hypolydian QT enharmonic scale
h. 8982 QT 9216 T 10368 TT 13122 QT 13473 QT 13824 TT 17496 QT 17964
Second associated a.Lydian QT enharmonic scale

4. a. 17496 QT 17964 QT 18432 TT 23328 QT 23952 QT 24576 T 27648 TT 34992
Third d.Hypophrygian QT enharmonic scale
b. 17496 QT 17964 QT 18432 TT 23328 QT 23952 QT 24576 TT 31104 T 34992
Third a.Mixolydian QT enharmonic scale
c. 17496 QT 17964 QT 18432 T 20736 TT 26244 QT 26946 QT 27648 TT 34992
Second d.Phrygian QT enharmonic scale
d. 17496 QT 17964 QT 18432 TT 23328 T 26244 QT 26946 QT 27648 TT 34992
Second a.Dorian QT enharmonic scale
e. 17964 QT 18432 TT 23328 QT 23952 QT 24576 T 27648 TT 34992 QT 35928
Third associated d.Hypolydian QT enharmonic scale
f. 17964 QT 18432 TT 23328 QT 23952 QT 24576 TT 31104 T 34992 QT 35928
Third associated a.Lydian QT enharmonic scale
g. 17964 QT 18432 T 20736 TT 26244 QT 26946 QT 27648 TT 34992 QT 35928
Second associated d.Lydian QT enharmonic scale
h. 17964 QT 18432 TT 23328 T 26244 QT 26946 QT 27648 TT 31104 QT 35928
Second associated a.Hypolydian QT enharmonic scale
i. 17496 T 19683 S 20736 TT 26244 S 27648 TT 34992
First a.Hypodorian ST enharmonic scale

5.
a. 34992 QT 35928 QT 36864 TT 46656 QT 47904 QT 49152 T 55296 TT 69984
Fourth d.Hypophrygian QT enharmonic scale
b. 34992 QT 35928 QT 36864 TT 46656 QT 47904 QT 49152 TT 62208 T 69984
Fourth a.Mixolydian QT enharmonic scale
c. 34992 QT 35928 QT 36864 T 41472 TT 52488 QT 53892 QT 55296 TT 69984
Third d.Phrygian QT enharmonic scale
d. 34992 QT 35928 QT 36864 TT 46656 T 52488 QT 53892 QT 55296 TT 69984
Third a.Dorian QT enharmonic scale
e. 35928 QT 36864 T 41472 TT 52488 QT 53892 QT 55296 TT 69984 QT 71856
Third associated d.Lydian QT enharmonic scale
f. 35928 QT 36864 TT 46656 T 52488 QT 53892 QT 55296 TT 69984 QT 71856
Third associated a.Hypolydian QT enharmonic scale
g. 35928 QT 36864 TT 46656 QT 47904 QT 49152 T 55296 TT 69984 QT 71856
Fourth associated d.Hypolydian QT enharmonic scale
h. 35928 QT 36864 TT 46656 QT 47904 QT 49152 TT 62208 T 69984 QT 71856
Fourth associated a.Lydian QT enharmonic scale
i. 34992 T 39366 QT 40419 QT 41472 TT 52488 QT 53892 QT 55296 TT 69984
First a.Hypodorian QT enharmonic scale

6.
a. 69984 QT 71856 QT 73728 T 82944 TT 104976 QT 107784 QT 110592 TT 139968
Fourth d.Phrygian QT enharmonic scale
b. 69984 QT 71856 QT 73728 TT 93312 T 104976 QT 107784 QT 110592 TT 139968
Fourth a.Dorian QT enharmonic scale
c. 69984 QT 71856 QT 73728 TT 93312 QT 95808 QT 98304 T 110592 TT 139968
Fifth d.Hypophrygian QT enharmonic scale
d. 69984 QT 71856 QT 73728 TT 93312 QT 95808 QT 98304 TT 124416 T 139968
Fifth a.Mixolydian QT enharmonic scale
e. 71856 QT 73728 T 82944 TT 104976 QT 107784 QT 110592 TT 139968 QT 143712
Fourth associated d.Lydian QT enharmonic scale
f. 71856 QT 73728 TT 93312 T 104976 QT 107784 QT 110592 TT 139968 QT 143712
Fourth associated a.Hypolydian QT enharmonic scale
g. 71856 QT 73728 TT 93312 QT 95808 QT 98304 T 110592 TT 139968 QT 143712
Fifth associated d.Hypolydian QT enharmonic scale
h. 71856 QT 73728 TT 93312 QT 95808 QT 98304 TT 124416 T 139968 QT 143712
Fifth associated a.Lydian QT enharmonic scale
i. 69984 T 78732 QT 80838 QT 82944 TT 104976 QT 107784 QT 110592 TT 139968
Second a.Hypodorian QT enharmonic scale

7.
a. 139968 QT 143712 QT 147456 T 165888 TT 209952 QT 215568 QT 221184 TT 279936
Fifth d.Phrygian QT enharmonic scale

b. 139968 QT 143712 QT 147456 TT 186624 T 209952 QT 215568 QT 221184 TT 279936
Fifth a.Dorian QT enharmonic scale

c. 139968 QT 143712 QT 147456 TT 186624 QT 191616 QT 196608 T 221184 TT 279936
Sixth d.Hypophrygian QT enharmonic scale

d. 139968 QT 143712 QT 147456 TT 186624 QT 191616 QT 196608 TT 248832 T 279936
Sixth a.Mixolydian QT enharmonic scale

e. 143712 QT 147456 T 165888 TT 209952 QT 215568 QT 221184 TT 279936 QT 287424
Fifth associated d.Lydian QT enharmonic scale

f. 143712 QT 147456 TT 186624 T 209952 QT 215568 QT 221184 TT 279936 QT 287424
Fifth associated a.Hypolydian QT enharmonic scale

g. 143712 QT 147456 TT 186624 QT 191616 QT 196608 T 221184 TT 279936 QT 287424
Sixth associated d.Hypolydian QT enharmonic scale

h. 143712 QT 147456 TT 186624 QT 191616 QT 196608 TT 248832 T 279936 QT 287424
Sixth associated a.Lydian QT enharmonic scale

i. 139968 T 157464 QT 161676 QT 165888 TT 209952 QT 215568 QT 221184 TT 279936
Third a.Hypodorian QT enharmonic scale

8. a. 279936 QT 287424 QT 294912 T 331776 TT 419904 QT 431136 QT 442368 TT 559872
Sixth d.Phrygian QT enharmonic scale

b. 279936 QT 287424 QT 294912 TT 373248 T 419904 QT 443136 QT 442368 TT 559872
Sixth a.Dorian QT enharmonic scale

c. 279936 QT 287424 QT 294912 TT 373248 QT 383232 QT 393216 T 442368 TT 559872
Seventh d.Hypophrygian QT enharmonic scale

d. 279936 QT 287424 QT 294912 TT 373248 QT 383232 QT 393216 TT 497664 T 559872
Seventh a.Mixolydian QT enharmonic scale

e. 287424 QT 294912 T 331776 TT 419904 QT 443136 QT 442368 TT 559872 QT 574848
Sixth associated d.Lydian QT enharmonic scale

f. 287424 QT 294912 TT 373248 T 419904 QT 443136 QT 442368 TT 559872 QT 574888
Sixth associated a.Hypolydian QT enharmonic scale

g. 287424 QT 294912 TT 373248 QT 383232 QT 393216 T 442368 TT 559872 QT 574848
Seventh d.Hypolydian QT enharmonic scale

h. 287424 QT 294912 TT 373248 QT 383232 QT 393216 TT 497664 T 559872 QT 574888
Seventh a.Lydian QT enharmonic scale

i. 279936 T 314928 QT 323352 QT 331776 TT 419904 QT 443136 QT 442368 TT 559872
Fourth a.Hypodorian QT enharmonic scale

9. a. 559872 QT 574848 QT 589824 T 663552 TT 839808 QT 862272 QT 884736 TT 1119744
Seventh d.Phrygian QT enharmonic scale

b. 559872 QT 574848 QT 589824 TT 746496 T 839808 QT 862292 QT 884776 TT 1119744
Seventh a.Dorian QT enharmonic scale

c. 559872 QT 574848 QT 589824 TT 746496 QT 766464 QT 786432 T 884736 TT 1119744
Eighth d.Hypophrygian QT enharmonic scale

d. 559872 QT 574848 QT 589824 TT 746496 QT 766464 QT 786432 TT 995328 T 1119744
Eighth a.Mixolydian QT enharmonic scale

e. 574848 QT 589824 T 663552 TT 839808 QT 862272 QT 884736 TT 1119744 QT 1149696
Seventh associated d.Lydian QT enharmonic scale

f. 574848 QT 589824 TT 746496 T 839808 QT 862272 QT 884736 TT 1119744 QT 1149696
Seventh associated a.Hypolydian QT enharmonic scale

g. 574848 QT 589824 TT 746496 QT 766464 QT 786432 T 884736 TT 1119744 QT 1149696
Eighth associated d.Hypolydian QT enharmonic scale

h. 574848 QT 589824 TT 746496 QT 766464 QT 786432 TT 995328 T 1119744 QT 1149696
Eighth a.Lydian QT enharmonic scale

i. 559872 T 629856 QT 646704 QT 663552 TT 839808 QT 862272 QT 884736 TT 1119744
Fifth a.Hypodorian QT enharmonic scale

10. d.Hypophrygian, a.Mixolydian, associated d.Hypolydian, and associated a.Lydian scale types vanish.

a. 1119744 QT 1149696 QT 1179648 T 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488
Eighth d.Phrygian QT enharmonic scale

b. 1119744 QT 1149696 QT 1179648 TT 1492992 T 1679616 QT 1724544 QT 1769472 TT 2239488
Eighth a.Dorian QT enharmonic scale

c. 1149696 QT 1179648 T 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392
Eighth associated d.Lydian QT enharmonic scale

d. 1149696 QT 1179648 TT 1492992 T 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392
Eighth associated a.Hypolydian QT enharmonic scale

e. 1119744 T 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488
Sixth a.Hypodorian QT enharmonic scale

11. a. 2239488 QT 2299392 QT 2359296 T 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976
Ninth d.Phrygian QT enharmonic scale

b. 2239488 QT 2299392 QT 2359296 TT 2985984 T 3359232 QT 3449088 QT 3538944 TT 4478976
Ninth a.Dorian QT enharmonic scale

c. 2299392 QT 2359296 T 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976 QT 4618944
Ninth associated d.Lydian QT enharmonic scale

d. 2299392 QT 2359296 TT 2985984 T 3359232 QT 3449088 QT 3538944 TT 4478976 QT 4618944
Ninth associated a.Hypolydian QT enharmonic scale

e. 2239488 T 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976
Seventh a.Hypodorian QT enharmonic scale

12. d.Phrygian, a.Dorian, d.Lydian, and a.Hypolydian enharmonic scales vanish.
4478976 T 5038848 QT 5173832 QT 5308816 TT 6718464 QT 6898176 QT 7077888 TT 8957952
Eighth a.Hypodorian QT enharmonic scale

The CF1-string enharmonic octave chains end at this point.

2.2.1.2 *Series 2*

This section articulates the CF2-string enharmonic octave chains beginning from 6561.

1. a. 6561 S 6912 TT 8748 S 9216 T 10368 TT 13122
   d.Hypophrygian ST enharmonic scale
   b. 6561 S 6912 TT 8748 S 9216 TT 11664 T 13122
   a.Mixolydian ST enharmonic scale

2. a. 13122 QT 13473 QT 13824 TT 17496 QT 17964 QT 18432 T 20736 TT 26244
   First d.Hypophrygian QT enharmonic scale
   b. 13122 QT 13473 QT 13824 TT 17496 QT 17964 18432 TT 23328 T 26244
   First a.Mixolydian QT enharmonic scale
   c. 13122 S 13824 T 15552 TT 19683 S 20736 TT 23328
   d.Phrygian ST enharmonic
   d. 13122 S 13824 TT 17496 T 19683 S 20736 TT 26244
   a.Dorian ST enharmonic scale
   e. 13473 QT 13824 TT 17496 QT 17964 QT 18432 T 20736 TT 26244 QT 26946
   First associated d.Hypolydian QT enharmonic scale
   f. 13473 QT 13824 TT 17496 QT 17964 QT 18432 TT 23328 T 26244 QT 26946
   First associated a.Lydian QT enharmonic scale

3. a. 26244 QT 26946 QT 27648 TT 34992 QT 35928 QT 36864 T 41472 TT 52488
   Second d.Hypophrygian QT enharmonic scale
   b. 26244 QT 26946 QT 27648 TT 34992 QT 35928 QT 36864 TT 46656 T 52488
   Second a.Mixolydian QT enharmonic scale
   c. 26946 QT 27648 TT 34992 QT 35928 QT 36864 T 41472 TT 52488 QT 53892
   Second associated d.Hypolydian QT enharmonic scale
   d. 26496 QT 27648 TT 34992 QT 35928 QT 36864 TT 46656 T 52488 QT 53892
   Second associated a.Lydian QT enharmonic scale
   e. 26244 QT 26496 QT 27648 T 31104 TT 39366 QT 40419 QT 41472 TT 52488
   First d.Phrygian QT enharmonic scale
   f. 26244 QT 26496 QT 27648 TT 34992 T 39366 QT 40419 QT 41472 TT 52488
   First a.Dorian QT enharmonic scale
   g. 26496 QT 27648 T 31104 TT 39366 QT 40419 QT 41472 TT 52488 QT 53892
   First associated d.Lydian QT enharmonic scale
   h. 26496 QT 27648 TT 34992 T 39366 QT 40419 QT 41472 TT 52488 QT 53892
   First associated a.Hypolydian QT enharmonic scale

4. a. 52488 QT 53892 QT 55296 TT 69984 QT 71856 QT 73728 T 82944 TT 104976
   Third d.Hypophrygian QT enharmonic scale

b. 52488 QT 53892 QT 55296 TT 69984 QT 71856 QT 73728 TT 93312 T 104976
Third a.Mixolydian QT enharmonic scale

c. 53892 QT 55296 TT 69984 QT 71856 QT 73728 T 82944 TT 104976 QT 107784
Third associated d.Hypolydian QT enharmonic scale

d. 53892 QT 55296 TT 69984 QT 71856 QT 73728 TT 93312 T 104976
Third associated a.Lydian scale QT enharmonic scale

e. 52488 QT 53892 QT 55296 T 62208 TT 78732 QT 80838 QT 82944 TT 104976
Second d.Phrygian QT enharmonic scale

f. 52488 QT 53892 QT 55296 TT 69984 T 78732 QT 80838 QT 82944 TT 104976
Second a.Dorian QT enharmonic scale

g. 53892 QT 55296 T 62208 TT 78732 QT 80838 QT 82944 TT 104976 QT 107784
Second associated d.Lydian QT enharmonic scale

h. 53892 QT 55296 TT 69984 T 78732 QT 80838 QT 82944 TT 104976
Second associated a.Hypolydian QT enharmonic scale

i. 52488 T 59049 S 62208 TT 78732 S 82944 TT 104976
a.Hypodorian ST enharmonic scale

5. a. 104976 QT 107784 QT 110592 TT 139968 QT 143712 QT 147456 T 165888 TT 209952
Fourth d.Hypophrygian QT enharmonic scale

b. 104976 QT 107784 QT 110592 TT 139968 QT 143712 QT 147456 TT 186624 T 209952
Fourth a.Mixolydian QT enharmonic scale

c. 107784 QT 110592 TT 139968 QT 143712 QT 147456 T 165888 TT 209952 QT 215568
Fourth associated d.Hypolydian QT enharmonic scale

d. 107784 QT 110592 TT 139968 QT 143712 QT 147456 TT 186624 T 209952
Fourth associated a.Lydian QT enharmonic scale

e. 104976 QT 107784 QT 110592 T 124416 TT 157464 QT 161676 QT 165888 TT 209952
Third d.Phrygian QT enharmonic scale

f. 104976 QT 107784 QT 110592 TT 139968 T 157464 QT 161676 QT 165888 TT 209952
Third a.Dorian QT enharmonic scale

g. 107784 QT 110592 T 124416 TT 157464 QT 161676 QT 165888 TT 209952 QT 215568
Third associated d.Lydian QT enharmonic scale

h. 107784 QT 110592 TT 139968 T 157464 QT 161676 QT 165888 TT 209952
Third associated a.Hypolydian QT enharmonic scale

i. 104976 T 118098 QT 121257 QT 124416 TT 157464 QT 161676 QT 165888 TT 209952
First a.Hypodorian QT enharmonic scale

6. a. 209952 QT 215568 QT 221184 TT 279936 QT 287424 QT 294912 T 331776 TT 419904
Fifth d.Hypophrygian QT enharmonic scale
b. 209952 QT 215568 QT 221184 TT 279936 QT 287424 QT 294912 TT 373248 T 419904
Fifth a.Mixolydian QT enharmonic scale
c. 215568 QT 221184 TT 279936 QT 287424 QT 294912 T 331776 TT 419904 QT 431136
Fifth associated d.Hypolydian QT enharmonic scale
d. 215568 QT 221184 TT 279936 QT 287424 QT 294912 TT 373248 T 419904
Fifth associated a.Lydian QT enharmonic scale
e. 209952 QT 215568 QT 221184 T 248832 TT 314928 QT 323352 QT 331776 TT 419904
Fourth d.Phrygian QT enharmonic scale
f. 209952 QT 215568 QT 221184 TT 279936 T 314928 QT 323352 QT 331776 TT 419904
Fourth a.Dorian QT enharmonic scale
g. 215568 QT 221184 T 248832 TT 314928 QT 323352 QT 331776 TT 419904 QT 431136
Fourth associated d.Lydian QT enharmonic scale
h. 215568 QT 221184 TT 279936 T 314928 QT 323352 QT 331776 TT 419904 QT 431136
Fourth associated a.Hypolydian QT enharmonic scale
i. 209952 T 236196 QT 242514 QT 248832 TT 314928 QT 323352 QT 331776 TT 419904
Second a.Hypodorian QT enharmonic scale

7. a. 419904 QT 431136 QT 442368 TT 559872 QT 574848 QT 589824 T 663552 TT 839808
Sixth d.Hypophrygian QT enharmonic scale
b. 419904 QT 431136 QT 442368 TT 559872 QT 574848 QT 589824 TT 746496 T 839808
Sixth a.Mixolydian QT enharmonic scale
c. 431136 QT 442368 TT 559872 QT 574848 QT 589824 T 663552 TT 839808 QT 862272
Sixth associated d.Hypolydian QT enharmonic scale

d. 431136 QT 442368 TT 559872 QT 574848 QT 589824 TT 746496 T 839808 QT 862272
Sixth associated a.Lydian QT enharmonic scale

e. 419904 QT 431136 QT 442368 T 497664 TT 629856 QT 646704 QT 663552 TT 839808
Fifth d.Phrygian QT enharmonic

f. 419904 QT 431136 QT 442368 TT 559872 T 629856 QT 646704 QT 663552 TT 839808
Fifth a.Dorian QT enharmonic scale

g. 431136 QT 442368 T 497664 TT 629856 QT 646704 QT 663552 TT 839808 QT 862272
Fifth associated d.Lydian QT enharmonic scale

h. 431136 QT 442368 TT 559872 T 629856 QT 646704 QT 663552 TT 839808 QT 862272
Fifth associated a.Hypolydian QT enharmonic scale

i. 419904 T 472392 QT 485028 QT 497664 TT 629856 QT 646704 QT 663552 TT 839808
Third a.Hypodorian QT enharmonic scale

8. a. 839808 QT 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 T 1327104 TT 1679616
Seventh d.Hypophrygian QT enharmonic scale

b. 839808 QT 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992 T 1679616
Seventh a.Mixolydian QT enharmonic scale

c. 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 T 1327104 TT 1679616 QT 1724544
Seventh associated d.Hypolydian QT enharmonic scale

d. 862272 QT 884736 TT 1119744 QT 1149696 QT 1179648 TT 1492992 T 1679616 QT 1724544
Seventh associated a.Lydian QT enharmonic scale

e. 839808 QT 862272 QT 884736 T 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616
Sixth d.Phrygian QT enharmonic scale

f. 839808 QT 862272 QT 884736 TT 1119744 T 1259712 QT 1293408 QT 1327104 TT 1679616
Sixth a.Dorian QT enharmonic scale

g. 862272 QT 884736 T 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544
Sixth associated d.Lydian QT enharmonic scale

h. 862272 QT 884736 TT 1119744 T 1259712 QT 1293408 QT 1327104 TT 1679616
Sixth associated a.Hypolydian QT enharmonic scale

i. 839808 T 944784 QT 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616
Fourth a.Hypodorian QT enharmonic scale

9. a. 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 T 2654208 TT 3359232
Eighth d.Hypophrygian QT enharmonic scale

b. 1679616 QT 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 TT 2985984 T 3359232
Eighth a.Mixolydian QT enharmonic scale

c. 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 T 2654208 TT 3359232 QT 3449088
Eighth associated d.Hypolydian QT enharmonic scale

d. 1724544 QT 1769472 TT 2239488 QT 2299392 QT 2359296 TT 2985984 T 3359232 QT 3449088
Eighth associated a.Lydian QT enharmonic scale

e. 1679616 QT 1724544 QT 1769472 T 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232
Seventh d.Phrygian QT enharmonic scale

f. 1679616 QT 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816 QT 2654208 TT 3359232
Seventh a.Dorian QT enharmonic scale

g. 1724544 QT 1769472 T 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088
Seventh associated d.Lydian QT enharmonic scale

h. 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816 QT 2654208 TT 3359232
Seventh associated a.Hypolydian QT enharmonic scale

i. 1679616 T 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232
Fifth a.Hypodorian QT enharmonic scale

10. d.Hypophrygian, a.Mixolydian, associated d.Hypolydian, and associated a.Lydian scale types vanish.

a. 3359232 QT 3449088 QT 3538944 T 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464
Eighth d.Phrygian QT enharmonic scale

b. 3359232 QT 3449088 QT 3538944 TT 4478976 T 5038848 QT 5173632 QT 5308416 TT 6718464
Eighth a.Dorian QT enharmonic scale

c. 3449088 QT 3538944 T 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464
Eighth d.Lydian QT enharmonic scale

d. 3449088 QT 3538944 TT 4478976 T 5038848 QT 5173632 QT 5308416 TT 6718464
Eighth a.Hypolydian QT enharmonic scale

e. 3359232 T 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464
Sixth a.Hypodorian QT enharmonic scale

11. a. 6718464 QT 6898176 QT 7077888 T 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436938
Ninth d.Phrygian QT enharmonic scale

b. 6718464 QT 6898176 QT 7077888 TT 8957952 T 10077696 QT 10347264 QT 10616832 TT 13436928
Ninth a.Dorian QT enharmonic scale

c. 6898176 QT 7077888 T 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436928 QT 13856832
Ninth associated d.Lydian QT enharmonic scale
Note: The last QT splits a tone as follows: [((13436928 + 15116544)/2) + 13436928]/2 = 13856832. The size of the quarter tone is 1.03125.

d. 6898176 QT 7077888 TT 8957952 T 10077696 QT 10347264 QT 10616832 TT 13436928 QT 13856832
Ninth associated a.Hypolydian QT enharmonic scale
Note: The last QT splits a tone as follows: [((13436928 + 15116544)/2) + 13436928]/2 = 13856832. The size of the quarter tone is 1.03125.

e. 6718464 T 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436928
Seventh a.Hypodorian QT enharmonic scale

12. The associated d.Lydian, associated a.Hypolydian, d.Phrygian, and a.Dorian scale types vanish.
13436938 T 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 TT 26873856
Eighth a.Hypodorian QT enharmonic scale

The CF2-string enharmonic octave chains end at this point.

2.2.2 Enharmonic Scales Beginning from Chromatic Nonfactors of 1719926784

2.2.2.1 *Series 1*

This section articulates the CNF1-string enharmonic octave chains beginning from 19683.

1. a. 19683 S 20736 TT 26244 S 27648 T 31104 TT 39366
   d.Hypophrygian ST enharmonic scale
   b. 19683 S 20736 TT 26244 S 27648 TT 34992 T 39366
   a.Mixolydian ST enharmonic scale

2. a. 39366 QT 40419 QT 41472 TT 52488 QT 53892 QT 55296 T 62208 TT 78732
   First d.Hypophrygian QT enharmonic scale
   b. 39366 QT 40419 QT 41472 TT 52488 QT 53892 QT 55296 TT 69984 T 78732
   First a.Mixolydian QT enharmonic scale
   c. 39366 S 41472 T 46656 TT 59049 S 62208 TT 78732
   First d.Phrygian ST enharmonic scale
   d. 39366 S 41472 TT 52488 T 59049 S 62208 TT 78732
   First a.Dorian ST enharmonic scale
   e. 40419 QT 41472 TT 53488 QT 53892 QT 55296 T 62208 TT 78732 QT 80838
   First associated d.Hypolydian QT enharmonic scale
   f. 40419 QT 41472 TT 52488 QT 53892 QT 55296 TT 69984 T 78732 QT 80838
   First associated a.Lydian QT enharmonic scale

3. a. 78732 QT 80838 QT 82944 TT 104976 QT 107784 QT 110592 T 124416 TT 157464
   Second d.Hypophrygian QT enharmonic scale
   b. 78732 QT 80838 QT 82944 TT 104976 QT 107784 QT 110592 TT 139968 T 157464
   Second a.Mixolydian QT enharmonic scale
   c. 80838 QT 82944 TT 104976 QT 107784 QT 110592 T 124416 TT 157464 QT 161676
   Second associated d.Hypolydian QT enharmonic scale
   d. 80838 QT 82944 TT 104976 QT 107784 QT 110592 TT 139968 T 157464
   Second associated a.Lydian QT enharmonic scale
   e. 78732 QT 80838 QT 82944 T 93312 TT 118098 QT 121257 QT 124416 TT 157464
   First d.Phrygian QT enharmonic scale
   f. 78732 QT 80838 QT 82944 TT 104976 T 118098 QT 121257 QT 124416 TT 157464
   First a.Dorian QT enharmonic scale
   g. 80838 QT 82944 T 93312 TT 118098 QT 121257 QT 124416 TT 157464 QT 161676
   First d.Lydian QT enharmonic scale
   h. 80838 QT 82944 TT 104976 T 118098 QT 121257 QT 124416 TT 157464 QT 161676
   First a.Hypolydian QT enharmonic scale

4. a. 157464 QT 161676 QT 165888 TT 209952 QT 215568 QT 221184 T 248832 TT 314928
   Third d.Hypophrygian QT enharmonic scale
   b. 157464 QT 161676 QT 165888 TT 209952 QT 215568 QT 221184 TT 279936 T 314928
   Third a.Mixolydian QT enharmonic scale
   c. 161676 QT 165888 TT 209952 QT 215568 QT 221184 T 248832 TT 314928 QT 323352
   Third associated d.Hypolydian QT enharmonic scale
   d. 161676 QT 165888 TT 202952 QT 215568 QT 221184 TT 279936 T 314928 QT 323352
   Third associated a.Lydian QT enharmonic scale
   e. 157464 QT 161676 QT 165888 T 186624 TT 236196 QT 242514 QT 248832 TT 314928
   Second d.Phrygian QT enharmonic scale
   f. 157464 QT 161676 QT 165888 TT 209952 T 236196 QT 242514 QT 248832 TT 314928
   Second a.Dorian QT enharmonic scale
   g. 161676 QT 165888 T 186624 TT 236196 QT 242514 QT 248832 TT 314928 QT 323352
   Second associated d.Lydian QT enharmonic scale
   h. 161676 QT 165888 TT 209952 T 236196 QT 242514 QT 248832 TT 314928 QT 323352
   Second a.Hypolydian QT enharmonic scale

5. a. 314928 QT 323352 QT 331776 TT 419904 QT 431136 QT 442368 T 497664 TT 629856
   Fourth d.Hypophrygian QT enharmonic scale
   b. 314928 QT 323352 QT 331776 TT 419904 QT 431136 QT 442368 TT 559872 T 629856
   Fourth a.Mixolydian QT enharmonic scale
   c. 323352 QT 331776 TT 419904 QT 431136 QT 442368 T 497664 TT 629856 QT 646704
   Fourth associated d.Hypolydian QT enharmonic scale
   d. 323352 QT 331776 TT 419904 QT 431136 QT 442368 TT 559872 T 629856 QT 646704
   Fourth associated a.Lydian QT enharmonic scale
   e. 314928 QT 323352 QT 331776 T 373248 TT 472392 QT 485028 QT 497664 TT 629856
   Third d.Phrygian QT enharmonic scale

f. 314928 QT 323352 QT 331776 TT 419904 T 472392 QT 485028 QT 497664 TT 629856
Third a.Dorian QT enharmonic scale

g. 323352 QT 331776 T 373248 TT 472392 QT 485028 QT 497664 TT 629856 QT 646704
Third associated d.Lydian QT enharmonic scale

h. 323352 QT 331776 TT 419904 T 472392 QT 485028 QT 497664 TT 629856 QT 646704
Third associated a.Hypolydian QT enharmonic scale

6. a. 629856 QT 646704 QT 663552 TT 839808 QT 862272 QT 884736 T 995328 TT 1259712
Fifth d.Hypophrygian QT enharmonic scale

b. 629856 QT 646704 QT 663552 TT 839808 QT 862272 QT 884736 TT 1119744 T 1259712
Fifth a.Mixolydian QT enharmonic scale

c. 646704 QT 663552 TT 839808 QT 862272 QT 884736 T 995328 TT 1259712 QT 1293408
Fifth associated d.Hypolydian QT enharmonic scale

d. 646704 QT 663552 TT 839808 QT 862272 QT 884736 TT 1119744 T 1259712 QT 1293408
Fifth associated a.Lydian QT enharmonic scale

e. 629856 QT 646704 QT 663552 T 746496 TT 944784 QT 970056 QT 995328 TT 1259712
Fourth d.Phrygian QT enharmonic scale

f. 629856 QT 646704 QT 663552 TT 839808 T 944784 QT 970056 QT 995328 TT 1259712
Fourth a.Dorian QT enharmonic scale

g. 646704 QT 663552 T 746496 TT 944784 QT 970056 QT 995328 TT 1259712 QT 1293408
Fourth associated d.Lydian QT enharmonic scale

h. 646704 QT 663552 TT 839808 T 944784 QT 970056 QT 995328 TT 1259712 QT 1293408
Fourth associated a.Hypolydian QT enharmonic scale

7. a. 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 T 1990656 TT 2519424
Sixth d.Hypophrygian QT enharmonic scale

b. 1259712 QT 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 T 2519424
Sixth a.Mixolydian QT enharmonic scale

c. 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 T 1990656 TT 2519424 QT 2586816
Sixth associated d.Hypolydian QT enharmonic scale

d. 1293408 QT 1327104 TT 1679616 QT 1724544 QT 1769472 TT 2239488 T 2519424 QT 2586816
Sixth associated a.Lydian QT enharmonic scale

e. 1259712 QT 1293408 QT 1327104 T 1492992 TT 1889568 QT 1940112 QT 1990656 TT 2519424
Fifth d.Phrygian QT enharmonic scale

f. 1259712 QT 1293408 QT 1327104 TT 1679616 T 1889568 QT 1940112 QT 1990656 TT 2519424
Fifth a.Dorian QT enharmonic scale

g. 1293408 QT 1327104 T 1492992 TT 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816
Fifth d.Lydian QT enharmonic scale

h. 1293408 QT 1327104 TT 1679616 T 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816
Fifth a.Hypolydian QT enharmonic scale

8. a. 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 T 3981312 TT 5038848
Seventh d.Hypophrygian QT enharmonic scale

b. 2519424 QT 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976 T 5038848
Seventh a.Mixolydian QT enharmonic scale

c. 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538949 T 3981312 TT 5038848 QT 5173632
Seventh associated d.Hypolydian QT enharmonic scale

d. 2586816 QT 2654208 TT 3359232 QT 3449088 QT 3538944 TT 4478976 T 5038848 QT 5173632
Seventh associated a.Lydian QT enharmonic scale

e. 2519424 QT 2586816 QT 2654208 T 2985984 TT 3779136 QT 3880224 QT 3981312 TT 5038848
Sixth d.Phrygian QT enharmonic scale

f. 2519424 QT 2586816 QT 2654208 TT 3359232 T 3779136 QT 3880224 QT 3981312 TT 5038848
Sixth a.Dorian QT enharmonic scale

g. 2586816 QT 2654208 T 2985984 TT 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632
Sixth associated d.Lydian QT enharmonic scale

h. 2586816 QT 2654208 TT 3359232 T 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632
Sixth associated a.Hypolydian QT enharmonic scale

9. a. 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 T 7962624 TT 10077696
Eighth d.Hypophrygian QT enharmonic scale
b. 5038848 QT 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 TT 8957952 T 10077696
Eighth a.Mixolydian QT enharmonic scale
c. 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 T 7962624 TT 10077696 QT 10347264
Eighth associated d.Hypolydian QT enharmonic scale
d. 5173632 QT 5308416 TT 6718464 QT 6898176 QT 7077888 TT 8957952 T 10077696 QT 10347264
Eighth associated a.Lydian QT enharmonic scale
e. 5038848 QT 51736322 QT 5308416 T 5971968 TT 7558272 QT 7760448 QT 7962624 TT 10077696
Seventh d.Phrygian QT enharmonic scale
f. 5038848 QT 5173632 QT 5308416 TT 6718464 T 7558272 QT 7760448 QT 7962624 TT 10077696
Seventh a.Dorian QT enharmonic scale
g. 51736322 QT 5308416 T 5971968 TT 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264
Seventh associated d.Lydian QT enharmonic scale
h. 5173632 QT 5308416 TT 6718464 T 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264
Seventh associated a.Hypolydian QT enharmonic scale

10. d.Hypophrygian, a.Mixolydian, associated d.Hypolydian, and associated a.Lydian enharmonic patterns vanish.
a. 1077696 QT 10347264 QT 10616832 T 11943936 TT 15116544 QT 15520896 QT 15925248 TT 20155392
Eighth d.Phrygian QT enharmonic scale
b. 10077696 QT 10347264 QT 10616832 TT 13436928 T 15116544 QT 15520896 QT 15925248 TT 20155392
Eighth a.Dorian QT enharmonic scale
c. 10347264 QT 10616832 T 11943936 TT 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528
Eighth associated d.Lydian QT enharmonic scale

d. 10347264 QT 10616832 TT 13436928 T 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528
Eighth associated a.Hypolydian QT enharmonic scale

11. Associated d.Lydian and associated a.Hypolydian enharmonic scale types vanish.
a. 20155392 QT 20694528 QT 21233664 T 23887872 TT 30233088 QT 31041792 QT 31850496 TT 40310784
Ninth d.Phrygian QT enharmonic scale type
b. 20155392 QT 20694528 QT 21233664 TT 26873856 T 30233088 QT 31041792 QT 31850496 TT 40310784
Ninth a.Dorian QT enharmonic scale type

The CNF1-string enharmonic octave chains end at this point.

2.2.2.2 *Series 2*

This section articulates the CNF1-string enharmonic octave chains beginning from 59049

1. a. 59049 S 62208 TT 78732 S 82944 T 93312 TT 118098
d.Hypophrygian ST enharmonic scale
b. 59049 S 62208 TT 78732 S 82944 TT 104976 T 118098
a.Mixolydian ST enharmonic scale

2. a. 118098 QT 121257 QT 124416 T 157464 QT 161676 QT 165888 T 186624 TT 236196
First d.Hypophrygian QT enharmonic scale
b. 118098 QT 121257 QT 124416 TT 157464 QT 161676 QT 165888 TT 209952 T 236196
First a.Mixolydian QT enharmonic scale
c. 121257 QT 124416 TT 157464 QT 161676 QT 165888 T 186624 TT 236196 QT 242514
First associated d.Hypolydian QT enharmonic scale
d. 121257 QT 124416 TT 157464 QT 161676 QT 165888 TT 209952 T 236196
First associated a.Lydian QT enharmonic scale

3. a. 236196 QT 242514 QT 248832 TT 314928 QT 323352 QT 331776 T 373248 TT 472392
Second d.Hypophrygian QT enharmonic scale
b. 236196 QT 242514 QT 248832 TT 314928 QT 323352 QT 331776 TT 419904 T 472392
Second a.Mixolydian QT enharmonic scale

c. 242514 QT 248832 TT 314928 QT 323352 QT 331776 T 373248 TT 472392 QT 485028
Second associated d.Hypolydian QT enharmonic scale

d. 242514 QT 248832 TT 314928 QT 323352 QT 331776 TT 419904 T 472392 QT 485028
Second associated a.Lydian QT enharmonic scale

4. a. 472392 QT 485028 QT 497664 TT 629856 QT 646704 QT 663552 T 746496 TT 944784
Third d.Hypophrygian QT enharmonic scale

b. 472392 QT 485028 QT 497664 TT 629856 QT 646704 QT 663552 TT 839808 T 944784
Third a.Mixolydian QT enharmonic scale

c. 485028 QT 497664 TT 629856 QT 646704 QT 663552 T 746496 TT 944784 QT 970056
Third associated d.Hypolydian QT enharmonic scale

d. 485028 QT 497664 TT 629856 QT 646704 QT 663552 TT 839808 T 944784
Third associated a.Lydian QT enharmonic scale

5. a. 944784 QT 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 T 1492992 TT 1889568
Fourth d.Hypophrygian QT enharmonic scale

b. 944784 QT 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 T 1889568
Fourth a.Mixolydian QT enharmonic scale

c. 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 T 1492992 TT 1889568 QT 1940112
Fourth associated d.Hypolydian QT enharmonic scale

d. 970056 QT 995328 TT 1259712 QT 1293408 QT 1327104 TT 1679616 T 1889568 QT 1940112
Fourth associated a.Lydian QT enharmonic scale

6. a. 1889568 1940112 1990656 TT 2519424 QT 2586816 QT 2654208 T 2985984 TT 3779136
Fifth d.Hypophrygian QT enharmonic scale

b. 1889568 QT 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232 T 3779136
Fifth a.Mixolydian QT enharmonic scale

c. 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208 T 2985984 TT 3779136 QT 3880224
Fifth associated d.Hypolydian QT enharmonic scale

d. 1940112 QT 1990656 TT 2519424 QT 2586816 QT 2654208 TT 3359232 T 3779136 QT 3880224
Fifth associated a.Lydian enharmonic scale

7. a. 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 T 5971968 TT 7558272
Sixth d.Hypophrygian QT enharmonic scale
b. 3779136 QT 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 T 7558272
Sixth a.Mixolydian QT enharmonic scale
c. 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 T 5971968 TT 7558272 QT 7760448
Sixth associated d.Hypolydian QT enharmonic scale
d. 3880224 QT 3981312 TT 5038848 QT 5173632 QT 5308416 TT 6718464 T 7558272 QT 7760448
Sixth associated a.Lydian QT enharmonic scale

8. a. 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832 T 11943936 TT 15116544
Seventh d.Hypophrygian QT enharmonic scale
b. 7558272 QT 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436928 T 15116544
Seventh a.Mixolydian QT enharmonic scale
c. 7760448 QT 7962629 TT 10077696 QT 10347264 QT 10616832 T 11943936 TT 15116544 QT 15520896
Seventh associated d.Hypolydian QT enharmonic scale
d. 7760448 QT 7962624 TT 10077696 QT 10347264 QT 10616832 TT 13436928 T 15116544 QT 15520896
Seventh associated a.Lydian QT enharmonic scale

9. a. 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 T 23887872 TT 30233088
Eighth d.Hypophrygian QT enharmonic scale
b. 15116544 QT 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 TT 26873856 T 30233088
Eighth a.Mixolydian QT enharmonic scale
c. 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 T 23887872 TT 30233088 QT 31041792
Eighth d.Hypolydian QT enharmonic scale

d. 15520896 QT 15925248 TT 20155392 QT 20694528 QT 21233664 TT 26873856 T 30233088 QT 31041792
Eighth a.Lydian QT enharmonic scale

The CNF2-string enharmonic octave chains end at this point.

Glossary of Musical Terms and Concepts

1 Ancient Interval Names

A. *Apotomē*: Ancient name for one of the two semitones occurring within octave species of the chromatic *genus*; characterized by the ratio 2187/2048 between the tone frequency marking the end of the subject interval and the tone frequency marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end).[1] The *apotomē* and the *leimma* (the other semitone—see the definition below) together comprise the interval of the ancient whole tone.[2]

B. *Diatessaron:* Ancient name for the musical fourth; ratio, known as the *sesquitertian* (4/3) ratio, between a number indexed to string vibration or impacts on air, marking the end of the subject interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end).[3]

C. *Diapente:* Ancient name for the musical fifth; ratio between a number indexed to string vibration or impacts on air, marking the end of the subject interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end), known as the *sesquialter* (3/2) ratio.[4]

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1 Thomas J. Mathiesen, "Greek Music Theory," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002; reprint, Cambridge: Cambridge University Press, 2004), 116; Reese, *Music*, 20, 21, 23; André Barbera, ed. and trans., *The Euclidean Division of the Canon: Greek and Latin Sources*, Greek and Latin Music Theory, eds. Thomas J. Mathiesen and Jon Solomon (Lincoln: University of Nebraska Press, 1991), 40, 48, 51, 60, 115, 231; Plutarch *De animae procreatione in Timaeo* 1021A–C; Flora R. Levin, *The Manual of Harmonics of Nicomachus the Pythagorean*, translated with annotations (Grand Rapids: Phanes Press, 1994), 61–71, 91–94, 174; Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, trans. Edwin L. Minar, Jr. (Cambridge: Harvard University Press, 1972), 380 n. 47; Proclus *in Timaeum* 3.2.168.19–20, 180.4 (and n. 268), 188.20–26, 189.10–11, 190.26–27.

2 Proclus *in Timaeum* 3.2.180.25–181.24, 188.20–26, 189.10–11.

3 Reese, *Music*, 20, 21, 42; Barbera, *Euclidean Division*, 18, 40, 48, 51, 60, 115, 231; Plutarch *De animae* 1018 D–E, 1020 F, 1021 A–C; Levin, *Manual*, 61–71, 91–94; Burkert, *Lore and Science*, 380 n. 47.

4 Ibid.

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D. *Diapason:* Ancient name for the octave; ratio between a number indexed to string vibration or impacts on air, marking the end of the subject interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end), known as the double (2/1) ratio.[5]

> *Note Bene*: An octave is comprised of a fourth + a fifth (4/3 × 3/2 = 12/6 = 2/1) or two disjunct fourths, i.e., two fourths separated by a whole tone (4/3 × 9/8 × 4/3 = 144/72 =2/1).[6]

E. *Diapason Diapente:* Ancient name for the interval comprised by an octave (*diapason*) and a fifth (*diapente*); corresponds to a musical twelfth; ratio between a number indexed to string vibration or impacts on air, marking the end of the subject interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end), known as the triple (3/1) ratio.[7]

F. *Diesis:* Ancient name, most accurately, for the intervals corresponding to quarter tones within the octave species of the enharmonic genus; also sometimes loosely used to refer to the semitone.[8]

G. *Disdiapason:* Ancient name for the interval spanned by two consecutive octaves; ratio between the a number indexed to string vibration or impacts on air, marking the end of the subject interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end), known as the double octave or quadruple (4/1) ratio.[9]

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5 Ibid., 20, 21, 42; Barbera, *Euclidean Division*, 18, 40, 48, 51, 60, 115, 231; Plutarch *De animae* 1018 D–E, 1021 A–C; Levin, *Manual*, 61–71, 75, 91–94; Burkert, *Lore and Science*, 380 n. 47.

6 Reese, *Music*, 23–24, 28–29, 42, 47–48; Plutarch *De animae* 1021 B–C; Levin, *Manual*, 80 n. 2, 100, 105–106 n. 7.

7 Christian Meyer, "L'Âme du monde dans la rationalité musicale: ou l'expérience sensible d'un order intelligible," in *Harmonia mundi, musica mondana e musica celeste fra antichità e medioevo*: *Atti del convegno internazionale di studi, Roma, 14–15 dicembre 2005*, a cura di Marta Cristiani, Cecilia Panti e Graziono Perillo, *Micrologus' Library* 19 (Firenze: Sismel—Edizione del Galluzzo, 2007), 69–70; Reese, *Music*, 20, 21; Barbera, *Euclidean Division*, 18, 40, 48, 51, 60, 115, 231; Plutarch *De animae* 1018 E, 1021 A; Levin, *Manual*, 61–71, 91–94; Burkert, *Lore and Science*, 380 n. 47; Proclus *in Timaeum* 3.2.168.4.

8 Proclus *in Timaeum* 3.2.168.15–25; Mathiesen, "Greek Music Theory," 119; Levin, *Manual*, 44 n. 9, 139.

9 Reese, *Music*, 20, 21, 42; Barbera, *Euclidean Division*, 18, 40, 48, 51, 60, 115, 162–163, 163 n. 50, 231; Plutarch *De animae* 1018 F, 1021 A–B; Levin, *Manual*, 61–71, 91–94; Burkert, *Lore and Science*, 380 n. 47.

H. *Komma* (*comma*)*:* The ratio of the *apotomē* (2187/2048) to the *leimma* (256/243), i.e., 531441/524288; measures the extent to which the *apotomē* exceeds the *leimma*.[10]

I. *Leimma:* Ancient name for one of the semitones occurring within the octave species of all three *genera*, diatonic, chromatic, and enharmonic; characterized by the ratio 256/243 between a number indexed to string vibration or impacts on air, marking the end of the subject interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end); sometimes referred to as *diesis*, in reference to its being the smallest interval in the relevant scale, namely the diatonic. Most specifically understood, this was the interval remaining over in an ancient tetrachord after it had been filled with two whole tones. Note that the *leimma* also occurred in the ancient fifth comprised of three whole tones and a *diesis* and that the *leimma* comprising the *diesis* was movable from the last position in the fourth or fifth, depending upon the particular *tonos* or octave species in which it occurred. A description of the different *tonoi* is provided further below.[11] The *apotomē* (the other semitone germane to the octave species of the chromatic genus) and the *leimma* together comprise the interval of the ancient whole tone.[12]

J. *Pyknon:* Name for the ditone consisting in two semitones in a chromatic scale or two quarter tones in an enharmonic scale.[13]

K. *Schisma:* The term used by Philolaus to designate ½ of a *komma*.[14]

L. *Tonos:* In the ancient whole tone interval, the applicable ratio between the a number indexed to string vibration or impacts on air, marking the end of the interval, and a number indexed to string vibration or impacts on air, marking its beginning (or alternatively, the string length marking the beginning and the string length marking the end), was the *sesquioctave* (9/8) ratio.[15]

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10 Proclus *in Timaeum* 3.2.183.30–184.1–10; Levin, *Manual*, 135, 185.

11 Mathiesen, "Greek Music Theory," 116; Reese, *Music*, 20, 21, 28; Barbera, *Euclidean Division*, 40, 48, 51, 60, 115, 231; Plutarch *De animae* 1020 C–F, 1021 A–1022 C; Levin, *Manual*, 44 n. 9, 61–71, 91–94, 115, 119, 125, 127, 131, 136, 139, 147, 175, 183, 189–191; Burkert, *Lore and Science*, 380 n. 47; Proclus *in Timaeum* 3.2.168.15–25, 168.19 n. 226, 177.5–179.5.

12 Proclus *in Timaeum* 3.2.180.25–181.24.

13 Reese, *Music*, 23, 32; Levin, *Manual*, 171; Barbera, *Euclidean Division*, 1, 177, 177 n. 65.

14 Levin, *Manual*, 135–136, 185n. 6.

15 Mathiesen, "Greek Music Theory," 115–116; Reese, *Music*, 20; Plutarch *De animae* 1020 E–F, 1021 A–C; Levin, *Manual*, 91; Barbera, *Euclidean Division*, 290n. 36.

2 Ways of Dividing Various Intervals[16]

A. *Sesquitertian Part* (*the fourth, Diatessaron*): Every sesquitertian (4/3) part contains two sesquioctave (9/8) intervals and one remainder over of 256/243; sesquitertian parts can be divided according to such intervals in various permutations. Thus:

4/3 = 9/8 × 9/8 × 256/243
4/3 = 9/8 × 256/243 × 9/8
4/3 = 256/243 × 9/8 × 9/8

B. *Sesquialter Part* (*the fifth, Diapente):* Every sesquialter part (3/2) contains three sesquioctave (9/8) intervals and one remainder over of 256/243 or one sesquitertian part and a sesquioctave interval; sesquialter parts can be divided according to such intervals in various permutations. Thus:

3/2 = 4/3 × 9/8
3/2 = 9/8 × 4/3
3/2 = 9/8 × 9/8 × 256/243 × 9/8
3/2 = 9/8 × 256/243 × 9/8 × 9/8
3/2 = 256/243 × 9/8 × 9/8 × 9/8
3/2 = 9/8 × 9/8 × 9/8 × 256/243

C. *The Double Interval* (*Octave; diapason*): Every double interval (2:1) contains a tetrachord plus a pentachord or, equivalently two disjunct tetrachords, i.e., two tetrachords separated by a sesquioctave (9/8) interval.

2/1 = 3/2 × 4/3
2/1 = 4/3 × 3/2
2/1 = 4/3 × 9/8 × 4/3
2/1 = 9/8 × 4/3 × 4/3
2/1 = 4/3 × 4/3 × 9/8

Any of the permutations listed above for 3/2 and 4/3 in paragraphs A and B can be substituted where 3/2 and 4/3 respectively appear in the patterns for the octave.

16 This section simply sets forth all mathematical possibilities; it does not imply that ancient Greek theorists or musicians used them all.

D. *The Triple Interval* (*musical twelfth; Diapason Diapente*)*:* Every triple interval contains three (4/3) sesquitertian parts and two (9/8) sesquioctave intervals or, equivalently two sesquialter parts and one sesquitertian part.

3/1 = 3/2 × 4/3 × 3/2
3/1 = 3/2 × 3/2 × 4/3
3/1 = 4/3 × 3/2 × 3/2
3/1 = 4/3 × 4/3 × 4/3 × 9/8 × 9/8
3/1 = 4/3 × 4/3 × 9/8 × 4/3 × 9/8
3/1 = 4/3 × 9/8 × 4/3 × 4/3 × 9/8
3/1 = 9/8 × 4/3 × 4/3 × 4/3 × 9/8
3/1 = 9/8 × 4/3 × 4/3 × 9/8 × 4/3
3/1 = 9/8 × 4/3 × 9/8 × 4/3 × 4/3
3/1 = 9/8 × 9/8 × 4/3 × 4/3 × 4/3
3/1 = 4/3 × 9/8 × 9/8 × 4/3 × 4/3
3/1= 4/3 × 4/3 × 9/8 × 9/8 × 4/3
3/1 = 4/3 × 9/8 × 4/3 × 9/8 × 4/3

Any of the permutations listed above for 3/2 and 4/3 in paragraphs A and B can be substituted where 3/2 and 4/3 respectively appear in the patterns for the triple (musical twelfth).

3 Systems and Associated Terms of Ancient Greek Music

A. *Diezeugmenon:* Term used to describe two fourths, disjunct from one another by a sesquioctave interval, rather than conjoined.[17]

B. *Greater Perfect System* (*"GPS"*)*:* Term designating an extended note system elaborated by fourth century B.C. theorists consisting of two pairs of conjunct fourths disjunct from each other by a whole tone, with the double octave completed at the bottom by an additional tone, *proslambanomenos*, necessary for theoretical, rather than musical reasons.[18]

The names of the four tetrachords of GPS, in descending order of pitch, were *hyperbolaion, diezeugmenon, meson*, and *hypaton*, with the disjunction in the system

17 Isobel Henderson, "Ancient Greek Music," in *Ancient and Oriental Music*, ed. Egon Wellesz, *The New Oxford History of Music*, vol. 1 (London: Oxford University Press, 1957; repr., London: Oxford University Press, 1969), 345; Levin, *Manual*, 110–111.

18 Henderson, "Ancient Greek Music," 345.

between *diezeugmenon* and *meson* and with *proslambanomenos*' not being counted as part of any fourth.[19]

The names of the notes of GPS in descending order of pitch were as follows: *nete hyperbolaion, paranete hyperbolaion, trite hyperbolaion, nete diezeugmenon, paranete diezeugmenon, trite diezeugmenon, paramese* (*diezeugmenon*), *mese* (*meson*), *lichanos meson, parhypate meson, hypate meson, lichanos hypaton, parhypate hypaton, hypate hypaton, and proslambanomenos*. Each note, as is evident, was named with its fourth. *Paramese* and *mese* were exceptional, in this regard, because their positions could not be confused by virtue of any similarity of name with the positions of other notes. The pitch values of the system were relative, not absolute; further the pitch values of particular notes, once a pitch range had been selected, could vary depending upon the genus (diatonic, chromatic, or enharmonic) of the scale with which one worked.[20]

C. *Lesser Perfect System* (*"LPS"*)*:* Term designating an extended note system elaborated by fourth century B.C. theorists consisting of a pair of conjunct fourths, immediately conjoined to a third fourth, with the system completed at the bottom by the additional tone, *proslambanomenos*, necessary for theoretical, rather than musical reasons.[21]

The names of the three conjunct fourths of the system in descending order of pitch were *synemmenon, meson*, and *hypaton. Proslambanomenos* did not count as the constituent of any fourth.[22]

The names of the notes of LPS were *nete synemmenon, paranete synemmenon, trite synemmenon, mese* (*synemmenon*), *lichanos meson, parhypate meson, hypate meson, lichanos hypaton, parhypate hypaton, hypate hypaton, and proslambanomenos*. As in the case of GPS, the names of the notes were given with the names of the fourths to which they belonged. *Mese* was exceptional, in this regard, because its position could not be confused by virtue of any similarity of name with the positions of other notes. The pitch values of the system were relative, not absolute; further the pitch values of particular notes, once a pitch range had been selected, could vary depending upon the genus (diatonic, chromatic, or enharmonic) of the scale with which one worked.[23]

19 Ibid., 346; Jan Herlinger, "Medieval Canonics," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002; reprint, Cambridge: Cambridge University Press, 2004), 172; Reese, *Music*, 22; Levin, *Manual*, 164.

20 Henderson, "Ancient Greek Music," 345–346; cf., Mathiesen, "Greek Music Theory," 122; Herlinger, "Medieval Canonics," 172; Reese, 22–23; Levin, *Manual*, 110–111.

21 Henderson, "Ancient Greek Music," 344.

22 Ibid., 346; Levin, *Manual*, 164.

23 Henderson, "Ancient Greek Music," 345–346; cf., Mathiesen, "Greek Music Theory," 122; Herlinger, "Medieval Canonics," 172; Levin, *Manual*, 110–111.

D. *Synemmenon*: Term used to describe two fourths conjunct with one another, rather than disjunct by any interval.[24]

4 Ancient Names of the Steps of the Diatonic Scale

From the lowest pitch to the highest pitch, the names of the steps in the diatonic scale were the following:

hypate, parhypate, lichanos, mese, paramese, trite, paranete, nete.[25]

These names are appropriate, even though "*hypate*" means "high" and "*nete*" means "low," because the names of the steps in Greek were not assigned with reference to pitch but, rather, with reference to the position of a note on the lyre. Ancient lyre players held their lyres with the narrow end down, so that the "highest string" was the lowest pitch. The "highest string," in other words, was the longest one. The longest string obviously produced the lowest tone. The narrow end of the lyre bearing the shortest strings occupied the "lowest" position. These shorter strings clearly produced the highest pitches.[26]

5 Sequence of Whole Tones and Dieses in an Ascending Lydian Diatonic Scale

Assuming some starting tone number for a particular octave, the sequence is as follows from lowest to highest pitch for an ascending Lydian diatonic scale:

TTSTTTS[27]

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24 Henderson, "Ancient Greek Music," 345; Levin, *Manual*, 110–111.

25 Levin, *Manual*, 45–53 (names of notes and derivation thereof), 138n. 16 (for usual descending order of the scale); Dirk Baltzly, "Introduction to Book 3, Part II," in *Book 3, Part II: Proclus on the World Soul*, trans. with an introduction and notes by Dirk Baltzly, vol. 4, *Proclus: Commentary on Plato's Timaeus* (Cambridge: Cambridge University Press, 2009), 13.

26 Reese, *Music*, 22.

27 Ibid., 30, 40–41.

6 Relation of Lydian Diatonic Scale to the Dorian Diatonic Scale

Assuming some starting tone number for a particular octave, the sequence is as follows from lowest to highest pitch for an ascending Dorian diatonic scale:

STTTSTT[28]

The descending Dorian scale, on the other hand, i.e., the scale proceeding from highest to lowest pitch, is the following:

TTSTTTS[29]

Clearly, an ascending Lydian has the same tone/diesis pattern as a descending Dorian. The Lydian and Dorian diatonic scales are, then, perfect reciprocals.

This study demonstrates that the primary *Timaeus* scale is an ascending Lydian diatonic scale, if the *Timaeus* numbers are indices to string vibration, but a descending Dorian diatonic scale if they, instead, represent string lengths. It is just possible that Plato allowed room for both interpretations. The analysis does not change one wit. One simply has to reverse the note names (proceeding from *nete* to *hypate*, rather than vice versa), so that they reflect the descending order for the Dorian scale.

7 Correlation of the Steps of an Ascending Lydian Scale or Descending Dorian Scale with the Ancient Names for the Steps of the Scale

One can represent the TTSTTTS pattern (descending Dorian or ascending Lydian) of the model octave string comprising the *Timaeus* scale, with Greek names for the steps. In the *ordo* of an ascending Lydian diatonic scale (ascending Lydian because descending pattern of Lydian is STTTSTT), the Greek names of the elements from lowest to highest pitch are, *hypate-T-parhypate-T-lichanos-S-mese-T-paramese*--T-*trite-T-paranete--S-nete*.

Assuming a descending order of the scale—so the descending Dorian diatonic scale—the Greek names of the elements from highest to lowest pitch in the scale are *nete-T-paranete-T-trite-S-paramese-T-mese-T-lichanos*-T-*parhypate-S-hypate*. Note that the intervals between elements with the same names in the ascending Lydian and the descending Dorian are different.[30]

28 Ibid.

29 Ibid.

30 The argument in this section is grounded in the preceding sections.

TABLE 64 List of patterns for the seven dominant octave species within the three genera in descending order[31]

Octave species	Diatonic	Chromatic	Enharmonic
Mixolydian	TTTSTTS	T [TS]SS [TS]SS	T [TT]QQ [TT]QQ
Lydian	STTTSTT	S T [TS]SS [TS]S	Q T [TT]QQ [TT]Q
Phrygian	TSTTTST	SS T [TS]SS [TS]	QQ T [TT]QQ [TT]
Dorian	TTSTTTS	[TS]SS T [TS]SS	[TT]QQ T [TT]QQ
Hypolydian	STTSTTT	S [TS]SS T [TS]S	Q [TT]QQ T [TT]Q
Hypophrygian	TSTTSTT	SS [TS]SS T [TS]	QQ [TT]QQ T [TT]
Hypodorian	TTSTTST	[TS]SS [TS]SS T	[TT]QQ [TT]QQ T

8 Model Octave

The model octave is the first series of whole numbers meeting the conditions for the octave set by the *Timaeus*. Those numbers are the following:

384 432 486 512 576 648 729 768
STN × 9/8 × 9/8 × 256/243 × 9/8 × 9/8 × 9/8 × 256/243
STN T T S T T T S[32]

Note that at least one ancient author, Severus, disagreed with beginning at 384. He found such a starting point offensive because it meant that the entire musical scale that Plato was thought to have been constructing, in his *Timaeus*, consisting of four octaves, a fifth, and an additional tone, would end with a tone.[33] Proclus reported that Severus thought it more appropriate that the indicated scale end with a semitone, since Plato had concluded his account of divisions within the soul with that ratio. Beginning the scale with 768, instead of 384, achieved such a result; in his view, the model octave would have been the first that made such a scale possible and, so, would have begun with 768 and ended with 1536.[34]

31 Reese, *Music*, 30–32, 41.
32 Proclus *in Timaeum* 3.2.178.1–179.8 and 185.3–187.16.
33 Ibid., 2.187.12–16.
34 Ibid., 2.191.1–192.14.

TABLE 65 Reciprocal relationships obvious upon inspection

A. Diatonic	B. Chromatic	C. Enharmonic
d.Mixolydian or a.Hypolydian	*No true reciprocals*	*No true reciprocals*
d.Lydian or a.Dorian		
d.Phrygian or a.Phrygian		
d.Dorian or a.Lydian		
d.Hypolydian or a.Mixolydian		
d.Hypophrygian or a.Hypodorian		
d.Hypodorian or a.Hypophrygian		

9 The *Timaeus* and Egyptian π

Not altogether unfittingly in light of Plato's project of constructing circles at *Timaeus* 36 C, the set of numbers comprising the *Timaeus* set includes the ratio for the Egyptian approximation to π, namely, 256/81.[35]

35 See Herbert Westren Turnbull, "The Great Mathematicians" in *The World of Mathematics: A Small Library of the Literature of Mathematics from A'h-Mosé the Scribe to Albert Einstein*, vol. 1, with commentaries and notes by James R. Newman and foreword by Philip and Phylis Morrison (New York: Simon and Schuster, 1956; repr., Redmond, Washington: Tempus Books, 1988), 87. Turnbull writes:

> "Recent investigations of the Rhind Papyrus, the Moscow Papyrus of the Twelfth Egyptian Dynasty, and the Strassburg Cuneiform texts have greatly added to the prestige of Egyptian and Babylonian mathematics. While no general proof has yet been found among these sources, many remarkable ad hoc formulae have come to light, such as the Babylonian solution of complicated quadratic equations dating from 2000 B.C., which O. Neugebauer published in 1929, and an Egyptian approximation to the area of a sphere (equivalent to reckoning π = 256/81)."

Selected Bibliography

Allen, Michael B. "The Ficinian *Timaeus* and Renaissance Science." In *Plato's* Timaeus *as Cultural Icon*, ed. Gretchen Reydams-Schils. Notre Dame: University of Notre Dame Press, 2003.

Allen, R.E. "Comment on Plato's Parmenides." In *Plato's Parmenides*. Translated with commentary by R.E. Allen. Revised edition. New Haven and London: Yale University Press, 1997.

Augustine. *On the Care to be had for the Dead*. In *Augustine: On the Holy Trinity, Doctrinal Treatises, Moral Treatises*. Nicene and Post-Nicene Fathers, First Series, ed. Philip Schaff, vol. 3. United States: Christian Literature Publishing Company, 1887; reprint, Peabody, MA: Hendrickson Publishers, Inc., 1994.

Baltzly, Dirk. "Introduction to Book 3, Part II." In *Book 3, Part II: Proclus on the Soul*. Vol. 4, *Proclus: Commentary on Plato's Timaeus*. Edited and translated with an introduction and notes by Dirk Baltzly. Cambridge: Cambridge University Press, 2009.

Barbera, André, ed. and trans. *The Euclidean Division of the Canon: Greek and Latin Sources*. Greek and Latin Music Theory, eds. Thomas J. Mathiesen and Jon Solomon. Lincoln: University of Nebraska Press, 1991.

Barker, Andrew. "Early *Timaeus* Commentaries and Hellenistic Musicology." In *Ancient Approaches to Plato's Timaeus*, eds. Robert W. Sharples and Anne Sheppard. Bulletin of the Institute of Classical Studies Supplement, ed. Geoffrey Waywell, no. 78. London: Institute of Classical Studies, School of Advanced Study of the University of London, 2003.

Barker, Andrew. *The Science of Harmonics in Classical Greece*. Cambridge: Cambridge University Press, 2007.

Boethius. *De institutione musica, libri quinque*. In *Anicii Manlii Torquati Sverini Boetii De institutione arithmetica, libri duo, De institutione musica libri quinque, accedit Geometria quae fertur Boeth*, ed. Godofredus Friedlein. Lipsiae: In Aedibus B.G. Teubneri, 1867.

Brisson, Luc and Meyerstein, Walter. *Inventing the Universe, Plato's Timaeus, The Big Bang, and the Problem of Scientific Knowledge*. Albany: State University of New York, 1995.

Brisson, Luc. *Le même et l'autre dans la structure ontologique du timée de Platon, un commentaire systématique du timée de Platon*. International Plato Studies, eds. Luc Brisson, Tomás Calvo, Livio Rossetti, Christopher J. Rowe, and Thomas A. Szlezák, no. 2. Sankt Augustine: Academia Verlag, 1994.

Brown, Peter. *Augustine of Hippo*. 2d ed. New York: Dorset Press, 1986.

Burkert, Walter. *Lore and Science in Ancient Pythagoreanism*. Translated by Edwin L. Minar, Jr. Cambridge: Harvard University Press, 1972.

Calcidius. *On Plato's Timaeus*. Translated by John Magee. London: Harvard University Press, 2016.

Calcidius. *Platonis Timaeus*. Interprete Chalcidio cum eiusdem commentario. Rescensuit Dr. Ioh. Wroel. Lipsiae: In Aedibus B.G. Teubneri, 1876.

Cornford, Francis MacDonald. *Plato's Cosmology: The Timaeus of Plato translated with a Running Commentary*. London: Routledge and Kegan-Paul, 1951.

Cornford, Francis MacDonald. *Plato's Theory of Knowledge, The Theaetetus and the Sophist of Plato*. Translated with a running commentary. London: Routledge and Kegan Paul, Ltd., 1935. Reprint, London: Routledge and Kegan Paul, Ltd., 1949.

Courcelle, Pierre. "La postérité chrétienne du songe de Scipion." *Revue des Études Latine* 36 (1958): 205–234.

Creese, Donald. *The Monochord in Ancient Greek Harmonic Science*. Cambridge: Cambridge University Press, 2010.

Critchlow, Keith. "Foreword." In *The Theology of Arithmetic, On the Mystical, Mathematical and Cosmological Symbolism of the Frist Ten Numbers Attributed to Iamblichus*. Translated by Robert Waterfield. Grand Rapids: Phanes Press, 1988.

Dickerson, Richard E. and Geis, Irving. *The Structure and Action of Proteins*. Menlo Park, CA: W.A. Benjamin, Inc., 1969.

Dillon, John. "Iamblichus' Commentary on the *Timaeus* of Plato: A Collection of the Fragments, with an Attempt at Reconstruction." Ph.D. diss., University of California, Berkeley, 1969.

Dillon, John, "The *Timaeus* in the Old Academy." In *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils. Notre Dame: University of Notre Dame Press 2003.

D'ooge, Martin Luther. "Studies in Greek Mathematics." In *Nicomachus of Gerasa*. Trans. Martin Luther D'ooge. University of Michigan Studies, Humanistic Series, vol. 16. Ann Arbor: University of Michigan Press, 1938.

Dupuis, J. "Note XII, On the Perfect Musical System Formed of Two Octaves." Note to Theon of Smyrna, *Mathematics Useful for Understanding Plato*, ed. Christos Toulis, et al. with an appendix of notes by Dupuis. Translated from the 1892 Greek/French edition of J. Dupuis by Robert and Deborah Lawler. Secret Doctrine Reference Series. San Diego, CA: Wizard's Bookshelf, 1979.

Eulogius, Favonius. *Disputatio de somnio Scipionis*. Edition et traduction de Roger E. Van Weddingen. Collection Latomus, vol. 27. Bruxelles: Latomus revue d' études latines, 1957.

Farmer, Henry George. "The Music of Mesopotamia." In *Ancient and Oriental Music*, ed. Egon Wellesz. *The New Oxford History of Music*, vol. 1. London: Oxford University Press, 1957; reprint ed., London: Oxford University Press, 1969.

Gerson, Lloyd P. *Plotinus*. London and New York: Routledge, 1994.

Gobry, Ivan. *Pythagore*. Collection les grande leçons de philosophie, dirigée par Henri Hude. Paris: Editions universitaires et editions, 1992.

Haar, James. "Musica mundana: Variations on a Pythagorean Theme." Ph.D. diss., Harvard University, 1960.

Hagel, Stephen. *Ancient Greek Music, A New Technical History*. Cambridge: Cambridge University Press, 2010.

Handschin, Jacques. "The Timaeus Scale." *Musica Disciplina* 4 (1950): 3–41.

Hawking, Stephen. *A Brief History of Time From the Big Bang to Black Holes*. With an Introduction by Carl Sagan. Toronto, New York, Sydney, Auckland: Bantam Books, 1988.

Henderson, Isobel. "Ancient Greek Music." In *Ancient and Oriental Music*, ed. Egon Wellesz. Vol. 1, *The New Oxford History of Music*. London: Oxford University Press, 1957; reprint, London: Oxford University Press, 1969.

Huffman, Carl A. *Philolaus of Croton, Pythagorean and Presocratic, a Commentary on the fragments and Testimonia with Intepretative Essays*. Cambridge: Cambridge University Press, 1993.

James, Jamie. *The Music of the Spheres: Music, Science and the Natural Order of the Universe*. New York: Copernicus/Springer-Verlag, 1995.

Kramer, Edna. *The Nature and Growth of Modern Mathematics*. United States: Hawthorn Books, 1970; reprint ed., Princeton: Princeton University Press, 1982.

Lasserre, François. "L'Education musicale dans la Grèce antique." Commentaire in Plutarque, *De la musique*. Olten and Lausanne: Urs Graf-Verlag, 1954.

Lawler, Robert. *Sacred Geometry*. New York: Crossroad; London: Thames and Hudson, 1982.

Levin, Flora. *The Manual of Harmonics of Nicomachus the Pythagorean*. Translated with annotations. Grand Rapids: Phanes Press, 1994.

Lippman, Edward A. *Musical Thought in Ancient Greece*. New York and London: Columbia University Press, 1964.

Mahon, Bruce. *University Chemistry*. 3rd edition. Reading, MA: Addison-Wesley Publishing Co., 1975.

Mathiesen, Thomas J. *Apollo's Lyre, Greek Music and Music Theory in Antiquity and the Middle Ages*. Publications of the Center for the History of Music Theory and Literature, vol. 2. Lincoln, Nebraska and London, England: University of Nebraska Press, 1999.

Mathiesen, Thomas J. "Greek Music Theory." In *The Cambridge History of Western Music Theory*, ed. Thomas Christensen. Cambridge: Cambridge University Press, 2002; reprint ed., Cambridge: Cambridge University Press, 2004.

McClain, Ernest. *The Pythagorean Plato, Prelude to Song Itself*. York Beach, Maine: Nicolas Hys, Inc., 1978.

Merlan, Philip. *From Platonism to Neoplatonism*. The Hague: Martinus Nijhoff, 1953.

Meyer, Christian. "L'Âme du monde dans la rationalité musicale: ou l'expérience sensible d'un order intelligible." In *Harmonia mundi, musica mondana e musica celeste fra*

antichità e medioeo: *Atti del convegno internazionale di studi, Roma, 14–15 dicembre 2005*, a cura di Marta Cristiani, Cecilia Panti, e Graziano Perillo. *Micrologus' Library* 19. Firenze: Sismel—Edizione del Galluzzo, 2007.

Miller, Mitchell. "The *Timaeus* and the 'Longer Way': God-Given Method and the Constitution of Elements and Animals." In *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils. Notre Dame: University of Notre Dame Press, 2003.

Moutsopoulos, Evanghélos. *La musique dans l'oeuvre de Platon*. Paris: Presses Universitaires de France, 1959.

Nicomachus of Gerasa. *Introduction to Arithmetic*. Translated by Martin Luther D'ooge. In *Nicomachus of Gerasa*. University of Michigan Studies Humanistic Series, vol. 16. Ann Arbor: University of Michigan Press, 1938.

Nicomachus of Gerasa. *Manual of Harmonics*. In "Nicomachus of Gerasa, Manual of Harmonics: Translation and Commentary," by Flora Rose Levin. Ph.D. diss., Columbia University, 1967.

Olsen, Scott. *The Golden Section, Nature's Greatest Secret*. New York: Walker & Company, 2006.

O'Meara, Dominic. *Pythagoras Revisited: Mathematics and Philosophy in Late Antiquity*. Oxford: Clarendon Press, 1989.

Plato. *Laws*. Translated by Trevor J. Saunders, In *Plato, Complete Works*, ed. John M. Cooper and D.S. Hutchinson. Indianapolis and Cambridge: Hackett Publishing Co., 1997.

Plato. "Letters VII, IX, and XII to Archytas the Tarentine." In *The Platonic Epistles*. Translated with an introduction and notes by J. Harward. New York: Arno Press, 1976.

Plato. *Parmenides*. Translated with a commentary by R.E. Allen. Revised Edition. New Haven and London: Yale University, 1997.

Plato. *Philebus*. In *Plato, Complete Works*. Edited by John M. Cooper and D.S. Hutchinson. Indianapolis: Hackett Publishing Company, 1997.

Plato. *Timaeus*. In *The works of Plato*, vol. 2. Translated by Thomas Taylor and Floyer Sydenham. The Thomas Taylor Series, vol. 10. England: Antony Rowe, Chippenham, Wiltshire, 1804. New, revised edition, Dorset: The Prometheus Trust, 1996. Reprint, Dorset: The Prometheus Trust, 2007.

Plato. *Timaeus*. Translated by Benjamin Jowett. In *The Collected Dialogues of Plato, Including the Letters*, eds. Edith Hamilton and Huntington Cairns. Bollingen Series, vol. 71. Princeton: Princeton University Press, 1963. Reprint, Princeton: Princeton University Press, 2009.

Plato. *Timaeus*. Translated by Donald J. Zeyl. In *Plato, Complete Works*, ed. John M. Cooper and D.S. Hutchinson. Indianapolis and Cambridge: Hackett Publishing Co., 1997.

Plato. *Timeo*. Testo greco a fonte. Direttore Giovanni Reale. Milano: Rusconi Libri, 1994.

Plutarch. *On the Generation of the Soul in the Timaeus*. With an English translation by Harold Cherniss. In *Plutarch: Moralia, vol. III, Part I*. Loeb Classical Library, ed. Jeffrey Henderson, no. 427. London, England and Cambridge, MA: Harvard University Press, 1976; reprint ed., London, England and Cambridge, MA: Harvard University Press, 2000.

Plutarch. *Plutarchi chaeroni de procreatione animi in Timaeo Platonis*. Adriani Turnebo Interprete (translator). In opera nunc primum ex bibliotheca amplissimi viri, Stephani Adriani F. Turnebii Senatoris Regy, unum collecta, emendata, aucta, & tributa, tomos 3. Argentorati: Sumptibus Lazari Zetzneri Bibliopolae, 1600.

Proclus. *Book 3, Part II: Proclus on the World Soul*. Vol. 4, *Proclus: Commentary on Plato's Timaeus*. Translated with an introduction and notes by Dirk Baltzly. Cambridge: Cambridge University Press, 2009.

Proclus. *The Commentary of Proclus on the Timaeus*. Translated from the Greek by Thomas Taylor. 2 vols. London: A.J. Valpy, 1820.

Pseudo-Plutarque. *De la musique*, Texte traduction commentaire précedé d'une étude sur l'éducation musicale dans la Grèce antique par François Lasserre. Biblioteca Helvetica Romana. Olten & Lausanne: Urs Graf-Verlag, 1954.

Ptolemaeus, Claudius. *Harmonicorum libri tres*. A facsimile of the Oxford, 1682 edition. Monuments of Music and Music Literature in facsimile. Second Series. Music Literature, no. 60. New York: Broude Brothers Limited, 1977.

Reese, Gustav. *Music in the Middle Ages*. With an introduction on music of ancient times. New York: W.W. Norton & Co., 1940.

Sallis, John. *Chorology: On Beginning in Plato's Timaeus*. Bloomington and Indianapolis: Indiana University Press, 1991.

Sayre, Kenneth. "The Multilayered Incoherence of *Timaeus*' Receptacle." In *Plato's Timaeus as Cultural Icon*, ed. Gretchen J. Reydams-Schils. Notre Dame: University of Notre Dame Press 2003.

Sayre, Kenneth. *Plato's Late Ontology*. Princeton: Princeton University Press, 1983.

Soulignac, Aimé. "Doxographies et manuels chez S. Augustin." *Recherches Augustiniennes* I—Supplement à La Revue des Etudes Augustiniennes (1958).

Tarán, Leo. *Speusippus of Athens, A Critical Study with a Collection of the Related Texts and Commentary*. Leiden: Brill, 1981.

Taylor, A.E. *A Commentary on Plato's Timaeus*. Oxford: Clarendon Press, 1928.

Taylor, Thomas. "Introduction to the Timaeus." In *The Works of Plato*, vol. 2. Translated by Thomas Taylor and Floyer Sydenham. The Thomas Taylor Series, vol. 10. England: Antony Row, Chippenham, Wiltshire, 1804. New, Revised Edition, Dorset: The Prometheus Trust, 1996; reprint ed., Dorset: The Prometheus Trust, 2007.

Theon of Smyrna. *Mathematics Useful for Understanding Plato*. Translated from the French edition by J. Dupuis by Robert and Deborah Lawler. Edited and annotated by Christos Toulis and others. With an appendix of notes by Dupuis. Secret Doctrine Reference Series. San Diego: Wizard's Bookshelf, 1979.

Treitler, Leo, gen. ed. *Source Readings in Music History*. Revised edition of compilation previously edited by Otto Strunk. New York and London: W.W. Norten and Company, 1998.

Turnbull, Herbert Westren. "The Great Mathematicians." In *The World of Mathematics, A Small Library of the Literature of Mathematics from A'h-Mosé the Scribe to Albert Einstein.* Vol. 1. Presented with commentaries and notes by James R. Newman. With a foreword by Philip and Phylis Morrison. New York: Simon and Schuster, 1956. Reprint, Redmond, Washington: Tempus Books, 1988.

Van Der Waerden, B.L. *Science Awakening*. Translated by Arnold Dresden with additions of the author. Groningen, Holland: P. Noordhoff, Ltd., 1954.

Van Weddingen, Roger E. "Introduction." In *Disputatio de somnium Scipionis Favonii Eulogii*. Edition et traduction de Roger E. Van Weddingen. Collection Latomus. Vol. 27. Bruxelles: Latomus revue d'études latines, 1957.

Vogel, C.J. *Pythagoras and Early Pythagoreanism, An Interpretation of Neglected Evidence on the Philosopher Pythagoras*. Assen: Van Gorcum and Company, N.V., 1966.

Wear, Sarah Klitenic, trans. *The Teachings of Syrianus on Plato's Timaeus and Parmenides*. Studies in Platonism, Neoplatonism, and the Platonic Tradition eds. Robert Berchman and John Finamore, vol. 10. Ancient Mediterranean and Medieval Texts and Contexts. Leiden: Brill, 2011.

West, M.L. *Ancient Greek Music*. Oxford: Clarendon Press, 1992.

Subject Matter Index

Note 1: The subject matter index finds both specific terms and related concepts. For example, for the term "composition," the index finds both "composition" and cognate material relating to the construction of things. There are many synonyms in the index constituting independent sources for recovering the same material.

Note 2: Page citations followed by "t," "f," "q," "n," and "Ap" indicate, respectively, that the reference is to a table, figure, quotation, note, or an appendix to the study, unless such item is itself the subject of the indexing. If one of these designations follows a range of pages, then the designation is relevant to all pages. Tables, figures, notes, and appendices are further designated by number for the reader's convenience, e.g., "t28," "Ap7," etc. Occasionally, to make locator designations more compact, there are combined references, such as 64&n33, rather than 64, 64n33, indicating that relevant material may be found both on a page of text and in a note to the page; 48&q, rather than 48, 48q, indicating that relevant material may be found both on a page of author text and in the quotation of a source on the same page; 75&f6, rather than 75, 75f6, or 109&t24, rather than 109, 109t24, indicating that relevant material occurs both in the text and in a table or figure on that page.

Note 3: References to "the study," "this study," or "study" indicate this work.

Note 4: "UPS," "GPS," and "LPS" mean, respectively, "Unmodulating Perfect System (also Unmodulating System)," "Greater Perfect System," and "Lesser Perfect System."

Note 5: References to LFT indicate the "List of Figures and Tables." Figures and Tables are generically indexed to the LFT, except for those that receive significant repeated reference throughout the text. The latter are indexed to specific pages as subheadings, under the generic main headings, respectively, for "Figures" and "Tables."

Note 6: References to GL indicate the "Glossary of Musical Concepts and Terms."

Note 7: Significant numbers are indexed only to define their special features.

Note 8: Indexed numbers are emboldened whenever distinguishing between the indexed item and a page number reference poses a difficulty. On occasion, page numbers are also emboldened for special emphasis.

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